QUANTIFYING DIFFRACTION IN TIME DOMAIN WITH FINITE ELEMENT METHOD

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Finite element method applied in frequency domain can show diffraction effects, but separation of the excitation from the diffracted wave fronts is difficult as the excitation dominates the pressure field. Time domain simulation overcomes these problems by enabling separation of the pressure fields in time. Time domain separation of the excitation and diffracted wave fronts enable not only time domain analysis but also specific frequency domain analysis of the diffracted sound. This paper demonstrates how time domain finite element analysis can be used to simulate diffraction in loudspeakers. The method is demonstrated in three geometries where the models are excited with one cycle of Gaussian windowed 5 kHz sine wave. The geometries demonstrate how rounded edges and a waveguide in front of the source can reduce diffraction.

1 INTRODUCTION

Historically the engineering problem of managing and minimizing diffraction effects has been tackled with physical prototypes, empirical rules of thumb, and various simulation methods. Already in 1957 Olson [1] presented an empirical study of frequency responses for ten geometries, showing that rounded and slanted enclosures have less diffraction than rectangular or cylindrical enclosures. In computational modelling, ray tracing traditionally used in room acoustics [2] has been enhanced also for modelling diffraction effects but the results do not always agree well with measurements [3]. Also image-source methods have been used by several authors [4][5] but the image-source method is only valid for high frequencies where the wavelength is short compared to dimensions of the geometry [4]. Today the finite element method (FEM) in the frequency domain is successfully used to predict acoustic pressure and diffraction [6][7].

Frequency domain FEM analysis result can present the effects of diffraction. Diffracted energy sums with the original sound source causing frequency specific level deviations in the pressure response. Separation of the excitation from the diffracted energy is difficult as the excitation typically dominates the pressure field.

This paper describes how time-domain FEM can be used to simulate diffraction. In time domain the excitation and diffracted wave front can be separated using time windowing. FEM allows visualization of the sound field progression in the simulated air space to improve understanding of the reasons and sources for diffraction in complex geometries. This enables optimizing the geometrical shapes before prototyping.

2 DIFFRACTION MODELIGN METHODS

A finite element method for pressure acoustics physics [8] is used for solving the problem. The precision of the time domain solution is limited by the mesh modeling. In this work, triangular mesh elements with maximum element size of 11 mm are used. Two-dimensional axisymmetric simulation is used for lowering the computational cost of calculation. Post-processing allows geometries to be presented in three dimensions.

The simulated length of time is 2 ms. The simulation time is long enough to enable the calculation of the propagation of the exciting wave front, and the first and second order diffractions. The time step used in calculations is 0.8 μs. A short time step is needed for simulation convergence and to achieve the required precision of the solved pressure values.

The time domain excitation is a bidirectional pressure pulse with one cycle of a 5 kHz sine wave, windowed with a Gaussian time window (Figure 1). The time window is essential for the convergence of calculations. Gaussian filtering broadens the frequency content of the excitation.

FEA modeling calculates propagation of the pressure pulse in the air around the enclosure. Because the bidirectional pulse has short duration, it is possible to isolate the diffraction pressure in the time domain for typical loudspeaker enclosure sizes, and to visualize and quantify the diffraction-related effects.

Diffraction is visualized using three types of post-processing plots. The first is the pressure as function of time on the acoustical axis of the source. The second is the pressure magnitude within the simulated air space (simulation domain) at a given time. The third is frequency domain analysis of the original impulse and the diffracted pressure waves. It is also possible to animate the pressure as a function of time in the simulation domain.

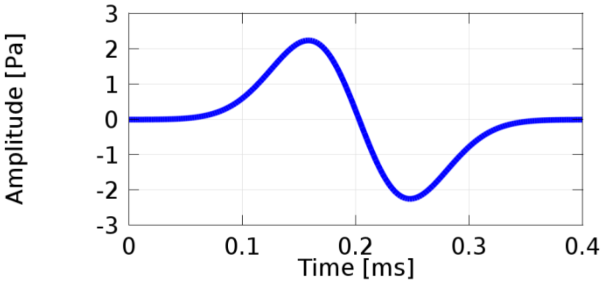

Figure 1. The stimulus is one cycle of Gaussian windowed 5 kHz sine wave

Table 1. Main dimensions of the three enclosures

Case	Face diameter [cm]	Depth [cm]	Rounding radius [cm]	Waveguide depth [cm]
A	26	24	-	-
B	25	24	3	-
C	25	24	3	1.3

The time domain method is demonstrated in three geometrically complex enclosure shapes. First geometry (Case A) is a cylinder with the exciting source at the centre of the circular plane (Figure 4). Second (Case B) is the cylinder with rounded edges (Figure 4). The third geometry (Case C) is the cylinder with rounded edges and the source located at the throat of a waveguide (Figure 4).

3 RESULTS

3.1 Case A: Cylindrical enclosure

Initial wave front is a hemispherical because of the flat plane on the cylinder (Case A, 0.2 ms and 0.6 ms, Figure 6). The cylindrical enclosure shows strong diffraction at the edges of the cylinder. Distance from the sound source to the edges is equal. Therefore diffracted wave fronts sum constructively on the acoustical axis of the source (Case A, 1.1 ms, Figure 6).

The diffracted wave fronts cause second and higher order diffractions from the cylinder edges later (Case A, 1.5 ms, Figure 6). Polarity of the first order diffraction effect is inverted as theory suggests, and scaled in level (Case A, Figure 5). Envelope of the diffracted wave front is similar to the initial impulse suggesting that the spectral content of the diffracted wave front remains similar to the excitation spectrum.

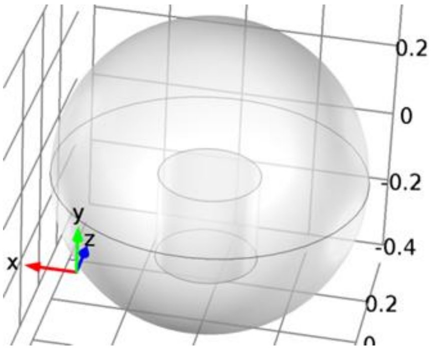

Figure 4. Cylindrical enclosure and spherical air space around it (Case A).

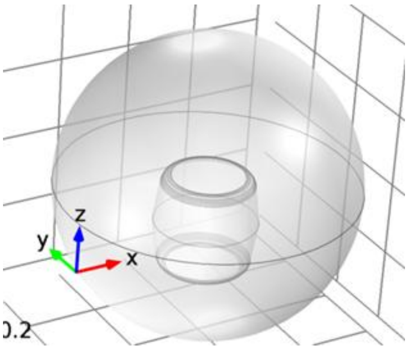

Figure 4. Cylindrical enclosure with rounded edges (Case B).

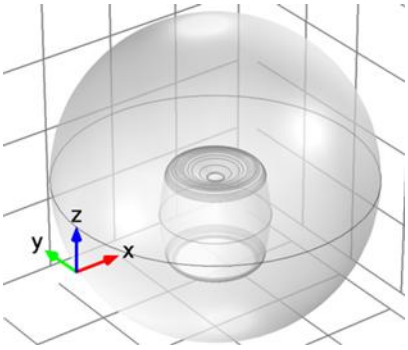

Figure 4. Cylindrical enclosure with rounded edges and source at throat of the waveguide (Case C)

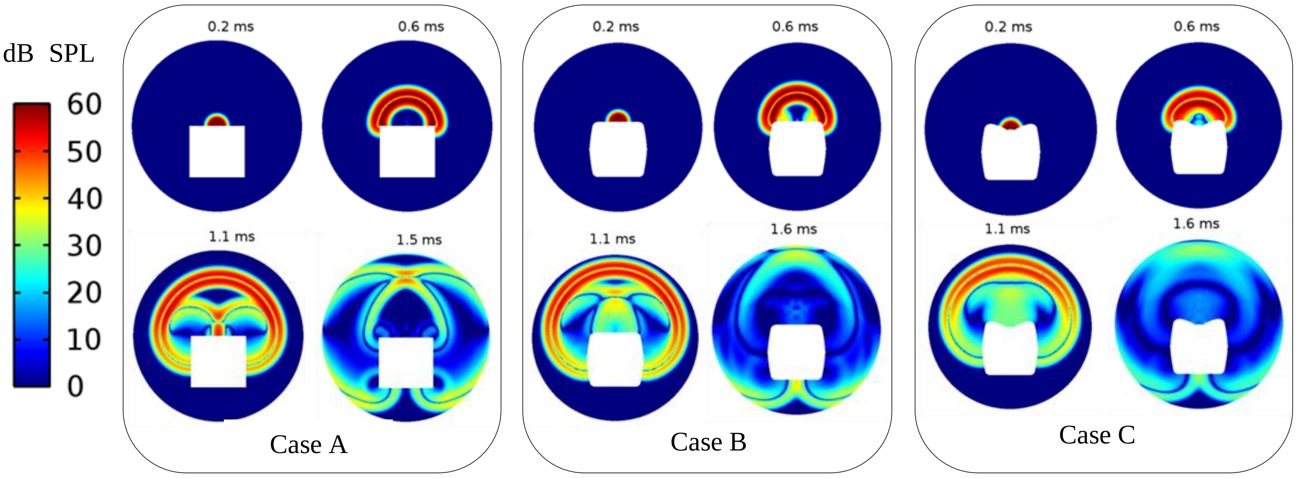

Figure 6. Pressure around enclosure. Three geometries.

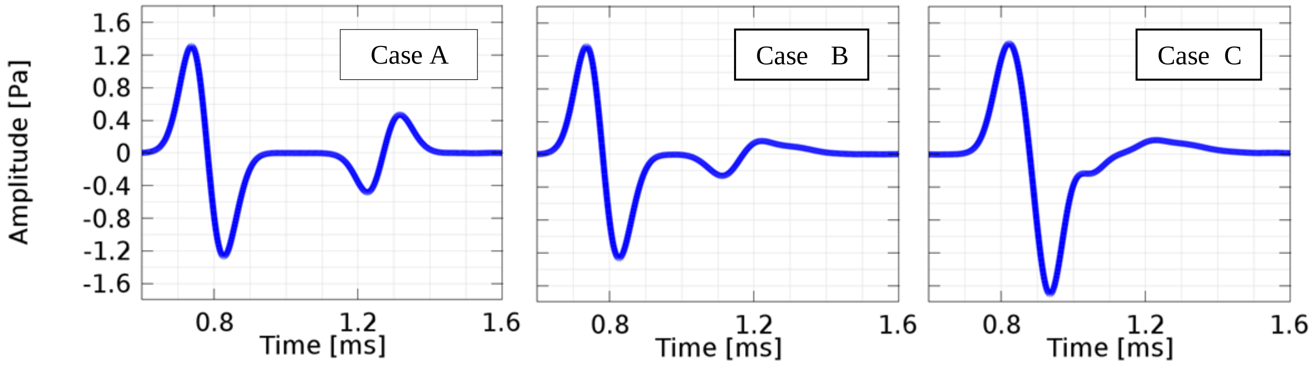

Figure 5. Pressure at 0.2 m on-axis of source.

3.2 Case B: Enclosure with rounded edges

Diffracted wave fronts sum on the acoustical axis but are lower in amplitude (Case B, 1.1 ms, Figure 6). The second order diffractions can still be seen (Case B, 1.6 ms, Figure 6). Diffracted wave amplitude reduces more than with Case A (Case B, Figure 5). Time domain shape of the diffracted pressure wave no longer corresponds with the pulse. This suggests larger changes in the spectrum of the diffracted pressure wave.

3.3 Case C: Rounded edges and waveguide

Addition of a waveguide affects the initial wave front shape. The wave front is no longer hemispherical (Case C, 0.6 ms, Figure 6). The amplitude of the wave front is lower at the sides (Case C, 1.1 ms, Figure 6). This suggests the waveguide increases directivity of radiation. Diffraction wave fronts can be seen but are low in amplitude and spread out in time. Higher order diffractions cannot be seen with the selected amplitude scale (Case C, 1.1 ms and 1.6ms, Figure 6).

Rounded corners and the waveguide change the envelope and magnitude of the diffracted waves. Diffractions are not visually distinguishable from the excitation anymore (Case C, Figure 5). This suggests altered spectrum of the diffracted wave.

3.4 Frequency domain results

The time domain waveforms can be analyzed for spectral content. A simple approach is to use time windowing to separate the diffractions from the excitation. The window function affects the frequency response in a known way, and time-frequency resolution trade-off has to be considered. For frequency domain analyses (Figure 7 and Figure 8) the diffracted wave has been separated using time windowing. High frequency content in the diffraction energy is reduced by rounded corners, as expected.

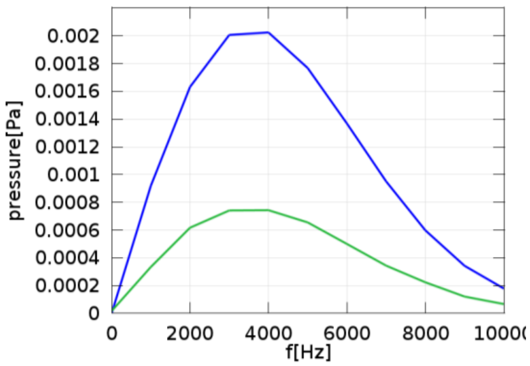

Figure 7. Case A. Frequency content in the excitation (blue) and in the first order diffraction (green).

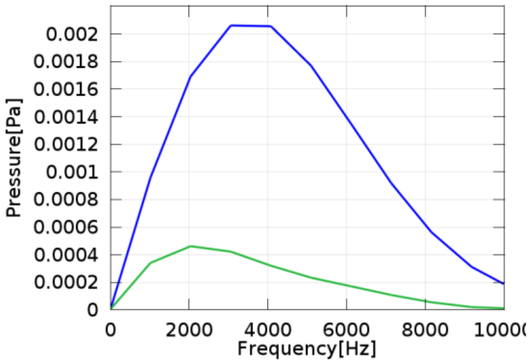

Figure 8. Case B. Frequency content in the excitation (blue) and in the first order diffraction (green).

4 CONCLUSIONS AND DISCUSSION

This paper demonstrates how FEM analysis in the time domain can be used in studying diffraction. Time domain modeling allows diffraction effects to be separated from the excitation.

Results agree with theory of diffraction. Inversion of polarity in the diffracted wave front is demonstrated [3]. Rounded corners of the enclosure can reduce the amplitude of the diffraction. Using a waveguide can increase the source directivity and further reduce diffraction. Rounded corners and a waveguide can alter the diffracted spectrum.

Diffraction products can be separated from the excitation waveform also by time windowing or deconvolution because the time-domain excitation waveform is precisely known in advance. Deconvolution can enable separation of diffraction products when the excitation overlaps with diffraction results in the time domain.

In applying finite element analysis there is a tradeoff between the geometry size and the highest simulated frequency. Size of the geometry limits the length of time domain system response that can be simulated. This then limits the order of diffraction that can be studied. Computational costs limit the size of the solvable problem. Three-dimensional models multiply the computational cost. Further limitation is posed by the sound source model. In this work, a 20-mm dome diaphragm was modeled by a point source. This is adequate as long as the source can be considered acoustically small compared to the wavelength. Modeling the source with a rigid piston may cause problems with model convergence. More realistic models of the audio source are possible with further increases in computational cost.

The demonstrated method can be used in simulating also other acoustical problems requiring time domain separation of the reflections from the excitation, such as acoustical reflection.

Finally, time domain FEM method presented in this paper should be compared with finite-difference time-domain modelling (FDTD) which has also been used in finding solutions for acoustical wave propagation problems in time domain, for example by several authors in simulating the seat-dip effect [9][10].

5 REFERENCES

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