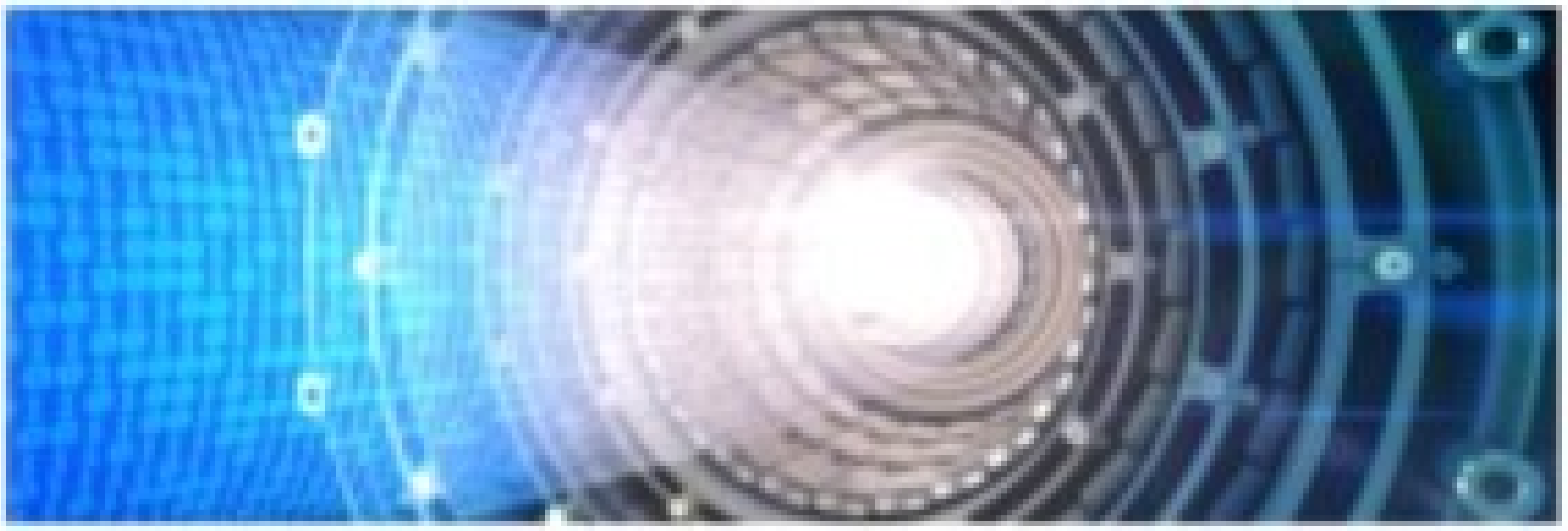
From Hilbert's Entscheidungsproblem to Valiant's counting problem

Nitin Saxena (Indian Institute of Technology Kanpur)



2014

Contents

- Hilbert
- Church & Turing
- Cook & Levin
- Valiant's permanent
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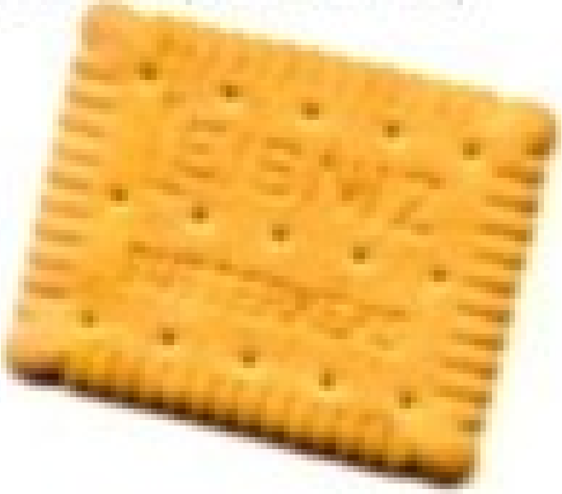

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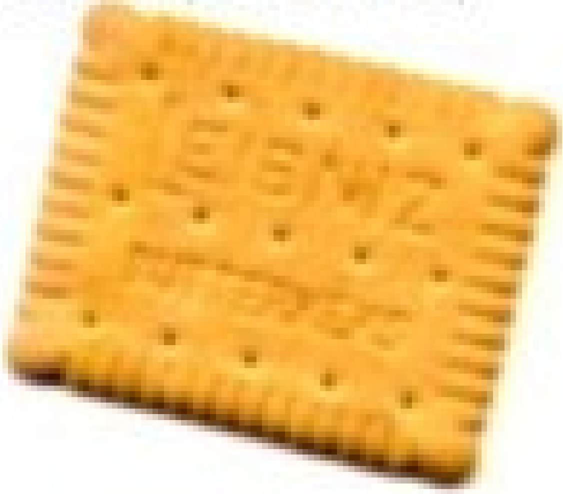

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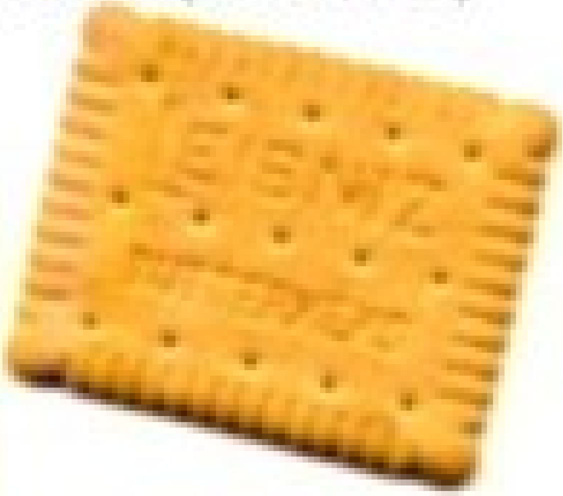

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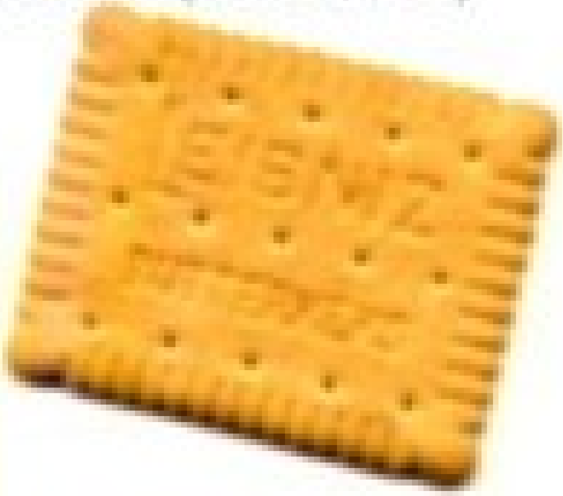

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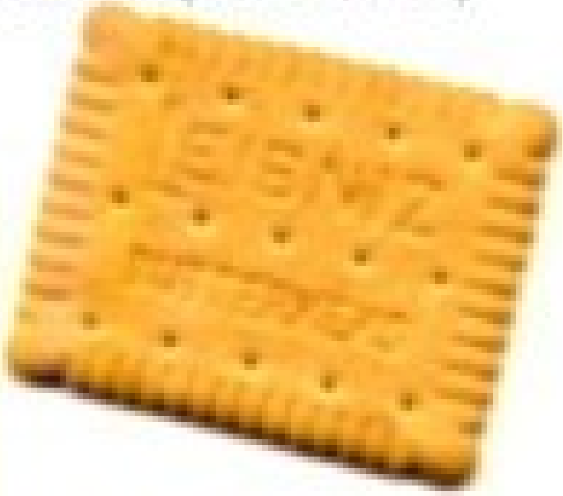

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  Example 1: Angles of a triangle sum to 180°.

  Example 2: Prime numbers are infinitely many.

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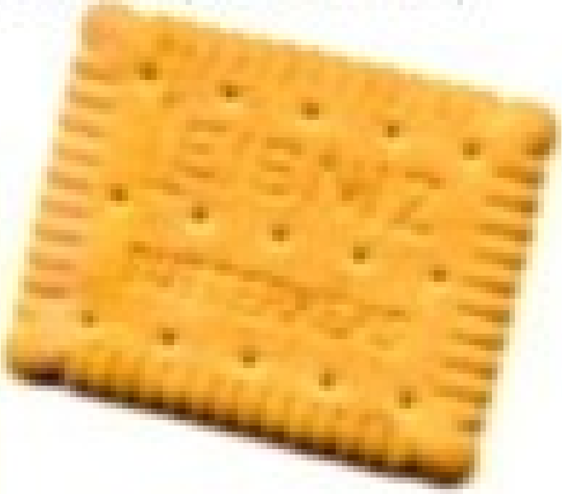

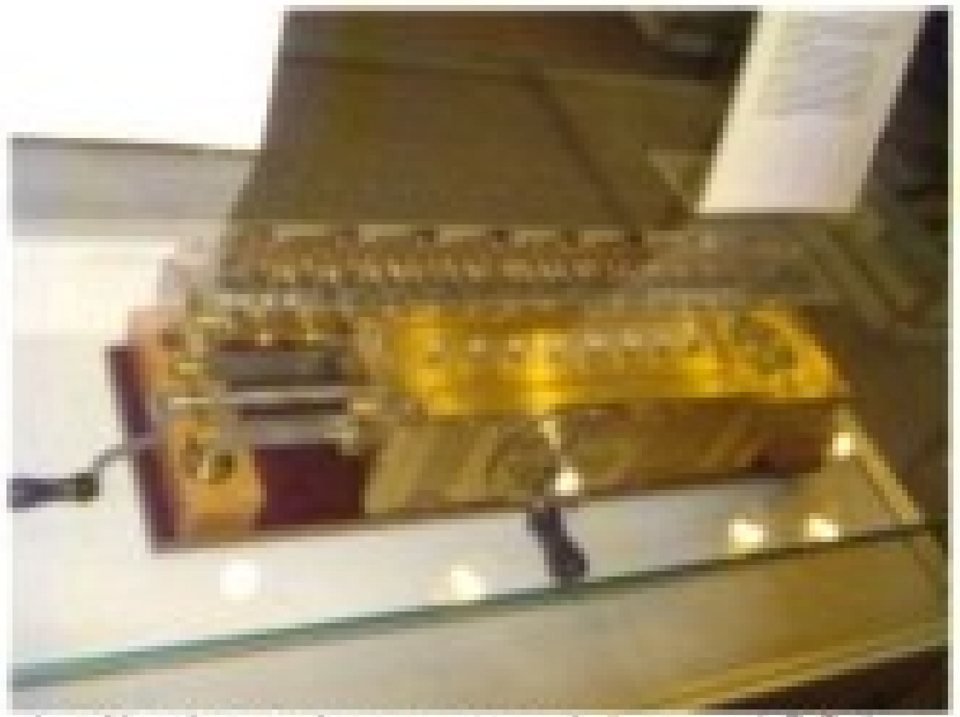

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  - hence, 'computation' requires a new mathematical framework.

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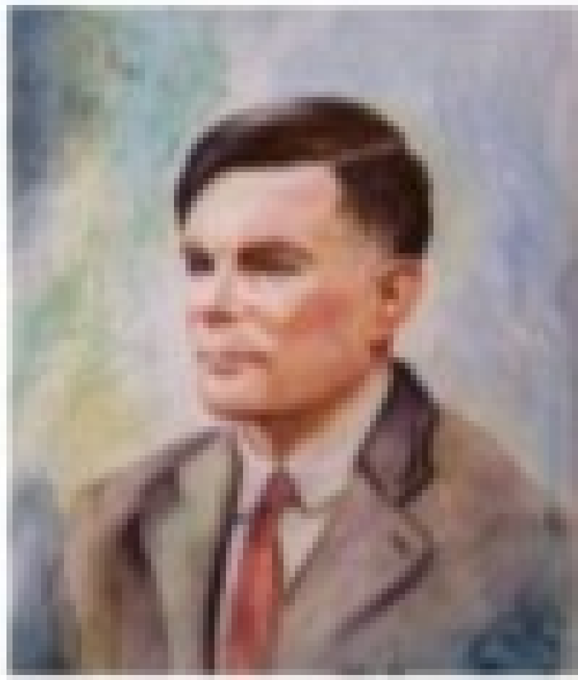
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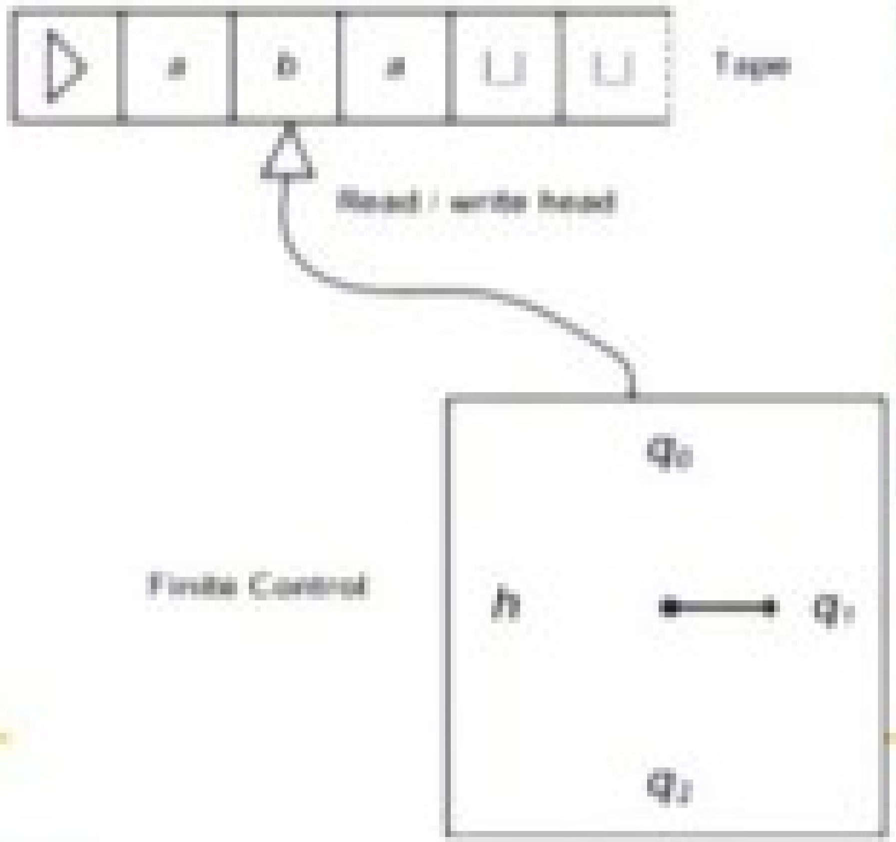

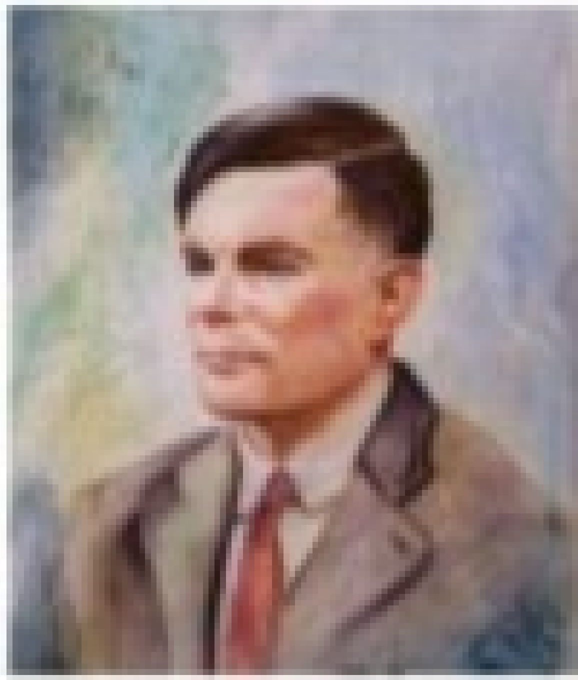

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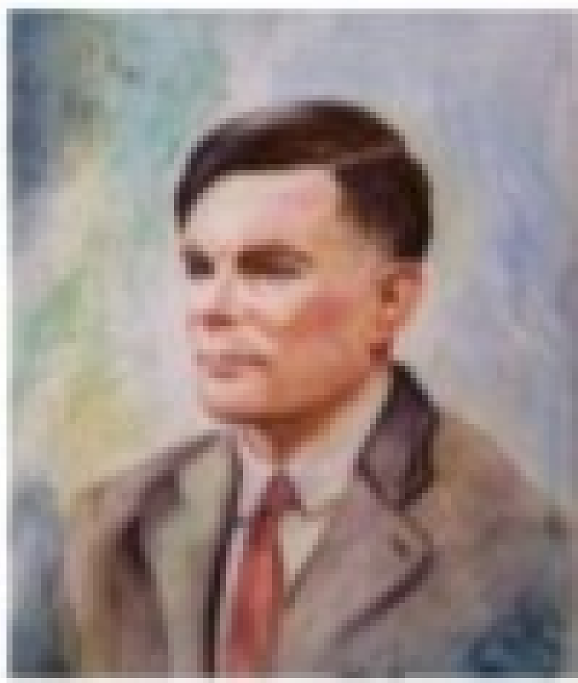

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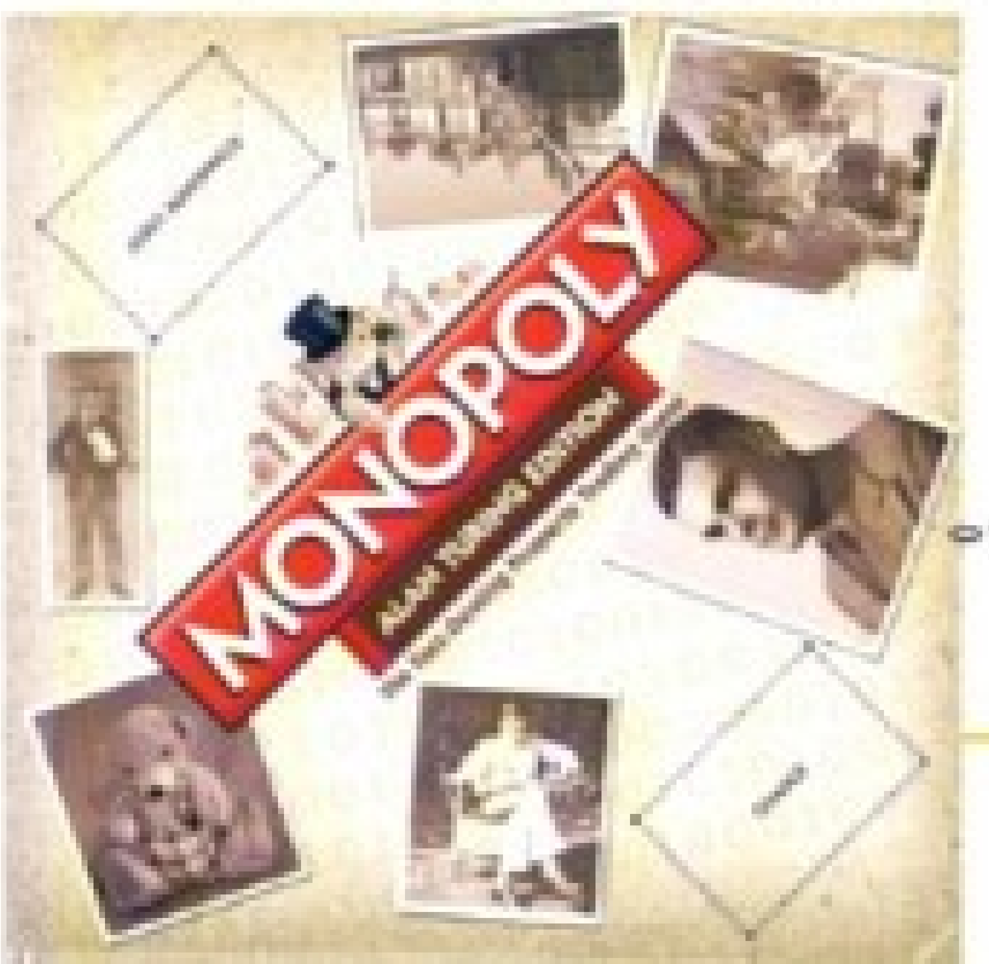

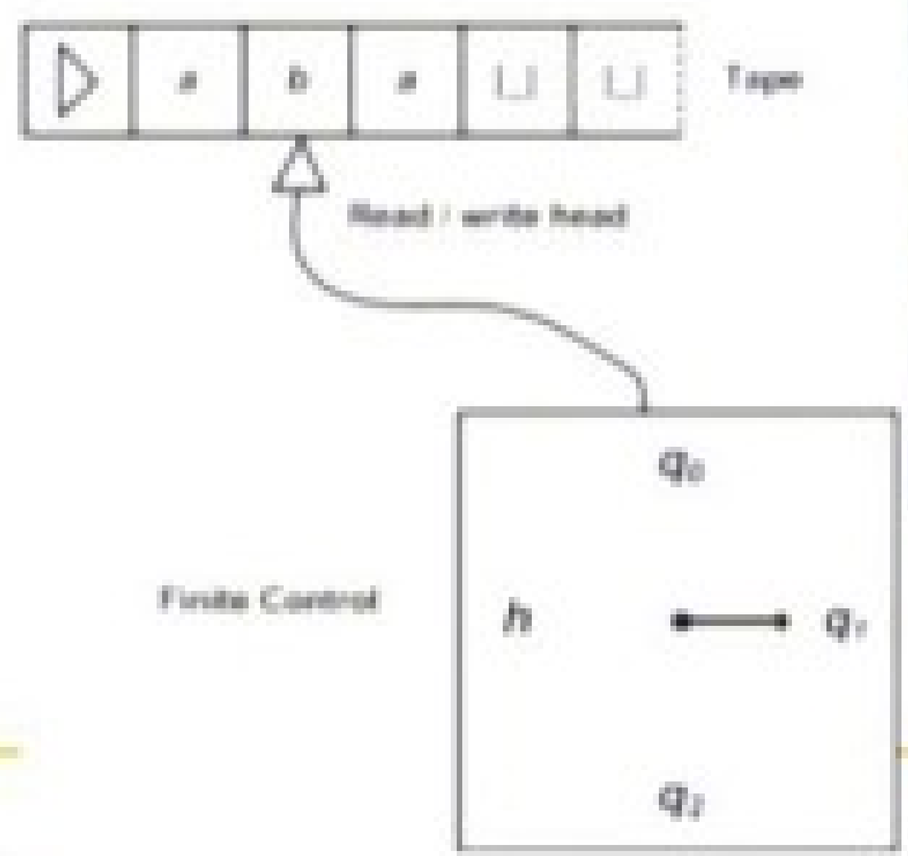

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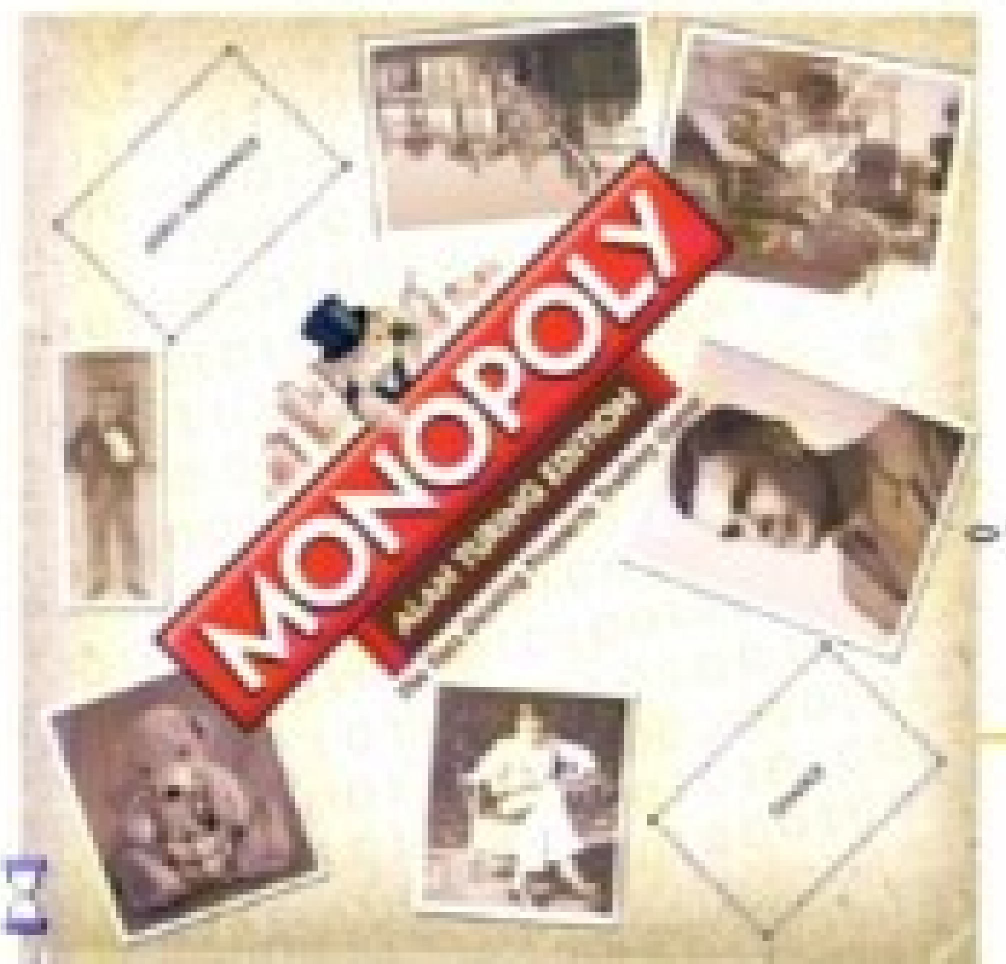

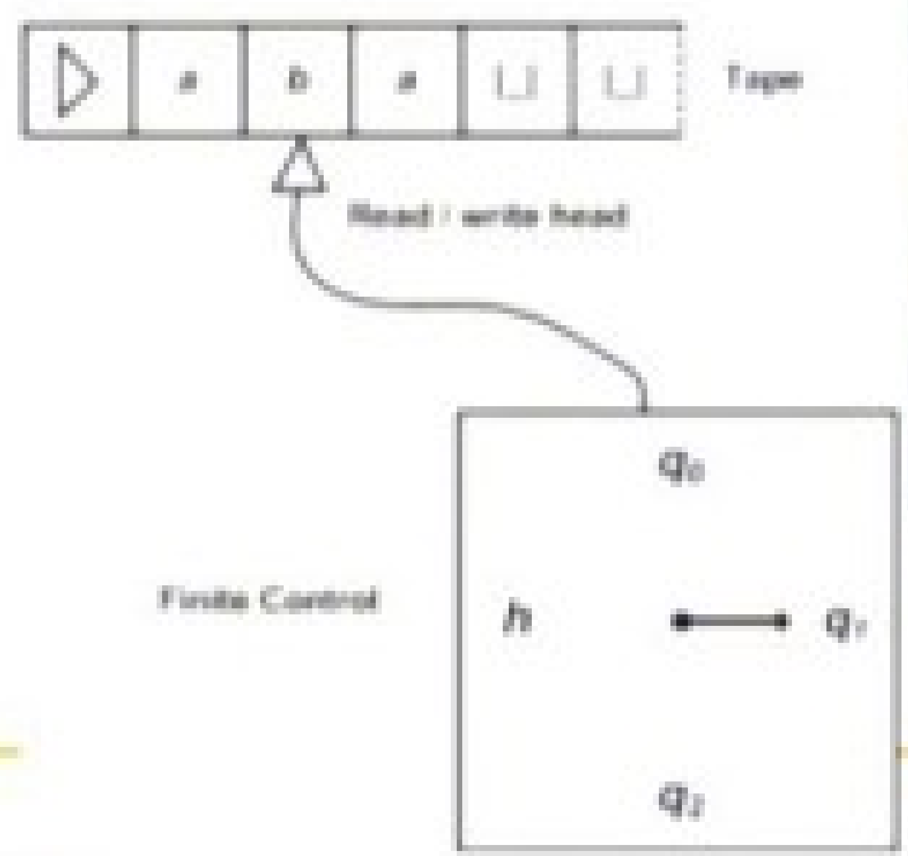

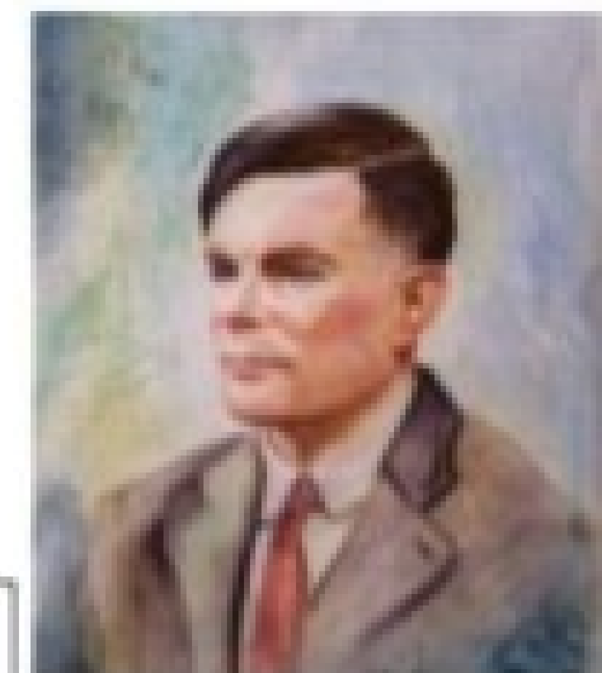

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  - Thus, Halting problem is undecidable.

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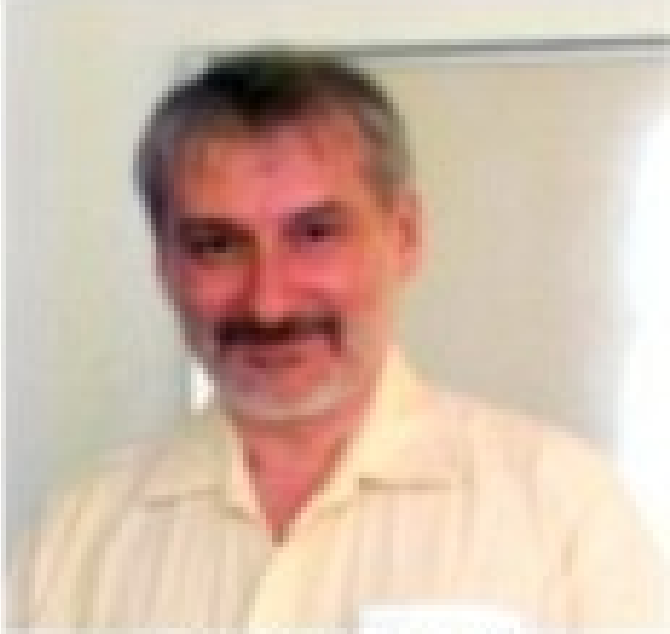

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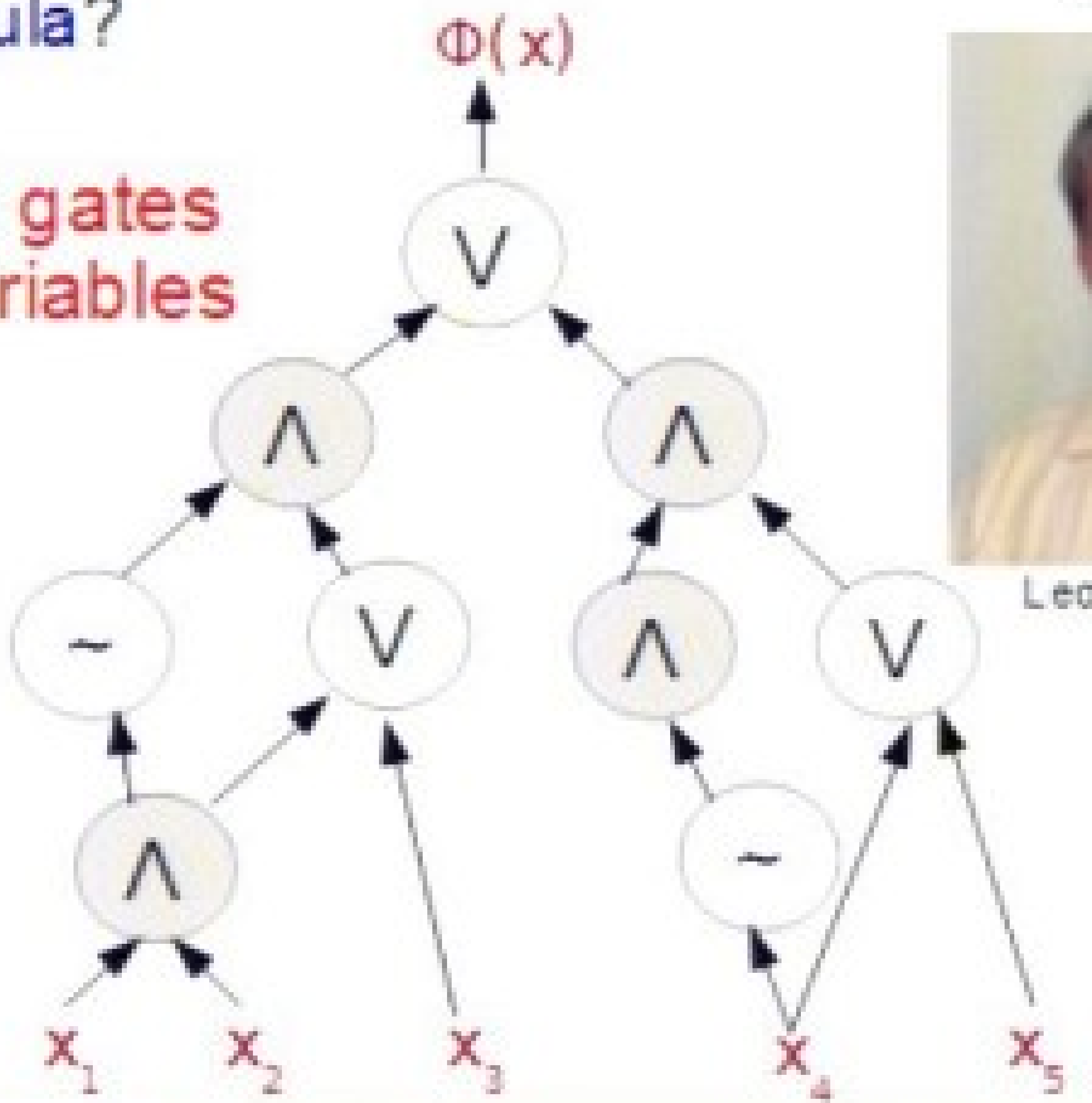

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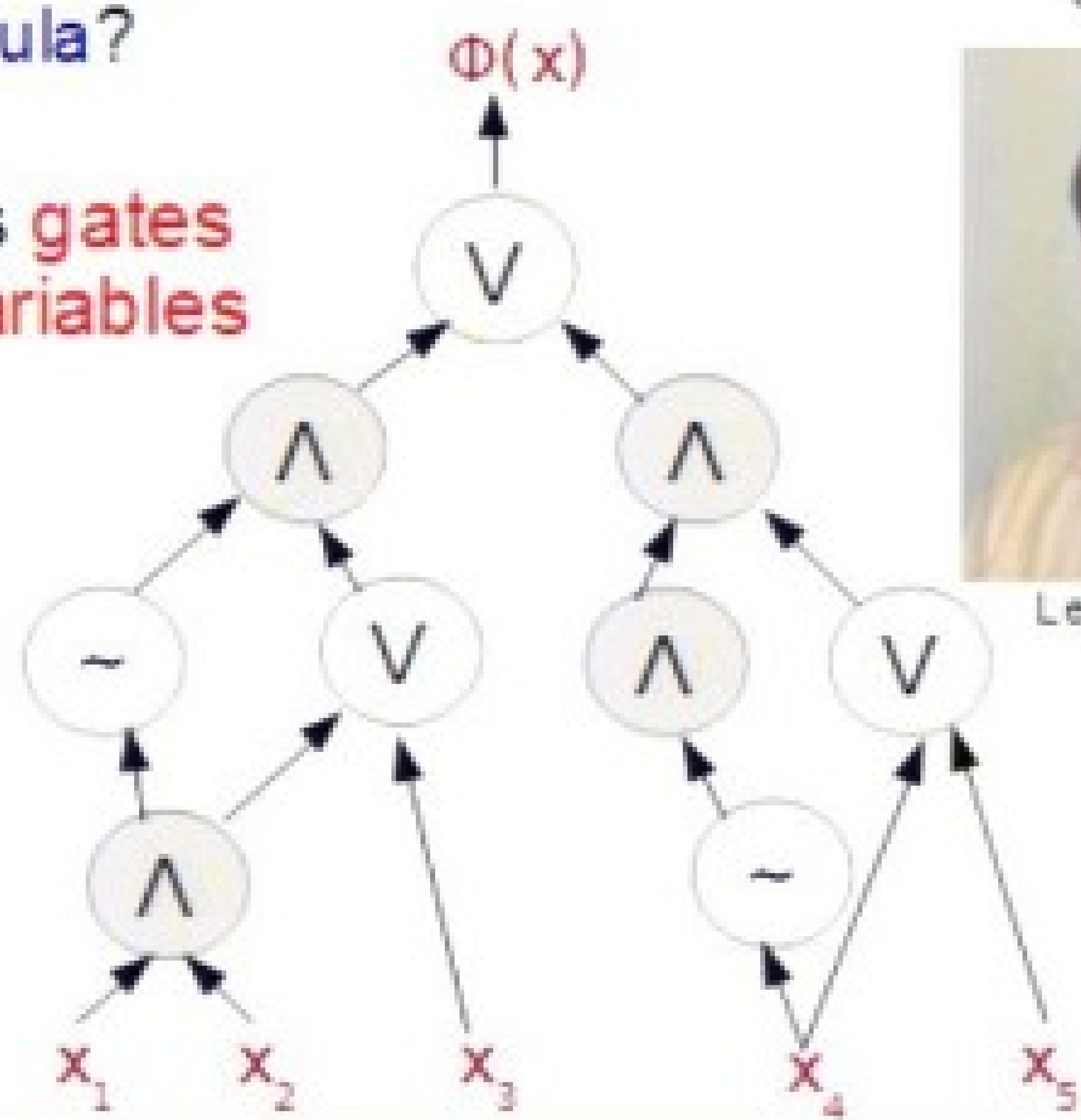


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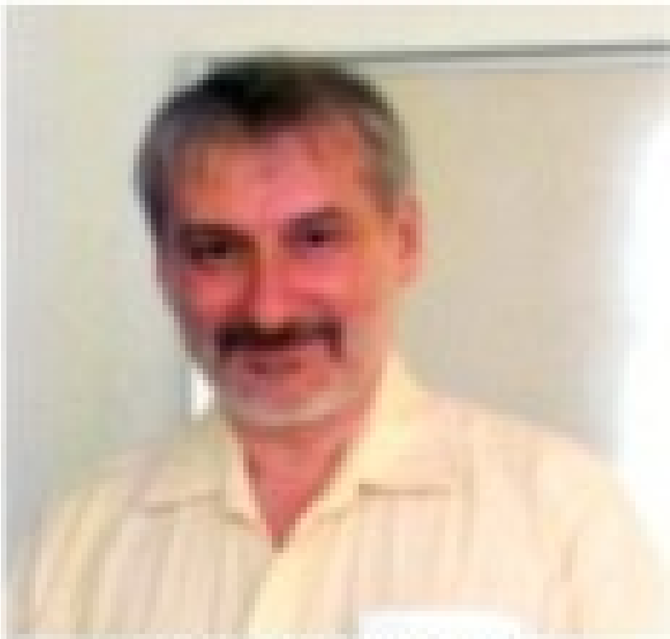
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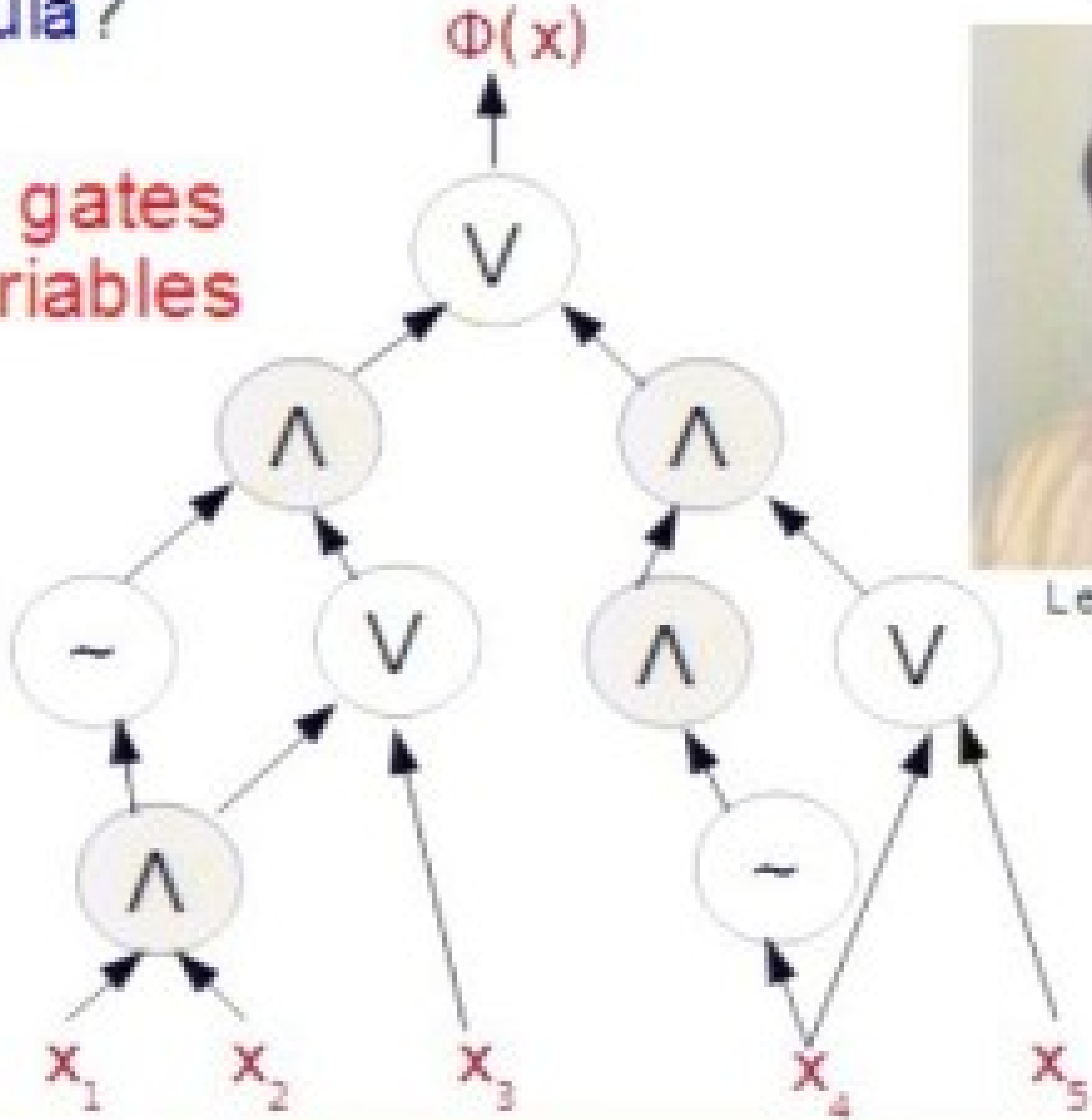

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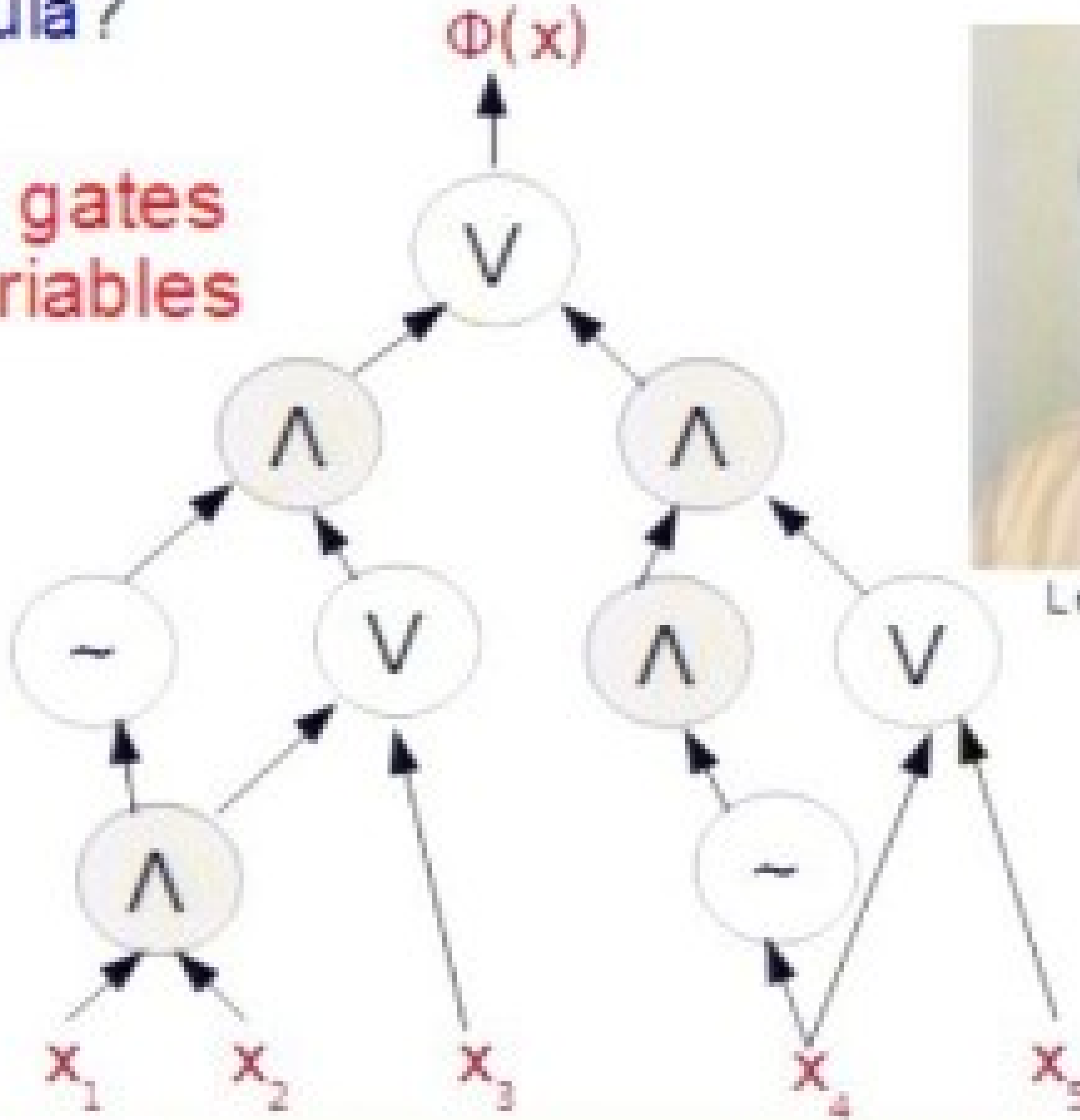
- Move from decidability to <u>efficiency</u>……

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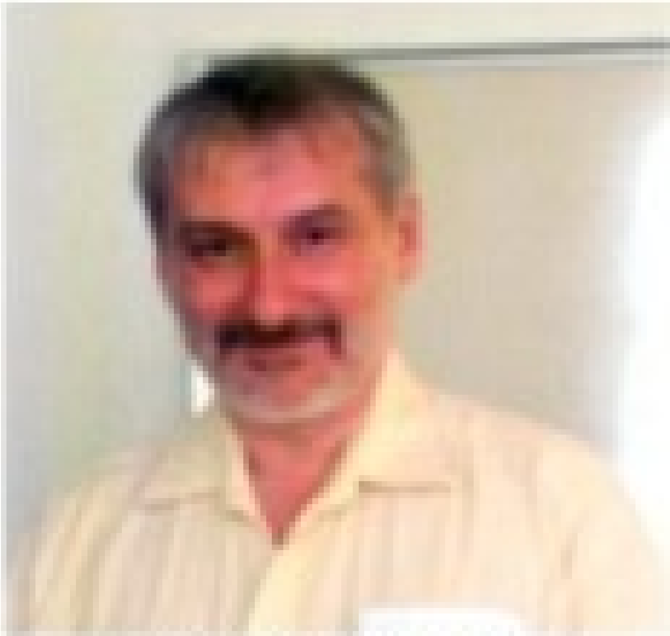
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Integer programming, set packing, vertex cover, feedback node set, hamiltonian cycle, chromatic number, clique, steiner tree, 3-dimensional matching, knapsack, job sequencing, partition, Max cut, independent set problem, Travelling salesmen problem

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Given a matrix A compute Per(A) ?

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- *Algebraic P vs NP question:*

  Is permanent efficiently computable ?

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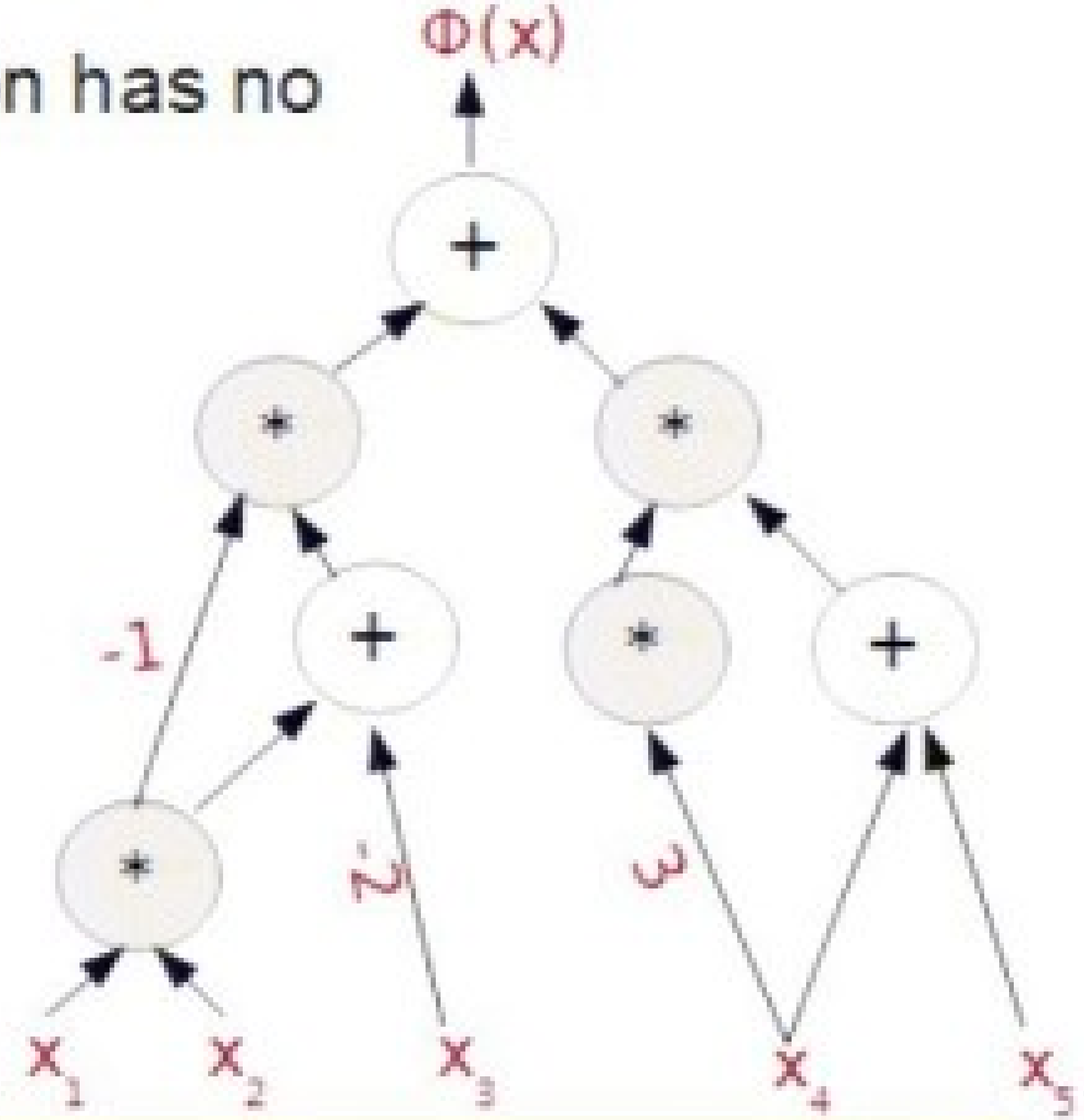

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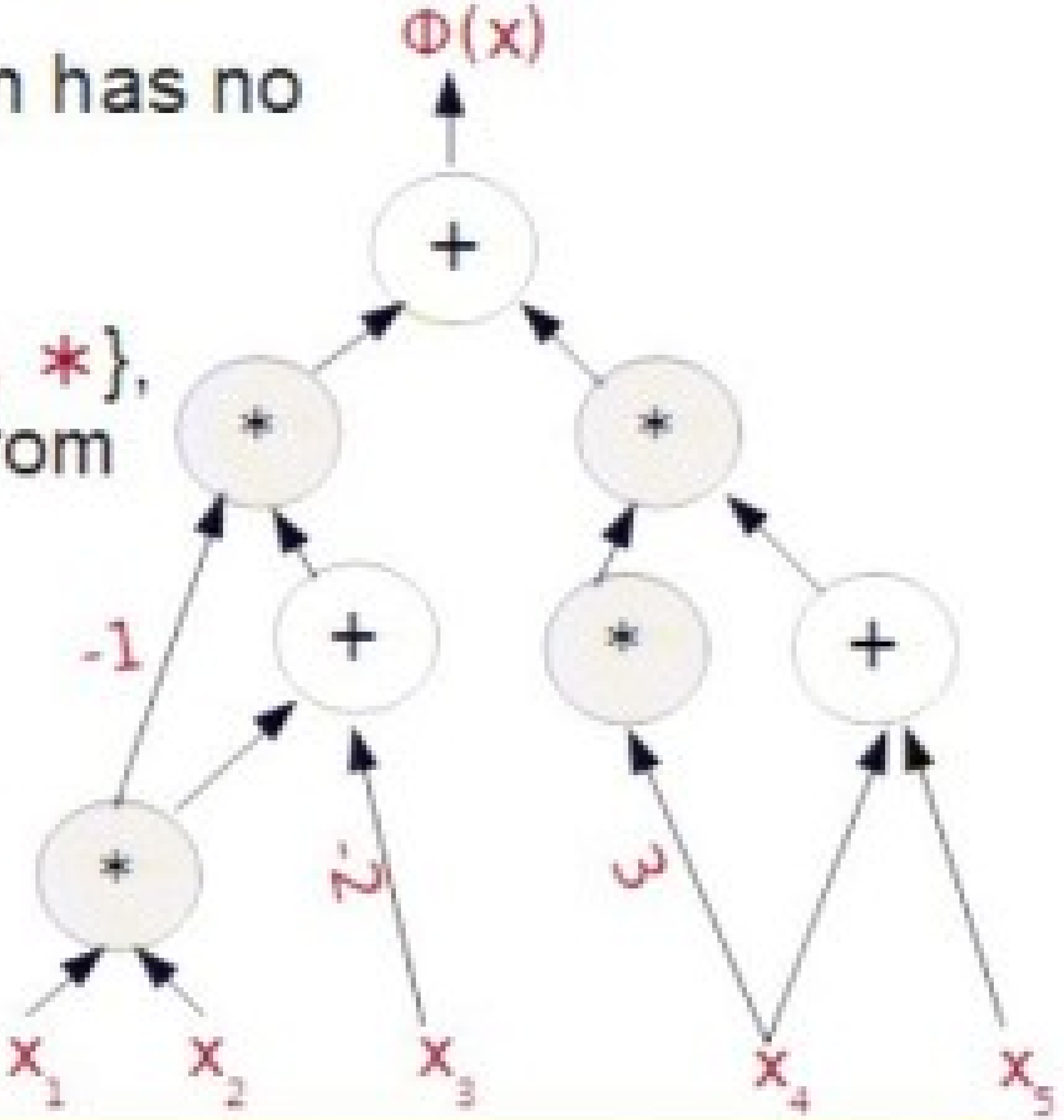
- I.e. show that the permanent function has no *small* arithmetic circuit.
- An arithmetic circuit Φ has gates $\{+, *\}$, variables $\{x_1, \ldots, x_n\}$ and constants from some field F.
- An arithmetic circuit is an algebraically neat model to capture real computation.



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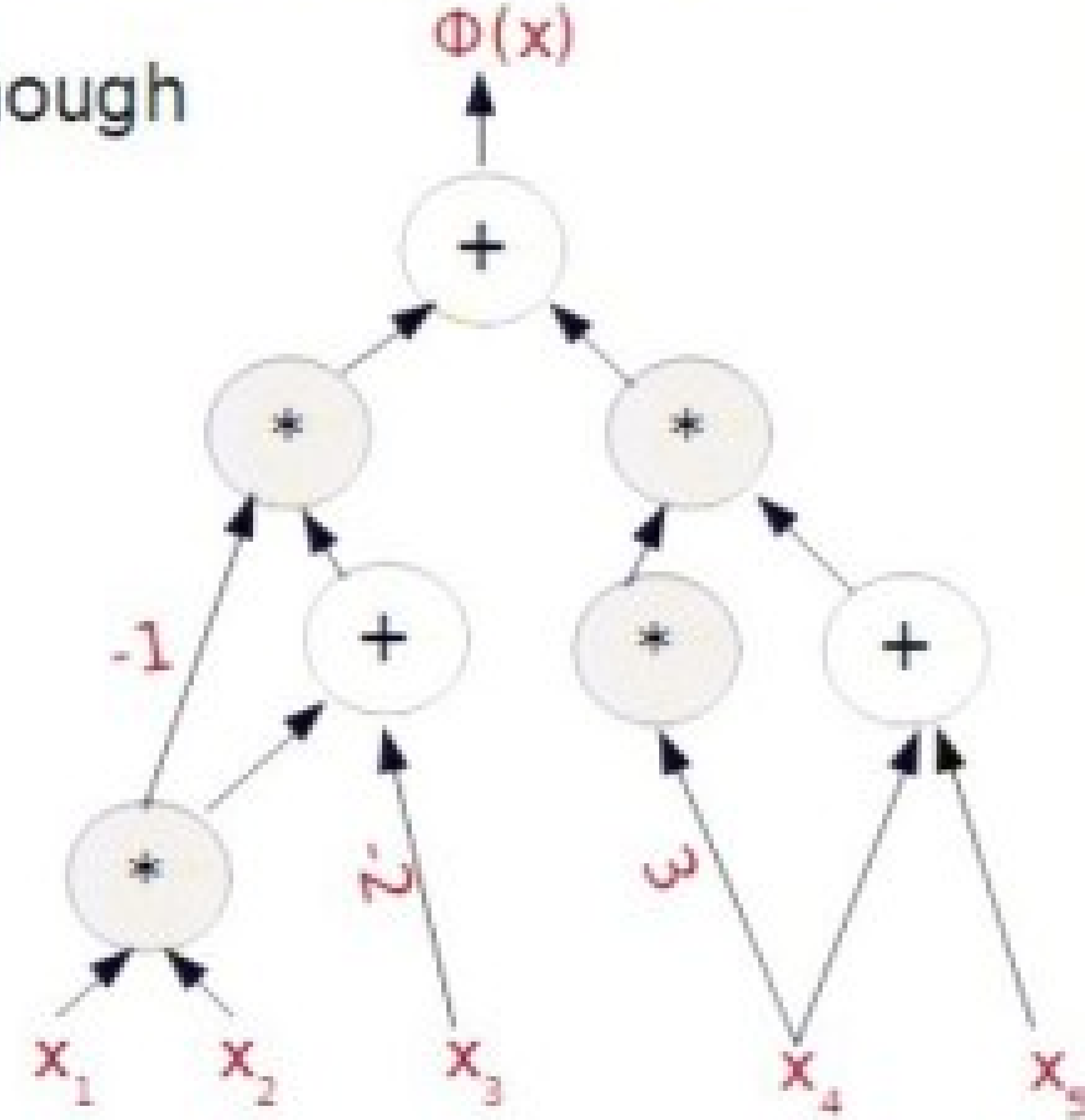

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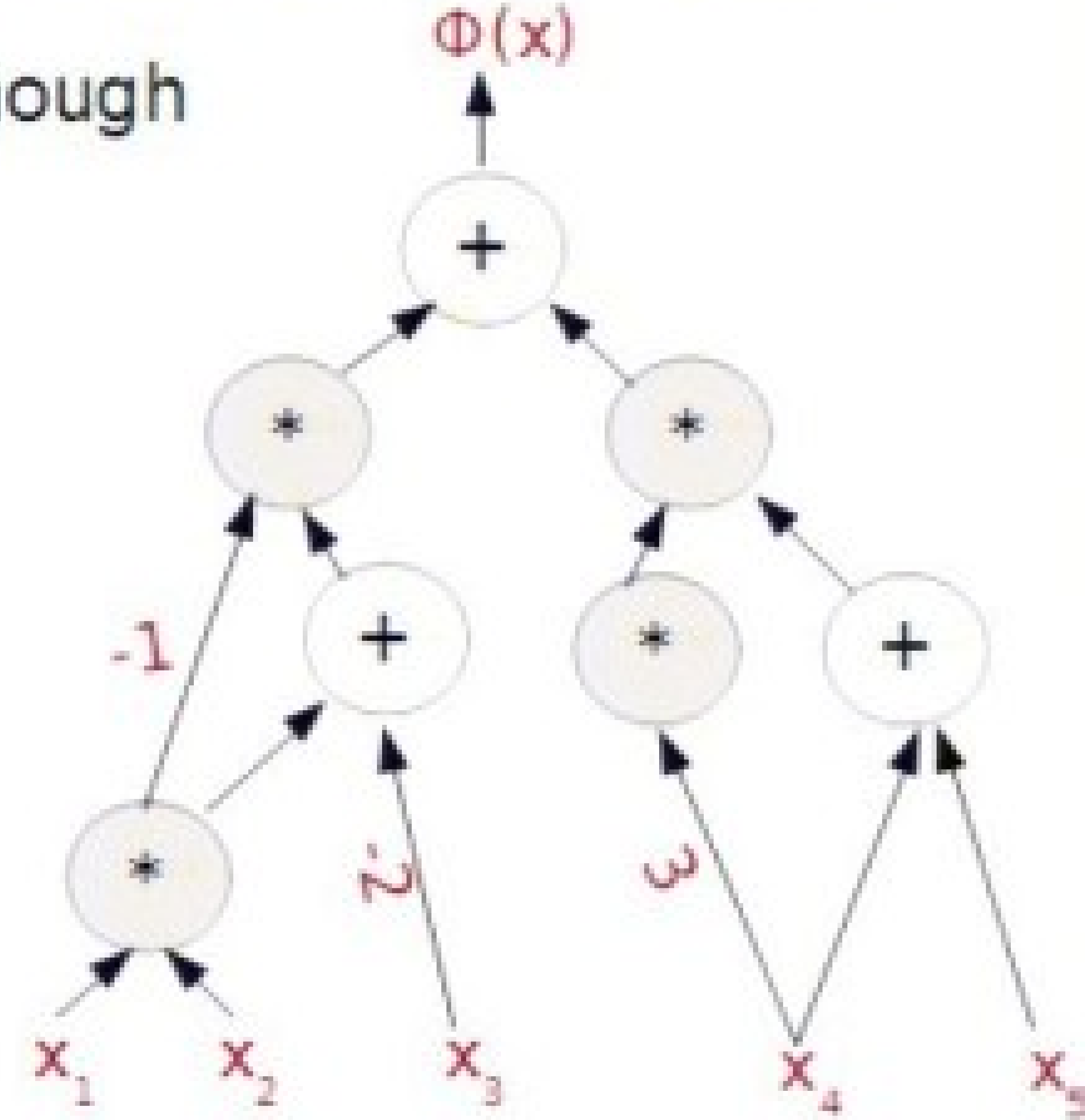

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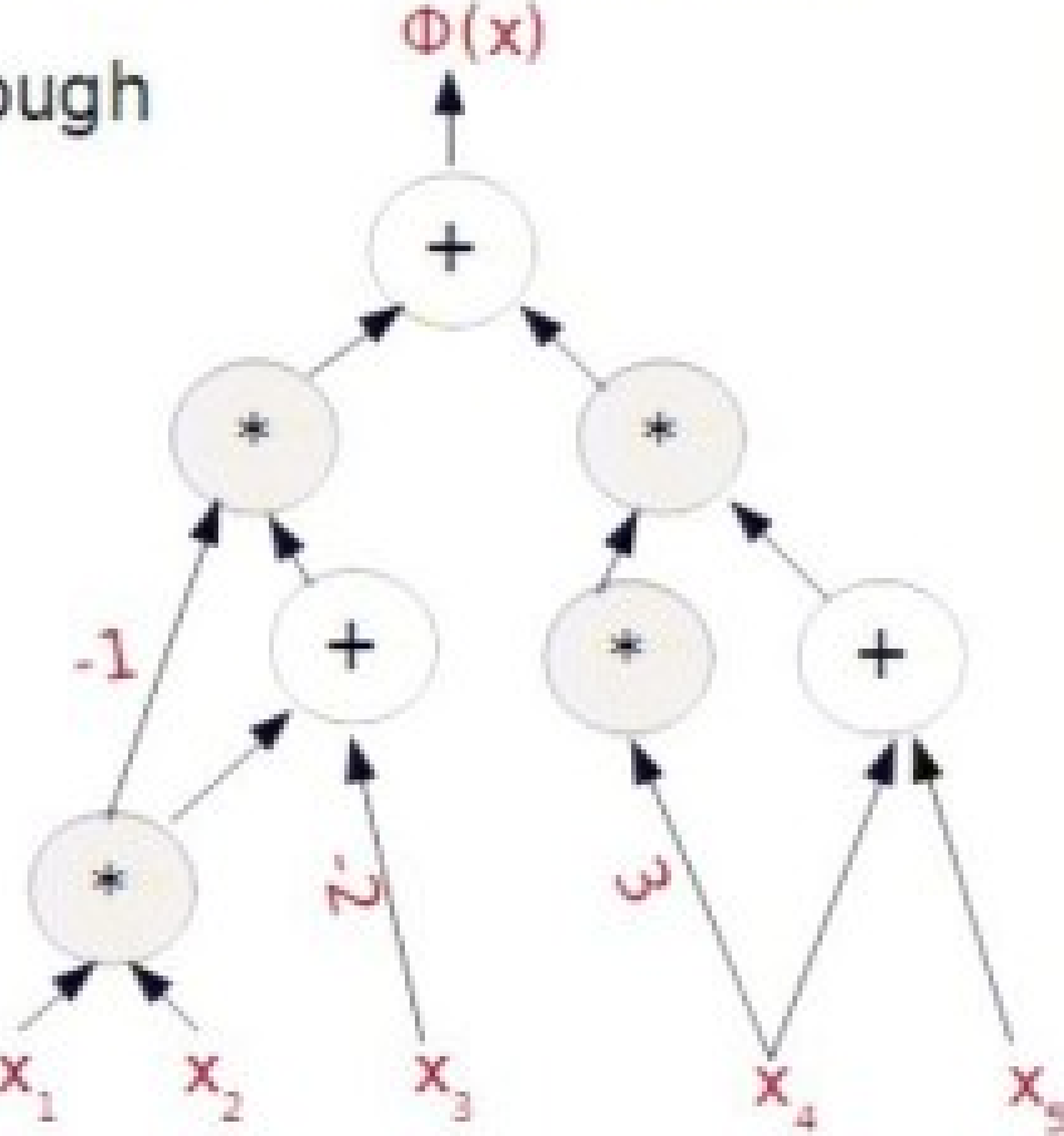

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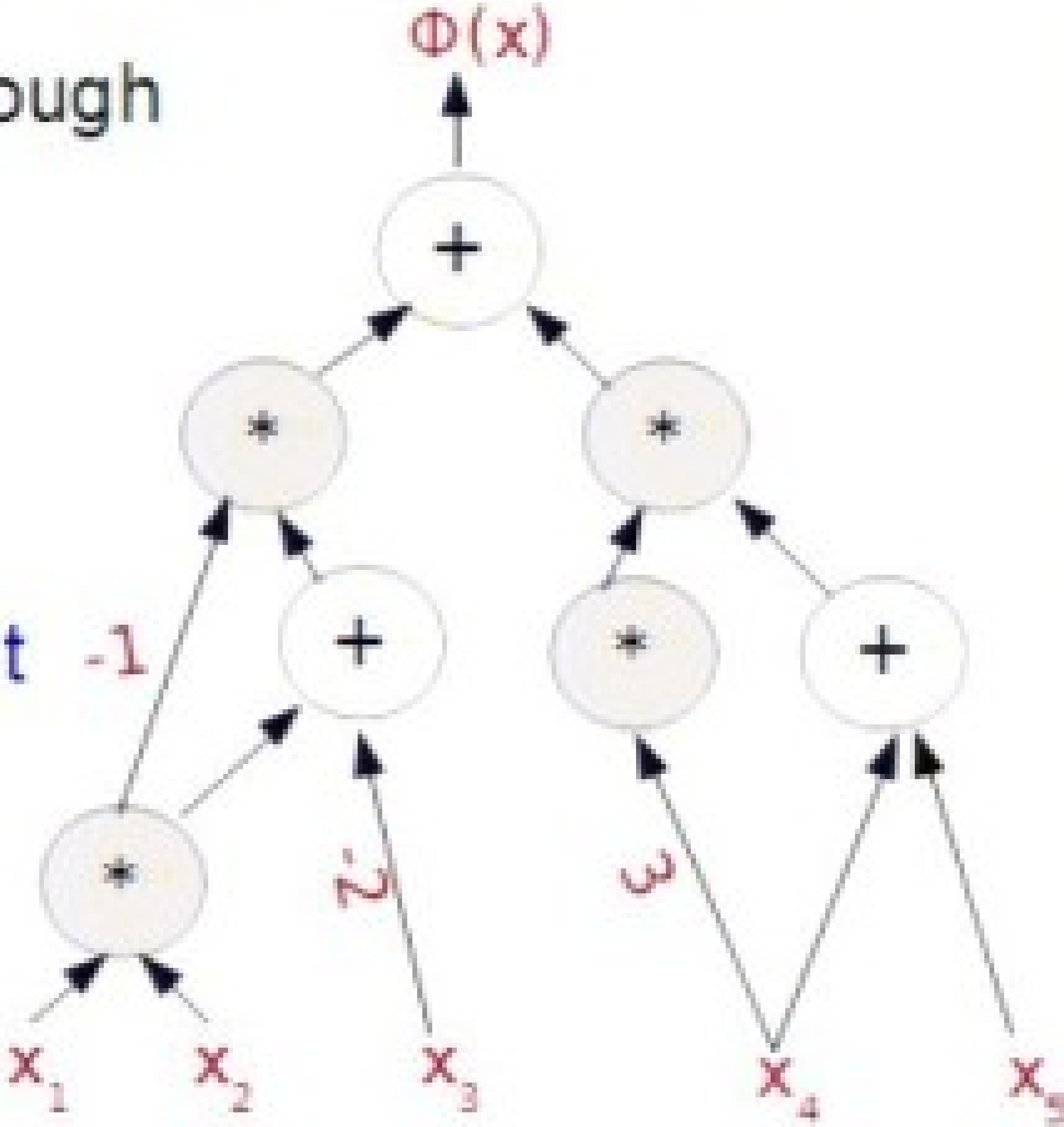

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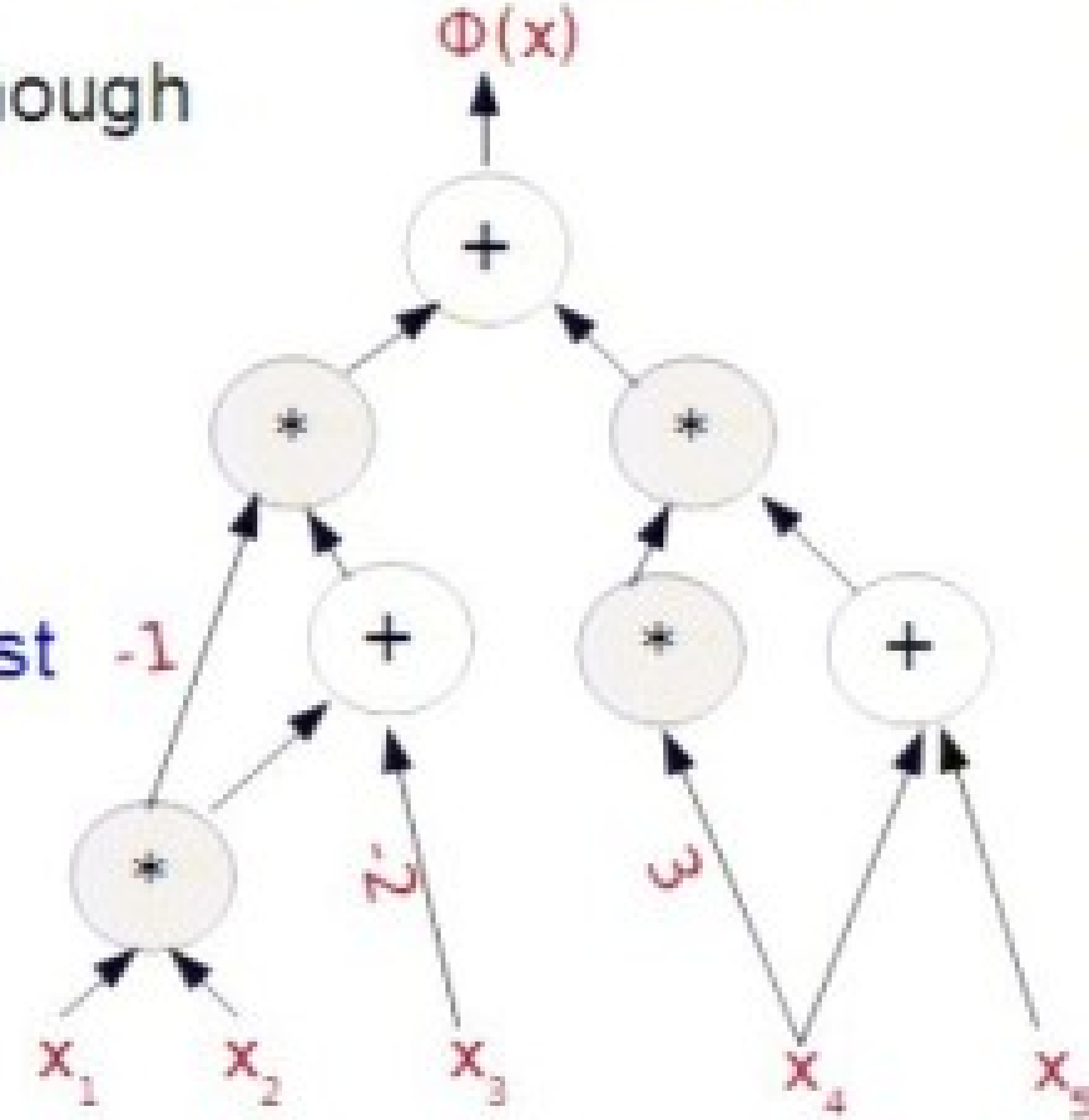
  - Identity testing.



Meta-Theorem: A solution of identity testing would answer the permanent question.

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Thank you!