# QUANTUM SIMULATION AND COMPUTING 

## A NEW WAY OF COMPUTING BEYOND SUPERCOMPUTERS

JENS EISERT, FU BERLIN HARDWARE HACKING, BIG
, Gordon Moore (Intel, 1965): Number of transistors in integrated circuits doubles approximately every two years


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ENIAC, EDVAC, ORDVAC, BRLESC-I (1945-62)


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Different physical laws matter


# QUANTUM MECHANICS 

, Quantum mechanics is a physical theory


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, Theory of atoms, molecules, and light quanta

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, Basis of semi-conductors, materials science, lasers

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- Fine structure constant: 7,297.352.566.4(17) $\times 10^{-3}$
, Radically different from classical mechanics


## RANDOMNESS

Measurement outcomes are random


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- The randomness of quantum mechanics is absolute

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- Bell inequality violated under assumption of local hidden variables

$$
P(a, b \mid A, B)=\int d \lambda p(\lambda) \chi_{A}(a, \lambda) \chi_{B}(b, \lambda)
$$

## UNCERTANTY

## UNCERTAINTY PRINCIPLE


, No measurement without disturbance


# SUPERPOSITION 

## SUPERPOSITION PRINCIPLE




, Systems can be in "many states at once"

## SUPERPOSITION PRINCIPLE

$|1\rangle$

- State space $\{\rho: \rho \geq 0, \operatorname{tr}(\rho)=1\}$ over complex vector space $\mathcal{H}$ For $n$ spins $\mathcal{H}=\mathbb{C}_{2}^{\otimes n}$


## QUANTUM

## TECHNOLOGIES

- Make use of quantum effects on the single quantum system level to think of new technologies in communication, sensing, computation, simulation
- Classical key distribution


010101
010101

- Classical key distribution


Classical key distribution

, Quantum key distribution for secure communication


010101
010101

## - Quantum key distribution for secure communication



No information gain without disturbance

## , Quantum key distribution for secure communication

Alice's bit<br>Alice's basis State<br>Bob's basis<br>Bob's result<br>Public part Key



- Basis +

$$
\uparrow=|1\rangle, \rightarrow=|0\rangle
$$

-Basis $\times$

$$
\rangle=|0\rangle+|1\rangle, \quad \nearrow=|0\rangle-|1\rangle
$$

## Quantum key distribution for secure communication

## Security can be proven

## Bug-proof communication: Quantum communication

Quantum communication uses quantum-cryptographically protected
 communication channels for the bug-proof transfer of information. Quantummechanically connected pairs of photons transport confidential information securely and reliably. Today, this method only allows information to be transported via glass fibres over a maximum of approximately 100 kilometres due to the absorption of the light used to convey the data. In order to achieve greater distances, the BMBF is funding research into quantum repeaters which use entanglement swapping to stationary quantum states over a distance of more than 100 kilometres.

## QUANTUM COMPUTERS

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E.g., 01010011 (bits) replaced by (qubits) $\alpha|0,1,0,1,0,0,1,1\rangle+\beta|1,1,0,0,1,1,1,0\rangle+\gamma|0,0,1,0,0,1,1,1\rangle+\ldots$

- Could solve some problems supercomputers cannot

- E.g., factoring of large products of prime numbers
- A factor of a large number $N$ can be found if the period $p$ of

$$
f(x)=a^{x} \bmod N
$$

can be identified

- Periods can be found using the quantum Fourier transform

$$
\sum_{i=0}^{n-1} x_{i}|i\rangle \mapsto \sum_{i=0}^{n-1} y_{i}|i\rangle \text { with } y_{k}=\frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} x_{j} e^{2 \pi i j k / n}
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- Solves NP problem in poly time: Runtime $O\left((\log N)^{3}\right)$
- Best known classical algorithm $\exp \left(O\left((\log N)^{1 / 3}(\log \log N)^{2 / 3}\right)\right)$
- Generalised to hidden subgroup problem
E.g., factoring of large products of prime numbers Shor, SIAM J Comp 26, 148 (1997)
Solving linear systems
Harrow, Hassidim, Lloyd, Phys Rev Lett 15, 150502 (2009)
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Solving linear systems
Harrow, Hassidim, Lloyd, Phys Rev Lett 15, 150502 (2009)
- Spectral analysis

Steffens, Rebenstrost, Marvian, Eisert, Lloyd, New J Phys 19, 033005 (2017)
, Semi-definite programming
Brandão, Kalev, Li, Lin, Svore, Wu, arXiv:1710.02581

- Can tolerate small errors in all steps (at high cost)

E.g., Litinski, Kesselring, Eisert, von Oppen, arXiv:1704.01589
- The race for building quantum computers
- Not there, but with 50 superconducting qubits taking shape

(Google)
(Rigetti)
(D-wave)


## QUANTUM SIMULATORS

Quantum simulators: Not all strongly correlated quantum systems/materials can be classically simulated

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, Idea: Simulate quantum systems with quantum systems


Richard Feynman

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- Idea: Simulate quantum systems with quantum systems


Cold atoms in optical lattices

## , Simulate interesting physical situations

Equilibration


Trotzky, Chen, Flesch, McCulloch, Schollwöck, Eisert, Bloch, Nature Physics 8, 325 (2012)

Pre-thermalization


Gring, Kuhnert, Langen, Kitagawa,
Rauer, Schreitl, Mazets, Smith, Demler Schmiedmayer, Science 337, 1318 (2012)
, Tai, Lukin, Rispoli Schittko, Preiss, Greiner, Science 353, 794 (2016)

Many-body localization


Choi, Hild, Zeiher, Schauß,Rubio-Abadal, Yefsah,
Khemani, Huse, Gross, Science 352, 1547 (2016)

- Some properties can be obtained beyond supercomputers
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- Imbalance as function of time for $|\psi(0)\rangle=|0,1, \ldots, 0,1\rangle$ under Bose-Hubbard Hamiltonian

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Best available classical matrix-product state simulation, bond dimension 5000

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- Imbalance as function of time for $|\psi(0)\rangle=|0,1, \ldots, 0,1\rangle$ under Bose-Hubbard Hamiltonian

- The approximation of dynamics with matrix-product states requires exponential resources in time

0


Best available classical matrix-product state simulation, bond dimension 5000

- Some properties can be obtained beyond supercomputers
- Simple Ising nearest-neighbor architectures


Random


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Relate to logical circuits


$U$ Additive error $\epsilon$

, Some properties can be obtained beyond supercomputers

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Random

- Present technology (basic) quantum simulators already outperform supercomputers on some tasks (and can be verified)


Relate to logical circuits


GETTING GOING. . .

## - 1G€ Euros-Flagship for quantum technologies

## Gartner Hype Cycle for Emerging Technologies, 2016


"Quantum computing is exciting even if you restrict
yourself to saying things that are true."
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## THANKS FOR YOUR ATIENTION

