Sampling Chess

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What Is This Talk About?

- complexity of chess = legendary
  - too big to fully explore by computer
  - still, humans can somehow navigate through chess games
- complexity of Go = even larger
  - *is this really the main difference?*
- *is size all that matters?*
- can we *explore the structure of the state space of chess?*
  can we make a map of chess games?
- sample huge state space = well-known topic in *statistical physics*

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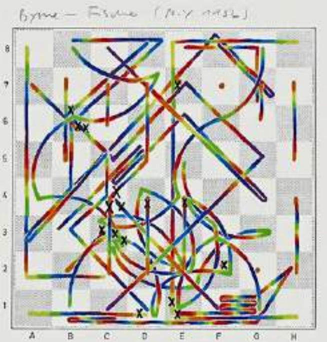

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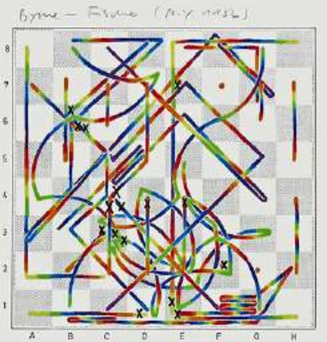

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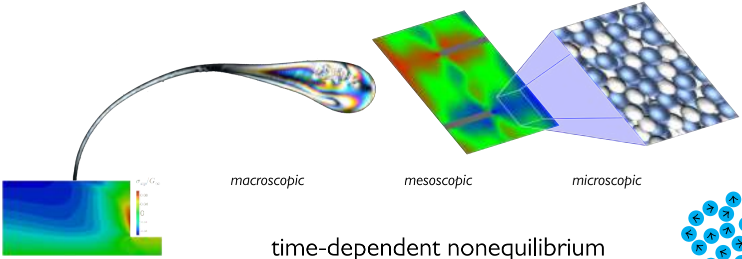
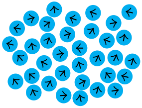
use statistical physics tools to explore chess

Disclaimer: What We Actually Do (Most of the Time)



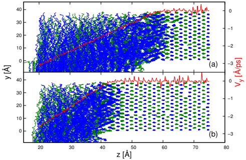
rheology of glasses

time-dependent nonequilibrium properties of materials

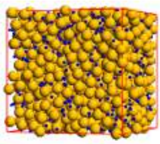


active particles/materials

metallic melts



nonequilibrium phase transitions



anomalous transport, porous media

Why Chess, Then?

- *new perspective* in trying to understand the game
- *non-trivial test case* for computer-physics tools
- *teach principles* of physics of complex systems
  - computer methods (Monte Carlo, biased sampling)
  - stochastic processes, abstract dynamical rules
- the real reason: motivate a good, but bored student (ELO 2200)
  - this, too, worked.

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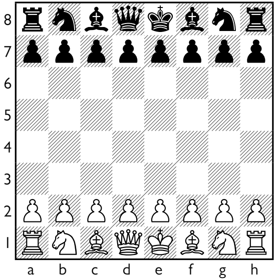
Chess: Quick Reminder

- 8×8 board, two players (black/white)
- 16 pieces each: ♖♖♘♘♗♗♕♔♙♙♙♙♙♙♙♙
- players move in turns, 1pc per turn

- each piece: specific move rule
  - pieces cannot pass through each other (exception: knights)
  - pieces can capture others (king must escape)
  - some special moves: pawns promote, castling, pawns can initially move 2 squares (subject to en-passant capture)
  - pawns only move forward



- goal: mate opponent

  attack king (“place in check”) such that it cannot escape

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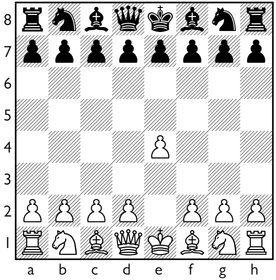

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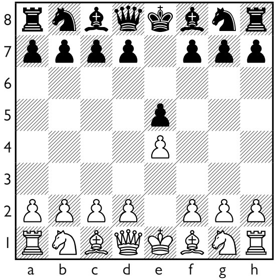

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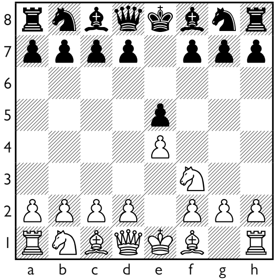

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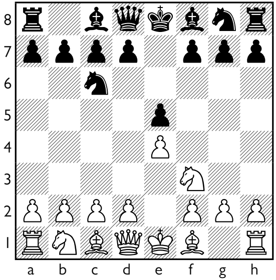

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Some History of Computers in Chess

- 1950 C. E. Shannon: *Programming a Computer for Playing Chess*
- 1951 D. Prinz: program *Matt in Zwei Zügen* (Mark I)
- 1958 A. Bernstein: first full chess program (IBM 704)
- 1970 first computer-chess tournament @ ACM
- 1989 Deep Thought challenges Kasparov
- 1997 Deep Blue defeats Kasparov
- 2008 initial Stockfish release

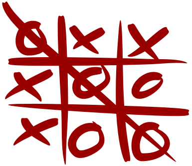
brute force + heuristics

Game Theory: “Solved”

- chess: finite two-player game of perfect information, alternating moves, no element of chance
- **Zermelo’s theorem**: each position is either a win, loss, or draw

  Ernst Zermelo, Über eine Anwendung der Mengenlehre auf die Theorie des Schachspiels, in: Proc. Fifth Internatl. Congress of Mathematicians II (Cambridge, **1913**), pp. 501–504
  often misquoted as “there exists a (unique) best strategy”

  - Tic tac toe, Checkers, Connect Four
  - Chess, Go, …



- explicit solutions: usually done by listing all possibilities
- for chess: done for *up to 7 chessmen*

  Lomonosov tablebases: 5×10^{14} positions up to symmetries, calculated on 75000-core supercomputer (#12 in 2009 TOP500)

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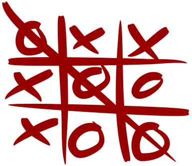

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Complexity

- *configuration* = placement of pieces + bits storing player's turn etc.
- *game tree* = graph with
  - nodes = individual configurations
  - edges = legal move between configurations
- complexity measures:
  - number of configurations $|\Omega|$
  - size of graph: game-tree complexity $|G|$

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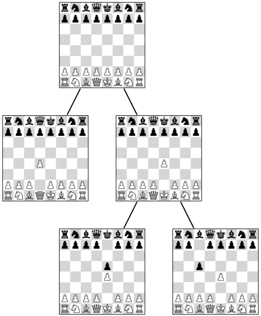

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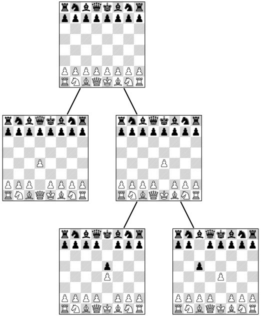


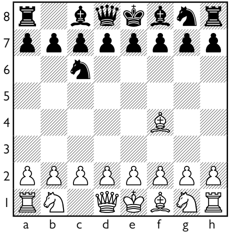
game	$\lvert\Omega\rvert$	$\lvert G\rvert$	
Tic tac toe	10^3	10^5	
Checkers	10^{20}	10^{42}	
Chess	**?**	**?**	# atoms in Earth: 10^{49}
Go	10^{170}	10^{360}	# atoms in universe: 10^{80}

[Tromp and Farnebäck, *Combinatorics of Go* (2016):
$a_{19} = 208168199381979984699478633344862770286522453884530548425639456820927419612738015378525648451698519643907259916015628128546089888314427129715319317557736620397247064840935$]

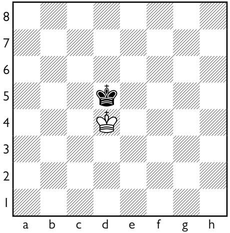
Chess Configurations

- *realizable* (some placement of pieces)
- *legal* (obeying the rules, e.g. kings not both in check)
- *reachable* from initial configuration
- *real-game* (not obviously bad for the player)

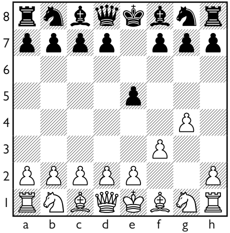
{realizable} $\supset$ {legal, reachable} $\supset$ {actually played}



legal, not reachable



reachable, not legal



reachable, not played

Chess Configurations: Estimates

configuration space:

- **Shannon (1948):** $|\Omega| \sim 10^{42}$ incl. illegal, no promotions / captures
- Steinerberger (2015): $|\Omega| \leq 2 \times 10^{40}$ legal, no promotions (strict)
- Chinchalkar (1996): $|\Omega| \sim 10^{50}$ including promotions

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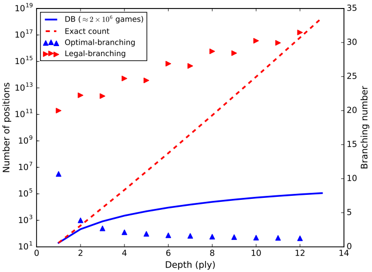

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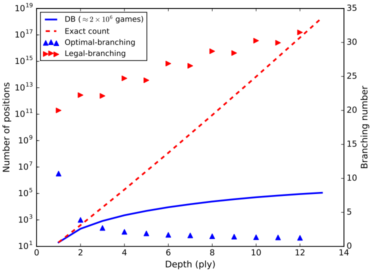
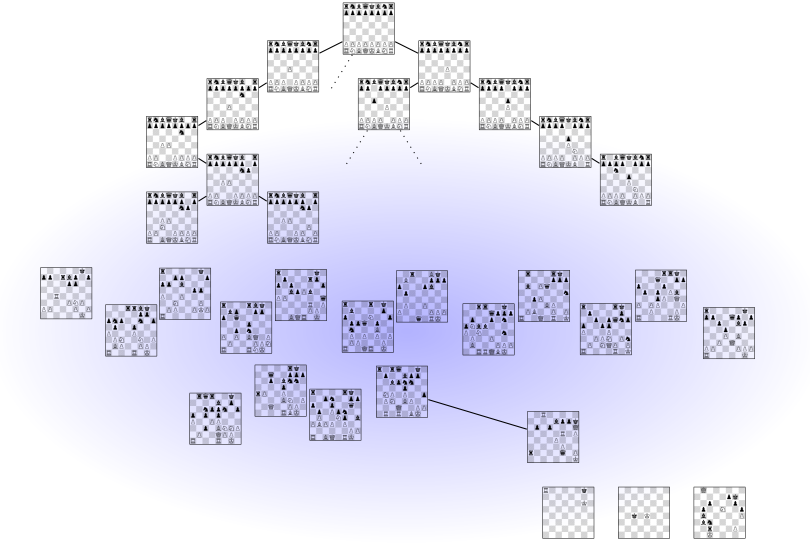


- Emmanuel Lasker: *"only one move per position, but a good one"*

Graphs and Sizes



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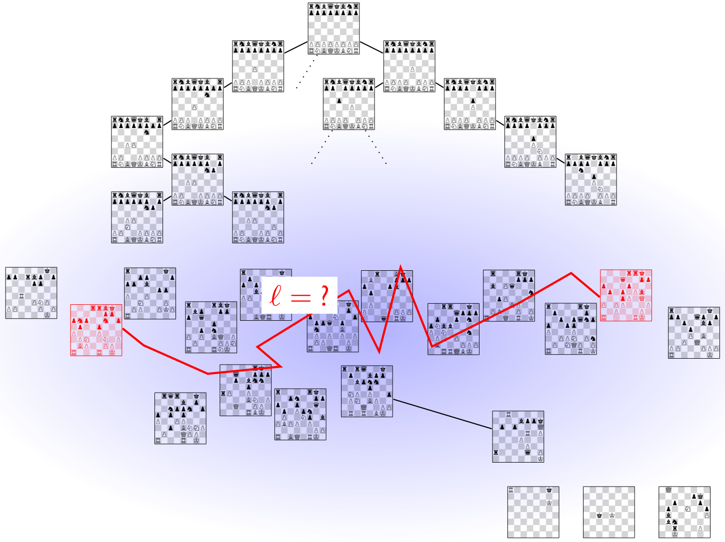

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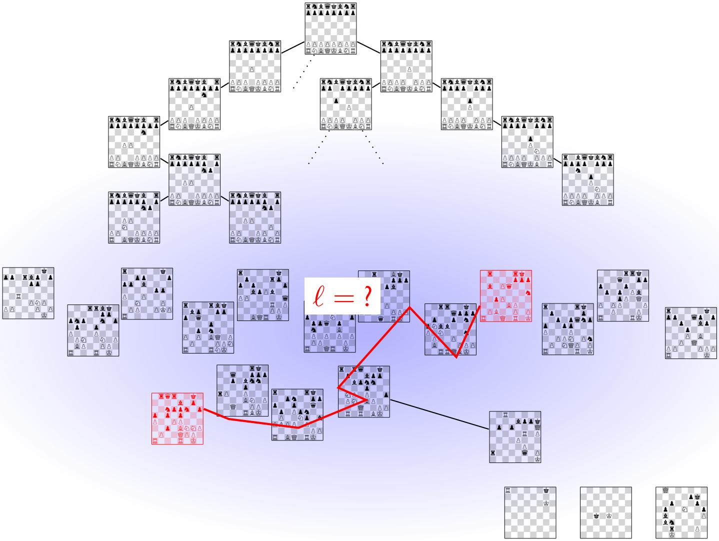
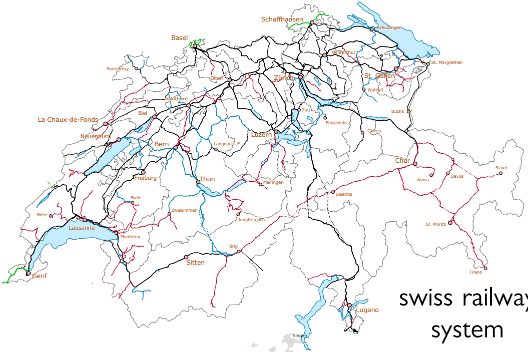
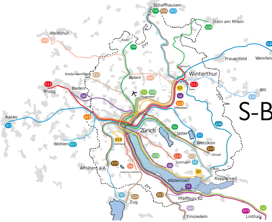


How to Measure Size?



swiss railway system

ICE trains



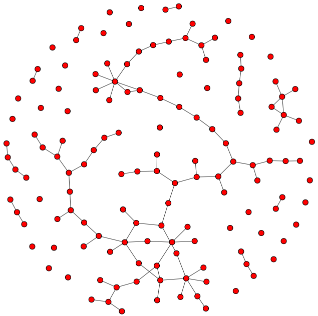
S-Bahn Zürich

S-Bahn München

[all images: wikimedia, CC-BY-SA]

How to Measure Size? – Random Graphs

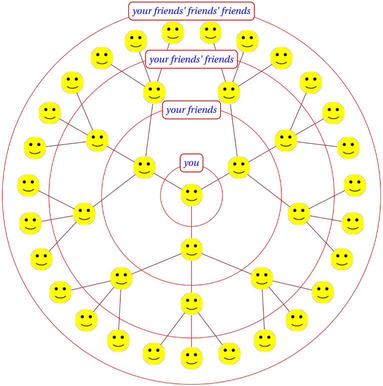
- Erdős-Rényi random graph model
  - graph with N nodes, edges with probability p
  - average branching number z
  - if Np big enough: *giant connected component*, almost surely



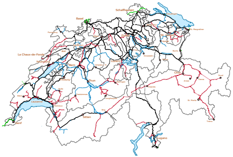
path length on connected component $\ell \sim \ln N / \ln z$

How to Measure Size? – Random Graphs

- related: *small-world networks* (Strogatz/Watts 1998)
  - $\ell \sim \ln N$ even when nodes cluster
- Milgram's *"six degrees of separation"* experiment (1967)
  - $N_{\text{world pop.}} \sim 7.5 \times 10^9$, $z \approx 30$



How to Measure Size? – Examples



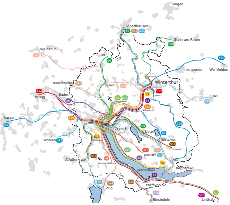
swiss trains:

$N \sim 100$ (Taktknoten), $z \sim 3$

$\rightsquigarrow \ell \sim 4$ nodes ~ 2 h travel time ~ 200 km

$\rightsquigarrow$ area $\sim$ $40000\,\text{km}^2$

in reality: area of Switzerland $\sim$ $41285\,\text{km}^2$



S-Bahn Zürich:

$N \sim 10$, $z \sim 5$

$\rightsquigarrow \ell \sim 1.4$ nodes ~ 42 min travel ~ 35 km

$\rightsquigarrow$ area $\sim$ $1225\,\text{km}^2$

in reality: area covered by ZVV $\sim$ $1840\,\text{km}^2$

Chess as a Random Graph

illustration: set of 1417 chess configurations, random-pair sampling

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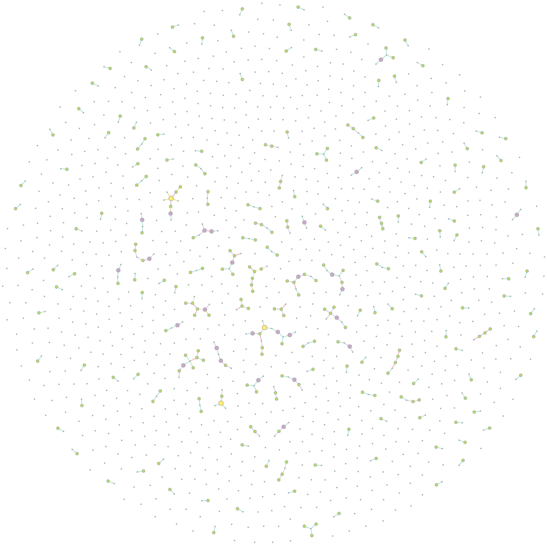

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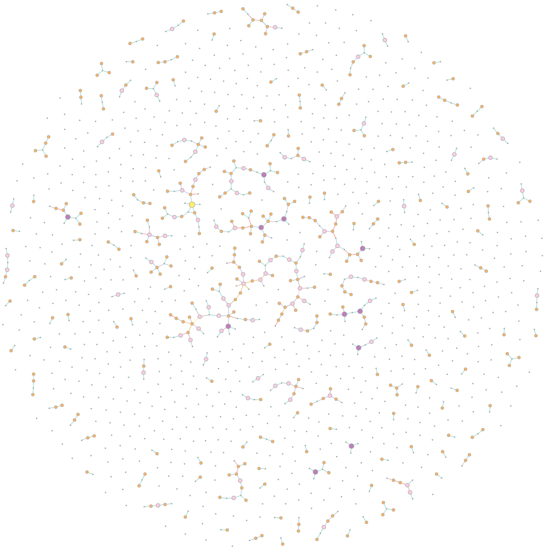

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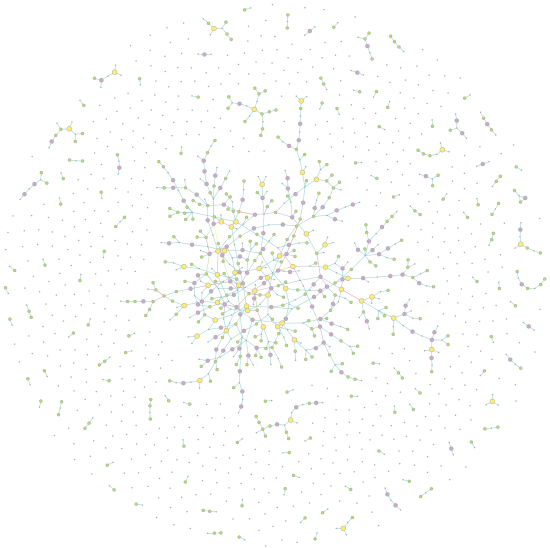

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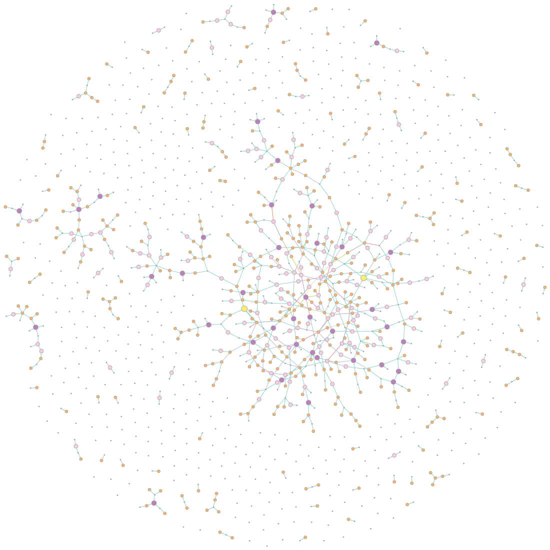

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Measuring Distances by Monte Carlo

idea:

1. pick configuration pairs (A, B) at random (some depth into game)
2. find path of legal moves $A \to B$, tabulate length ℓ
3. infer size of connected component to which (A, B) belong

variations:

- pre-condition (A, B)
  - all reachable (MC generated), actually played (TWIC database)
  - related to specific opening moves (same *vs.* distinct)

Random Chess: Monte Carlo Simulations

pick a random move per ply

sampling issue:

- "$A \to B$" is a *rare event*
- each step branches 30-fold: after 10 steps, $\sim 10^{15}$ possibilities…

$\Rightarrow$ **Monte Carlo with importance sampling**

- sample moves not equi-probably,
- higher probability on those that help in $A \to B$

PLEASE CHECK LAST MOVE

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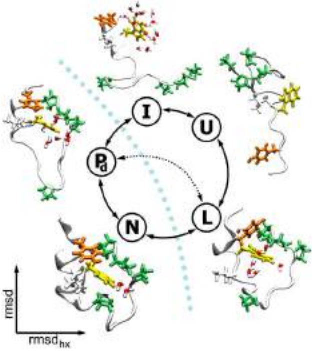
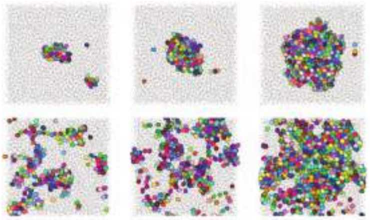
Rare Events



protein folding

folding of Trp-cage mini-protein
[Juraszek and Bolhuis, PNAS (2006); Biophys. J. (2008)]

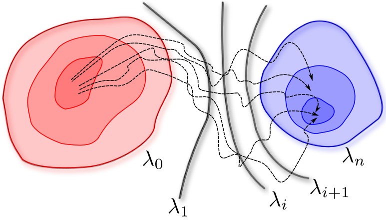


nucleation and growth of hard-sphere crystals
[Dorosz and Schilling, J. Chem. Phys. (2013)]

crystal nucleation

any kind of
(non-equlibrium)
rare fluctuation

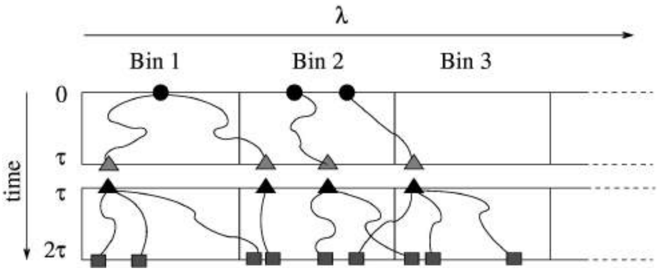
Transition Path Sampling



- high-dimensional state space $\{\vec{r}^N\}$, some (stochastic) dynamics
- define states A and B by some condition
- define **reaction coordinate** $\lambda(\{\vec{r}^N\})$ with $\lambda(A) = 0$, $\lambda(B) = 1$
- task: sample paths $A \to B$ *randomly, with proper weight*

examples for λ:
size of nucleus (crystallization), bond angles/distances (protein folding)

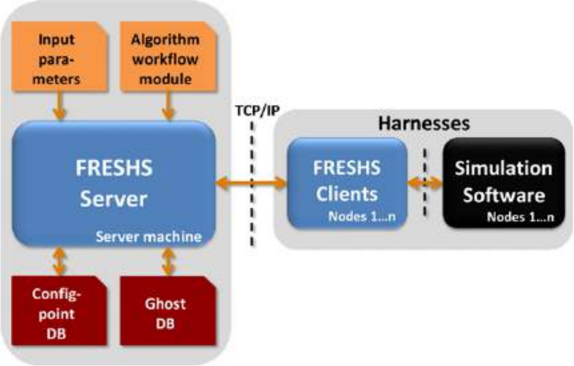
Stochastic-Process Rare Event Sampling (SPRES)



- "shoot" short trajectories (length τ) – same number per bin
- samples transitions $\lambda_i \mapsto \lambda_{i+1}$ and their weight
- works for *any dynamics* *(chess!)*
- advantage: does not rely on optimal reaction coordinate

[Berryman and Schilling, J Chem Phys **133**, 244101 (2010); DOI:10.1063/1.3525099]

FRESHS – A Modular Rare-Event Sampler



- server implements shooting strategy
- provides trivial parallelization (many independent trajectories)
- works with any “black-box” simulation software

[Kratzer *et al.*, Comput. Phys. Commun. **185**, 1875 (2014); DOI:10.1016/j.cpc.2014.03.013]
[http://www.freshs.org/]

A Reaction Coordinate for Chess

5	4	3	2	2	2	2	2
5	4	3	2	1	1	1	2
5	4	3	2	1	♔	1	2
5	4	3	2	1	1	1	2
5	4	3	2	2	2	2	2
5	4	3	3	3	3	3	3
5	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5

Chebyshev distance

7	6	5	4	3	2	3	4
6	5	4	3	2	1	2	3
5	4	3	2	1	♖	1	2
6	5	4	3	2	1	2	3
7	6	5	4	3	2	3	4
8	7	6	5	4	3	4	5
9	8	7	6	5	4	5	6
10	9	8	7	6	5	6	7

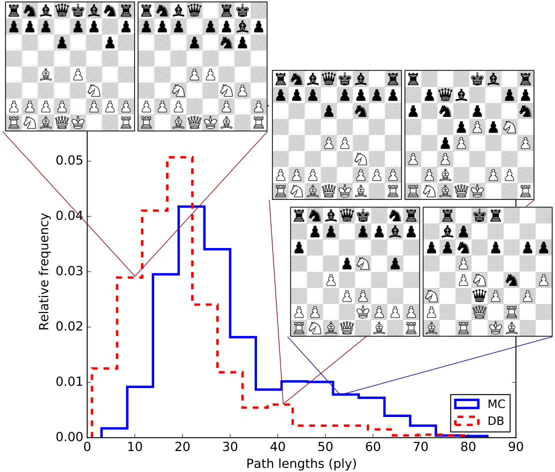
taxicab distance

3	2	3	4	1	2	1	4
4	3	2	1	2	3	2	1
3	2	3	2	3	♘	3	2
4	3	2	1	2	3	2	1
3	2	3	4	1	2	1	4
4	3	2	3	2	3	2	3
3	4	3	2	3	2	3	2
4	3	4	3	4	3	4	3

(knight moves)

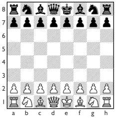
- r.c. = purely geometrical construction
- need not be ideal for SPRES

Results: Path Length Histograms

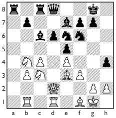


- path length distribution $p(\ell)$: *two peaks*
- real-game pairs are closer

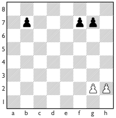
Overlap Correlations



time 0

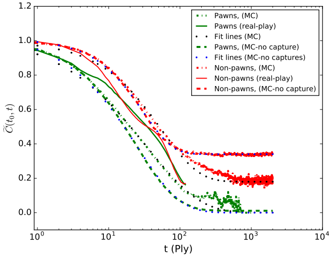


time t



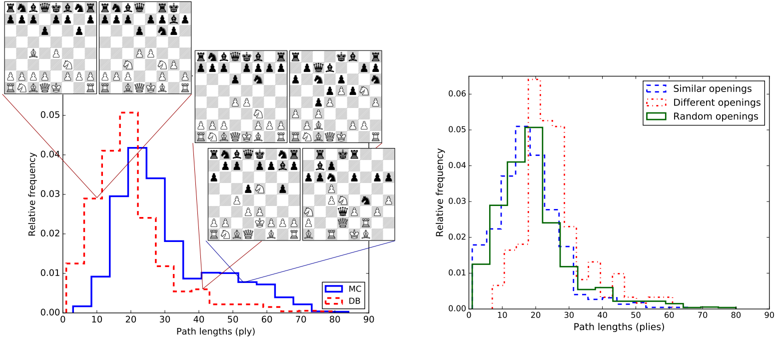
pawn overlap

- for real games *vs.* MC-generated games
- real games: *pawn persistance*
- GM Nimzowitsch: opening, middle game, endgame



$\Rightarrow$ *pawns make the difference* – their moves are irreversible

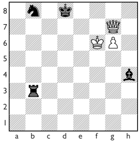
Results: Path Length Histograms



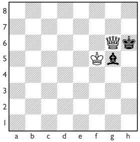
- traditional chess-opening theory maps to graph structure
- “same opening” = “more closely connected positions”

Real-Game Paths

sample transition



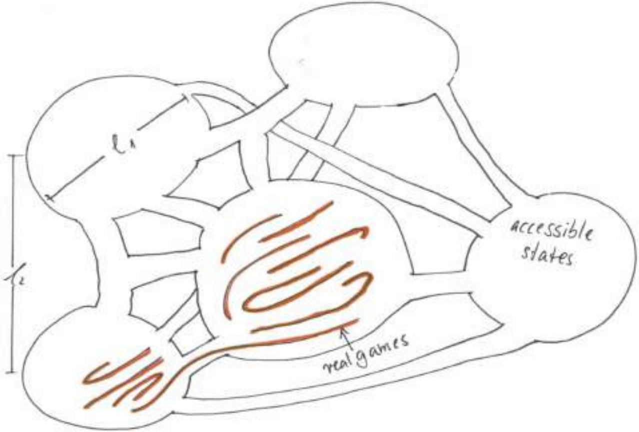
$\mapsto$



shortest path:
$\leq$13 moves

optimal play:
549 moves

Interpretation



$$N_{\text{accessible}} \sim 10^{42} \gg N_{\text{relevant}} \sim 10^{22} \gg N_{\text{played}} \sim 10^{6}$$

opening = pawn structure = fixes pocket

Summary / Outlook

- *structure of chess* probed by statistical physics
- chess' configuration space *decomposes into pockets*
  - pockets are "pinheads compared to Mt Everest"
  - real-games are "single polymers compared to pinhead"
  - playing 10^6 games/second since beginning of time: explores one pocket
- *combine chess and computer physics:*
  - SPRES + Stockfish for targeted look-ahead?
- *teach statistical physics* using chess
  - also works with other games
  - but that's another story…

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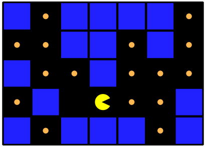

Thanks

- Tanja Schilling, U Luxembourg
- Arshia Atashpendar, U Luxembourg

  Europhysics Letters **116**, 10009 (2016)

- Mark Crowther (TWIC database of chess games)
- Andreas Hirstein, NZZ; Patrick Illinger, SZ

NZZ am Sonntag 4. Dezember 2016 | **Wissen** | 61

Worauf es ankommt

Simulationsrechnungen zeigen, warum Bauern im Schachspiel so wichtig sind. **Von Tanja Schilling und Thomas Voigtmann**

THANK YOU FOR AN INTERESTING GAME.