IT ALL FITS TOGETHER:

Number Patterns that Foster

Number Sense in K-2 Students—

A Brain Based Model

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Abstract

The goal of this study is to improve the teaching of number sense to first-grade students. Mathematics activities in elementary school currently employ number patterns that are incompatible with the current research on brain-based learning and children's learning styles. Therefore, the purpose of this research is to explore whether a new set of number patterns can benefit students in developing good number sense. The author developed a logical and consistent set of number patterns that work like a puzzle. The study integrates these new patterns called *Number Puzzle Patterns* into an existing first-grade mathematics curriculum to find if the use of these patterns facilitates students' development of number sense. The students in the study are from a multicultural, socially and economically diverse public school, in which 48% of the students are ESL and 54% qualify for Free and Reduced Lunch. Data collection strategies include assessment interviews with each student, review of student work, and an interview with the teacher. The results show that after a year of exposure to *Number Puzzle Patterns,* students exhibit a higher level of understanding than the teacher and researcher expect in these students regarding the part-part-whole relationships of numbers and place value concepts. The data also show that students use *Number Puzzle Patterns* as figural representations of numbers and that they are able to recognize the numerosity of patterned arrangements of objects without counting. Finally, the study demonstrates that the consistency that *Number Puzzle Patterns* provides to the curriculum facilitates students' internalization and retention of their learning.

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RATIONALE

For twenty years I have worked in classrooms, tutored, and watched students struggle to understand math. I frequently asked myself, "Why are so many students having trouble? What do they have in common that makes math difficult for them to learn? Why do they dislike math?" With these questions in mind and after years of observations and discussions with teachers and students, I arrived at the following conclusions:

1. The students I assisted could not memorize number combinations and operations.
2. These children could not construct their own, effective math methods.
3. The students needing help were tactual/kinesthetic learners who resorted to solving math problems by counting on their fingers. Doing so, they made errors in their counting and became discouraged.
4. These students did not exhibit good *number sense.*

Number sense as defined by Sharon H. Ross, an assistant professor of mathematics at California State University; Chico is the frequent and flexible use of numerical part-whole relationships and place value to perform mental computations and numeric estimates (Ross, 1989). The development of number sense is central to the National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics* (NCTM 2000). Students, who struggle with basic number sense will, in turn, struggle with higher mathematics learning. It is like expecting someone to compose or play a melody without understanding the notes. Knowing the difficulty of such an undertaking and concerned with my students' disabilities in the areas of memorization and number sense, I racked my brain and searched my own experiences to ascertain what I could possibly do to help my students build the foundation for their future mathematics understanding.

Numerous activities in elementary mathematics classes employ dice, playing cards, and dominoes. When I was young, my family frequently engaged in games of this nature so the number patterns used on them were familiar to me. As I watched students use these materials, I wondered if the dot configurations helped students to "see" numbers, to form figural representations for numbers in their minds. I deduced that not all students were doing this because many were simply counting the dots they saw. Perhaps the opportunity to "see" number patterns is not enough for some students. Tactual/kinesthetic learners benefit from resources "that include writing, manipulative games, and puzzles"(Dunn, 1994, p.20). Maybe the common number patterns that represent one to ten do not make sense to the students I tutor. Imagine for yourself the common dice pattern for five:  . Now visualize the dice pattern for one:  . Without rearranging the dots, can you fit these two patterns together like a puzzle and construct the pattern for six:  ? No. Try another problem. Can you combine the dice patterns for three:  and one:  to build the dice pattern for four:  again without rearranging the dots? It is impossible. This trial exemplifies that the patterns on dice, playing cards, and dominoes are impractical to manipulate like a puzzle, a preferred learning activity of the tactual/kinesthetic learner. Because of this, I wonder, could number patterns that work like a puzzle benefit students in developing good number sense? Brain research on learning sheds light on this possibility.

The assertion of recent brain research that our minds search for meaning through patterning provides the impetus for my project. The brain innately perceives and generates patterns. It resists the imposition of meaningless patterns, such as isolated pieces of information that are unrelated to what makes sense to a person. Learners pattern, perceive, and create meaning all the time in one way or another. We cannot stop them, but we can influence their direction (Caine and Caine as cited in Bamburg, 1997). In light of this, are the common number patterns "meaningless" to tactual/kinesthetic learners? Do students' brains innately reject them? Could number patterns that work like a puzzle facilitate the direction in which students "perceive and create" meaning for numbers?

Another area of brain research that applies to this study is that the brain processes parts and wholes simultaneously. Therefore, when teachers overlook either parts or wholes, students experience enormous difficulty in learning. "Parts and wholes are conceptually interactive. They derive meaning from and give it to each other" (Caine and Caine as cited in Bamburg, 1997, p.5). When two of our common number patterns are put together, i.e. five and one, they are not "conceptually interactive". They do not result in the complete six pattern. Perhaps this makes it difficult for a child's brain to use these patterns to develop the numerical part-whole component of number sense.

Early elementary math instruction incorporates number patterns that seem to be incompatible with current brain research. Could a set of logical, consistent number patterns more in line with how young children's brains function i.e. that work like a puzzle, benefit students in developing good number sense? By addressing this question in my research, I hope to help those students who struggle to understand mathematics, as well as provoke thought on a common practice that may not be beneficial to all students. In my opinion, the current methods of teaching number sense promote inequity since they do not meet the learning needs of all students. Perhaps if we resolve this bias we can prevent more students from joining the "I hate math" group, stifling further development, and limiting their future contributions to society.

LITERATURE REVIEW

Introduction

"Research in mathematics education needs the new discoveries of those who roam the frontiers gathering empirical data. It also needs the discoveries of those who sift and aggregate the artifacts of the tribe. They, too, are our scouts and hunters" (Kilpatrick, 1985, p. 82). This literature review looks at some of the "artifacts" from the "tribe" of mathematics researchers. What has already been uncovered? The first artifact (section) is a historical overview of mathematics education. Following this is a section on cognitive development theories as they relate to mathematics understanding. The third section expounds on recent brain research concerning mathematics learning. The last two sections relate more specifically to young children's development of number sense. The first covers the currently available manipulative models to teach number sense. The second covers mathematics disabilities that inhibit the development of number sense. This literature review concludes with a reflection on these research findings as they relate to the number patterns used in early elementary mathematics education.

History of Mathematics Education

Behaviorism was the predominant learning theory driving mathematics education in the United States, until the early 1960s. The idea that formed its foundation was; "bonds" formed between a stimulus in the environment and the response of the student. For example, the stimulus 4 + 4 would elicit the response 8 and a bond of learning would form. Drill and practice, the major focus of mathematics instruction strengthened these bonds of learning (Knuth & Jones, 1991), and led to memorization of the number facts (Baroody, 1983). The National Research

Council dubbed the "learning" produced by this model—"mindless mimicry mathematics" (NRC, p.4 as cited in Battista, 1999).

The 1960s featured an abstract and conceptual focus of mathematics education termed "new math." Teachers introduced these methods to young children and found their impact to be less than expected. Thus, the era of "new math" was short lived. "Back to basics" became the cry of parents. Drill and practice became the focus again until the end of the 1970s (Knuth & Jones, 1991).

In the late 1970s, under the influence of cognitive science, the view of mathematics education shifted from "numbers and computations" to "math as problem-solving." The goals of learning became "problem solving, understanding the conceptual nature of the problem, (and) knowing when to employ skills and facts as tools to solve the problem" (Knuth & Jones, 1991, p.1).

During the 1970s and 1980s, researchers and practitioners developed a better idea of what it means for children to learn and understand math (Knuth & Jones, 1991). With this knowledge, a new movement to reform mathematics education began "in response to the documented failure of traditional methods of teaching mathematics, to the curriculum changes necessitated by the widespread availability of computing devices, and to a major paradigm shift in the scientific study of mathematics learning" (Battista, 1999, p. 426).

As part of this movement, two renowned educational groups released documents in 1989, which addressed the problems with past efforts in mathematics education and articulated a vision for the future. The National Council of Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics*—to "establish a broad framework to guide reform in school mathematics in the next decade" (NCTM, 1989, p. v). The

Mathematical Sciences Education Board (MSEB) released *Everybody Counts: A Report to the Nation on the Future of School Mathematics* (MSEB, 1989).

In 2000, the NCTM published an updated document titled *Principles and Standards for School Mathematics*. Its goal is to supply guidance and vision for today's K-12 mathematics education (NCTM, 2000, p. 11). It is with reflection on the following *Principles* of this document that I undertake this study:

> ***Equity.*** Excellence in mathematics education requires equity—high expectations and strong support for all students.
>
> ***Learning.*** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, 2000, p. 11).

Connecting the new to the known and developing understanding in <u>all</u> students are important components of today's mathematics education. The *Standards* of the NCTM document provide guidance to this end. Those that apply to my study are:

> ***Number and Operations.*** Instructional programs should enable students to understand numbers, ways of representing numbers, relationships among numbers, and number systems. All students should use multiple models to develop initial understandings of place value and the base-ten number system. All students should develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers (NCTM, 2000, p. 78)
>
> ***Representation.*** Instructional programs should enable all students to create and use representations to organize, record, and communicate mathematical ideas (NCTM, 2000, p. 136).

Today, the NCTM's *Principles and Standards for School Mathematics* provides a lighthouse in the historical storm affecting mathematics education. As will be seen in the following sections, knowledge of children's cognitive development and brain function will influence the future piloting of the ship of mathematics instruction.

Cognitive Development of Mathematics Understanding

Many theories exist about children's cognitive development. The three most frequently mentioned in current literature regarding mathematics education are Piagetian, Constructivist, and Learning Styles theory. John A. Van de Walle, Professor of Education at Virginia Commonwealth University and a member of the Board of Directors of the NCTM (1998) cites the applicable aspects of Piagetian theory:

> When the things we perceive are familiar to us or are things that we "know," we know them because they fit our previously developed understanding. Piaget referred to this mental activity as *assimilation*, fitting our experiences or perceptions into existing ideas. At other times, the things we perceive do not fit; something does not quite make sense. To understand this conflicting input and to relieve the *dissonance* (or *disequilibrium*), our ideas must be modified or new ideas must be created to make the current idea fit. This is the action of *accommodation*, a modification or growth of our cognitive framework that permits assimilation of the new idea (Labinowicz, 1980, 1985 as cited in Van de Walle, 1998, p. 22).

Debate permeates the literature about the validity of many of Piaget's theories and their application to mathematics education (Geary, 1994; Dehaene, 1997). At least, in his theory of accommodation and assimilation, Piaget supports the NCTM's emphasis on connecting the new to the old. The following paragraph explains how the cognitive development theory of constructivism applies Piaget's theory of accommodation and assimilation.

There is wide acceptance of the use of constructivist theory in mathematics education (Battista, 1999; NCTM, 2000; Clement, 1991; Cobb, et al., 1991; Cobb et al., 1992, as cited in Battista & Larson, 1994). Van de Walle (1998) expresses constructivism clearly:

> Networks of ideas that presently exist in the learner's mind determine how an idea might be constructed. These integrated networks, frequently referred to as cognitive schemas, are both the product of constructing knowledge and the tools with which new knowledge is constructed. The more connections with the existing network of ideas, the better the new ideas are understood. As learning occurs, the networks are rearranged, added to, or otherwise modified. When there is active, reflective thought, schemas are constantly being modified or changed so that ideas fit better with what is known (p. 25).

Mathematics makes more sense and is easier to remember and apply when students connect new knowledge to existing knowledge. The result is that these well-connected, conceptually grounded ideas are more readily accessible in new situations (Schoenfeld 1988, & Skemp 1976, as cited in NCTM, 2000). Through constructivist curricula, students retain ideas, in an ever-increasing store of well-integrated knowledge (Battista & Larson, 1994).

In spite of the apparent growth in influence of the constructivist view of mathematics learning there are some holdouts. David C. Geary, an associate professor in Experimental Psychology at the University of Missouri—Columbia, writes:

> To be sure, there is much to be gained by understanding social contextual influences on mathematical development (e.g., Saxe, 1991), but to assume that all development follows this route and to reject outright the idea that there are mechanistic changes in children's

cognitive growth is naïve. Moreover, in addition to building basic skills, mechanistic approaches to mathematical tasks probably do influence children's conceptual development (p. 264).

Geary (1994) points out that psychological research suggests that children need different teaching techniques to acquire both procedural skills and conceptual knowledge. Other researchers agree—students often forget or remember incorrectly over-practiced computational methods that are without understanding (Hiebert 1999; Kamii, Lewis, and Livingston, 1993; Hiebert and Lindquist 1990, as cited in NCTM, 2000) and, understanding without fluency can inhibit the problem-solving process (Thornton 1990, as cited in NCTM, 2000). As popular as constructivist theory is, it does not paint the whole picture of the development of mathematics understanding.

In need of mentioning is another area of cognitive development, Learning Styles theory: a multidimensional construct that includes each person's environmental, emotional, sociological, physiological, and psychological processing preferences. It suggests exactly how each person is likely to concentrate, process, internalize, and retain new and difficult information (Shaughnessy, 1998). The realms of education and psychology generally accept the existence of individual learning styles (Keefe et al., 1986, as cited in Languis, 1998). My study focuses on the physiological (auditory, visual, tactual, and/or kinesthetic) processing preferences underlying the development of number sense.

According to Rita and Kenneth Dunn, two prominent theorists and researchers on learning styles, young children are physiologically tactual/kinesthetic processors who learn by touching, feeling, moving, and experiencing. Unfortunately, most classroom instruction focuses on auditory and visual teaching strategies--telling (auditory), assigning readings (visual), or explaining and writing on a chalkboard (auditory and visual). Little of what happens instructionally in most classes responds to the tactual and kinesthetic learner. Once these students begin to fall behind scholastically, they lose confidence, feel defeated, or begin to resent school because of their repeated failures (Dunn et al., 1994).

Two decades of research (Dunn, 1990c, as cited in Dunn et al., 1994) have verified that many students who do not do well in school are tactual or kinesthetic learners. Tactual learners remember by handling or manipulating resources that teach required information and kinesthetic learners remember by moving and experiencing (Dunn et al., 1994). Dunn and Dunn believe that "teachers do not need to adapt to each child's style. Rather, they need to do the following…Have alternative instructional methods and resources to teach the identical information differently to students with diverse learning styles" (Shaughnessy, 1998, p. 141-145).

Discussions of learning styles theory have often left the impression that it is a complex, impractical, and costly way to individualize. Attention to learning styles is more than a way of individualizing. It can be a significant step towards the NCTM's Principle of ***equity*** in schools. "If the tactual/kinesthetic learner has less opportunity to learn that the auditory/visual learner, not only has that learner been shortchanged, but our society has been deprived of the optimum talents of that individual" (Hughes, in Guild & Garger, 1985, p. Forward). Consciously accommodating learning styles through variety in curriculum and instructional practice is a genuine acceptance of diversity (Guild & Garger, 1985).

Brain Research and Mathematics Learning

Advocates of brain-based education and learning styles convincingly demonstrate that accommodating the students' learning strengths and attending to ways the brain absorbs and processes information result in more effective learning (Guild, 1997). The ten years, 1990 to 2000, generated a significant increase in our understanding of the brain and how it works (Bamburg, 1997). Three areas of brain research are important to this study. The first is the finding that the brain has two different types of memory—a spatial memory system and a system for rote learning. The spatial memory system does not rest, is inexhaustible, does not need rehearsal, and allows for "instant memory" of experiences. The system for rote learning stores relatively unrelated information. The more separated information and skills are from prior knowledge and actual experience, the more dependence there needs to be upon rote memory and repetition (Caine and Caine, 1991, as cited in Bamburg, 1997). Teaching young children addition with paper, pencil, and Arabic numbers emphasizes this last type of memory.

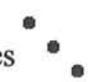
The second area of brain research is hemispheric specialization theory: the two hemispheres of the cerebral cortex process stimuli differently. The left hemisphere is better at linguistic processing tasks such as reading, speaking, analytical reasoning, and mental arithmetic (calculation). The right hemisphere is better at spatial tasks, such as recognizing faces, and music (Butler & Glass, 1974; Hines, 1975; Kimura, 1967; Levy, Trevarthen, & Sperry, 1972; Marcel, Katz, & Smith, 1974; McGlone & Davidson, 1973; Yeni-Homshian, Isenberg, & Goldberg, 1975; as cited in Wheatley, Mitchell, Frankland & Kraft, 1978).

Using an electroencephalograph (EEG) to highlight brain activity, researchers found that formal-operational persons (11 to 12 years old and onwards) showed significantly more left hemisphere processing than concrete-operational, younger persons (7 to 11 years old) showed (Dilling, Wheatley, & Mitchell, 1976 as cited in Wheatley et al., 1978). The authors hypothesize that pre-operational children (2 to 7 years old) would be similar to concrete-operational children in EEG activity on spatial tasks. In contrast, they expect concrete-operational, older children would show more left hemisphere processing than pre-operational children would on logical tasks. This study supports the idea that the younger the child, the more he, or she uses the right or spatial side of the brain.

An analysis of the literature suggests that the right, spatial oriented hemisphere is the "leading hemisphere" in most children (Wheatley et al., 1978). The authors cite Harris (1975) who states, "…we might conclude that the human infant is neuroanatomically (sic) disposed to effective spatial learning" (p. 24). The importance of this finding for mathematics education suggests that programs that provide significant opportunities for exploration, nonverbal expression, hands-on activities, and multi-sensory learning, because they employ the right hemisphere, may enhance students' mathematical understanding (Wheatley et al., 1978).

The third area of brain research stems from the work of Prof. Stanlis Dehaene and Prof. Elizabeth Spelke of the Massachusetts Institute of Technology. Using brain scanner techniques, they uncovered the first hard evidence that two very different modes of activity—handled by different brain areas—underlie our inborn capacity for mathematics. Exact calculations—which tapped the brain's verbal mathematical mode—lit up the volunteers' left frontal lobe, the brain area known to make associations between words. However, mathematical estimation—the analogue mode—involved the left and right parietal lobes, responsible for visual and spatial representations (Highfield, 1999). Two recent books: *The Number Sense*, by Stanlis Dehaene and *What Counts: How Every Brain is Hardwired for Math*, by Brian Butterworth, a British cognitive neural-psychologist, discuss in depth these areas of brain development. The research application for my study is the mathematical ability- *subitizing,* found to be in the left parietal lobe.

The term *subitize*, from the Latin word meaning *suddenly*, refers to the quick, confident, and accurate report of the numerosity of a group of objects presented for a short time (Mandler & Shebo, 1982). In elementary terms, it is instantly seeing how many. Subitizing implies that the quantity awareness of groups of one to three is declarative knowledge in a way that differs from counting. If a person looks very quickly at a picture of three dots, he or she can declare that there are three, without being conscious of having to count the dots. The definition can extend to include the recognition of familiar geometric patterns such as four in a square or five in the die configuration (Fischer, 1992).

There is debate over *how* human beings subitize, at what age this ability manifests itself, and at what numerosity the ability disappears. "Subitizing is defined by systematic changes in the slope for judgements of numerosity; it is not a concept disputable on theoretical grounds… to determine what processes are involved during subitizing is indeed a theoretical enterprise" (Mandler & Shebo, 1982, p. 2). The scope of this study does not include such an undertaking of theory. However, one important foundation is the indisputable, innate nature of subitizing. According to Brian Butterworth, our perception of small numerosities is as innate and as automatic as is our perception of color (Sherman, 1999). As one declares the fire-truck red without conscious thought of how he or she decided upon red, one similarly declares  to be three without stopping to count. Butterworth (1999) claims that humans are born with brain circuits specialized for identifying small numerosities. He calls these circuits the "Number Module." In a review of Butterworth's book, John Yandell writes, "So is this yet another attempt to explain human difference in terms of the genetic coding which we receive? Not at all. 'Everyone counts' is the central, democratic and enthusiastically delivered message… this awareness of number is the foundation-stone from which subsequent mathematical ability is built" (Yandell, 1999).

Douglas H. Clements, a mathematics education professor at the State University of New York, divides subitizing into two categories: *perceptual subitizing* and *conceptual subitizing*. Perceptual subitizing includes the innate ability found in animals and humans to "see 3" without any prior mathematical learning. Conceptual subitizing refers to ones ability to see an eight-dot domino and "just know" it is eight. People recognize the number pattern for eight as a composite of parts, and as a whole. Conceptual subitizing is a valuable component of number sense. Students can use pattern recognition to discover essential properties of number. They can develop such capabilities as unitizing, and composing and decomposing numbers, as well as their understanding of arithmetic and place value (Clements, 1999). Children "may subitize only small numbers at first. Such actions, however, can be stepping-stones to constructing more sophisticated procedures with larger numbers" (p. 401). Clements claims that students who cannot subitize conceptually are "handicapped" in learning mathematics.

In contrast to how they perceive three, 6-year-olds cannot subitize four dots in a row. When presented with such a linear array, there is no immediate comprehension of four. Instead, young children will usually count one by one. Four is easier to identify when presented in a geometric pattern such as a square (Fischer, 1992). The spatial arrangement of sets influences how difficult they are to subitize. Children usually find rectangular arrangements easiest (Beckwith and Restle 1966; Wang, Resnick, and Boozer 1971 as cited in Clements, 1999). If the arrangement does not lend itself to grouping, people of any age have more difficulty with larger sets (Brownell 1928 as cited in Clements, 1999).

A study by Mandler and Shebo shows that learned patterns can improve the accuracy of subitizing above three. Furthermore, it should be possible to teach the use of larger canonical patterns for even more efficient approaches to the numerosity task (Mandler & Shebo, 1982). This understanding of young children's apprehension of three and four can help pinpoint the easiest patterns to learn, which remains an area of investigation for educators today despite its long history (Fischer, 1992).

Across all classroom activities, students need pictures of numerosities that encourage conceptual subitizing. Groups to be subitized should consist of simple forms, such as groups of circles or squares, regular mostly symmetrical arrangements, and good figure-ground contrast (Clements, 1999; Elkind, 1964). Unfortunately, mathematics textbooks often present sets that discourage subitizing (Carper 1942; Dawson 1953 as cited in Clements, 1999). Complexity that hinders conceptual subitizing increases errors and encourages simple one-by-one counting.

In the effort to foster conceptual subitizing, Clements advocates setting up experiences, guiding investigations, and sometimes telling. He suggests that students gain a familiarity with regular patterns by playing games that use dice or dominoes. However, he gives no rational for how these patterns meet his requirements for developing conceptual subitizing (Clements, 1999).

Research claims that subitizing is an important element in children's development of mathematics understanding. However, a hole exists in how best to use this ability in mathematics instruction. Perhaps the integration of the three areas of brain research, two memory systems, hemispheric specialization, and subitizing could fill this void.

Number Sense

The last three sections of this literature review deal with the specific topic of number sense. Number sense is a skill supporting more advanced mathematics concepts. Students with good number sense recognize multiple ways to solve a problem. This leads to greater confidence in their mathematical ability, which protects them from the all too familiar math anxiety (Howden, 1989). Mathematics education must build a strong understanding of number sense. A deficit in this area negatively affects a student throughout life (Baroody, 1983).

The National Council of Teachers of Mathematics and the Mathematical Sciences Education Board view the development of number sense as the most important foundation of mathematics learning (NCTM, 1989, 2000; MSEB, 1989). Flexibility in composing and decomposing numbers, ways to represent numbers, and the understanding of place-value are the number sense elements addressed in my study.

How Number Sense Develops

Counting is the major activity in children's early mathematics learning (Fuson, 1988; Carpenter & Moser, 1983; Ilg & Ames, 1951; Siegler, 1986, 1987; Washburne & Vogel, 1928; Wheeler, 1939; Geary et al., 1991; as cited in Geary, 1994 and Ashcraft, 1982; Resnick & Ford, 1981; as cited in Baroody, 1983). However, does it contribute to the development of number sense as outlined by the NCTM? Some researchers do not think so. "More relationships must be created for children to develop what is known as number sense, a flexible concept of number not completely tied to counting" (Van de Walle, 1998, p. 100). "Might it be that when it comes time to transcend the early knowledge built on the counting principles these principles will hinder progress almost as much as they first promoted it?" (Gelman & Meck, 1992, p. 187). "Very early teaching of counting can be fairly costly… In general, children construct number regardless of path, but in some cases, the long-term consequences of initial choices may weigh heavily in the balance" (Brissiaud, 1992, p.65). "Counting is at best a narrow and limited type of activity within which to "see" the rich relations that are number in its fullest sense. A program based largely on counting is simply inadequate to help children construct a broad sense of number (Van de Walle, 1988, p. 15). Even the NCTM encourages teachers to move students away from counting as a strategy for problem solving (NCTM, 2000).

If not counting, what do students need to develop number sense? According to Van de Walle, students need to develop:

(1) *Spatial relationships*: Recognizing the numerosity of sets of objects in patterned arrangements without counting. This enables thinking of a number as a "single unit" rather than a count.

(2) *One and two more, one, and two less:* Children should *know* that seven, for example, is one more than six and two less than nine.

(3) *Anchors or "benchmarks" of 5 and 10:* Because 10 plays such a large role in our numeration system and two fives make up 10; it is very useful to relate the numbers 1 to 10 to the important anchors of 5 and 10.

(4) *Part-part-whole relationships:* To conceptualize a number as formed of two or more parts is the most important relationship a child can develop about numbers. For example, 7 separates into 4 + 3, 5 + 2, 6 + 1, and 7 + 0 (Van de Walle, 1998, p. 100).

Sharon H. Ross describes the impact of understanding this fourth relationship:

> Eventually their (first-graders') thinking allows them mentally to compose wholes from their component parts. They can also decompose whole quantities into parts, and perhaps rearrange the parts and recompose the whole quantity, confident all the while that the quantity of the whole has not changed (Ross, 1989, p. 47).

This is the major conceptual achievement of the early school years (Resnick, 1983, as cited in Van de Walle, 1998). The more of these relationships children create in their minds, the broader is their concept of the number or the better is their number "sense" (Van de Walle, 1998).

Unfortunately, many curriculums place almost no emphasis on these relationships. When children can count, and read and write numerals, the next step is to begin addition and subtraction. Counting a set of objects will never cause children to either focus on the relationship of two parts, or on what the size of those parts might be. Therefore, activities that focus on quantities in terms of two or more parts are crucial to children's development of number sense. Otherwise, many children simply continue to use counting as their principal means of accessing quantity (Van de Walle, 1998). The following section of this literature review illustrates how the current manipulative models for early elementary mathematics education encourage counting instead of the part-part-whole relationships advocated by mathematics researchers.

Common Spatial Activities to Teach Number Sense

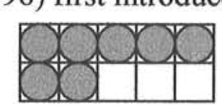
Lesh, Post, and Behr (1987, as cited in Van de Walle, 1998) talk about five "representations" for mathematical concepts: manipulative models, pictures, written symbols, oral language, and real-world situations. A model for a mathematical concept refers to any object, picture, or drawing on which a student imposes the relationship for that concept. Models give learners something to visualize, manipulate, think about, talk about, and reason with (Thompson, 1984). Although the materials themselves do not show number, the activities using these representations enhance the construction of varied relations in children's minds (Van de Walle, 1998). According to the "Representation Standard" of the *Principles and Standards for School Mathematics*, "Instructional programs ... should enable all students to – use representations to model and interpret physical, social, and mathematical phenomena" (p. 67). These representations can make mathematical ideas more concrete, helping students to organize their thinking and reflect on their work (NCTM, 2000).

The intent of activities with models is to have children recognize set number patterns without counting. These patterns begin to be related mentally one to another and enhance part-part-whole relations. Offering a rich variety of activities can help children construct these many relations (Van de Walle, 1998).

What are these models used in the activities of elementary mathematics education? As can be seen in the following section, they come in many forms. The most common are: children's own fingers; paper and pencil drawings of ten frames or sticks and dots; and manufactured items such as colored rods, base ten blocks, the abacus, dice, dominoes, and playing cards.

The most basic manipulative readily available to students is their fingers. Children use their fingers for counting, however, as a model for mathematical relationships, students should use their hands for naming quantities, not for counting (Cotter, 2000; Brissiaud, 1992).

In a study supporting first graders' ten-structured thinking in an urban classroom Fuson, Smith, & Lo Cicero (1997) used another convenient model, pencil drawn ten-sticks and dots. Students drew vertical sticks to represent the tens and horizontal dots to represent the ones. The teacher modeled various ways to count in order for students to check their work. Because this model necessitated counting, several children made minor execution (miscounting) errors each day.

In 1974, Robert Wirtz (as cited in Van de Walle, 1998) first introduced the model of ten-frames. A ten-frame is composed of a five-by-five grid  with counters or dots placed in adjacent spaces. Counters mark the squares one at a time, beginning at the upper left and proceeding across until filling the first row. The second row receives counters in the same manner. Ten frames become cumbersome for quantities greater than about thirty. However, according to Van de Walle (1984), this model has several advantages. One is that it encourages children to think of ten as two fives. Another is that every number displayed in the frame references both five and ten. "A set of seven constructed in a ten frame is instantly recognized as two more than five, as a set of two and five, and as three less than ten" (p. 19). In light of brain research, the facts cannot be "instantly recognized" because young children are unable to subitize five dots in a row. They still have to count.

Wirtz developed kits to "help children use the patterns and connections in arithmetic to successfully memorize the basic facts and move beyond" (Wirtz, 1977, Introduction). He compared memorizing basic facts to memorizing "Roses are red, violets are blue; ________ is sweet, ____ ____ are you." If you have memorized this rhyme, you can fill in the blanks. In the same way, you can fill in the blank for "Six + seven = ____" if you have memorized that fact (Wirtz, 1977). Historically, this behaviorist approach to learning lost its support in the 1960's. In addition, this type of learning relies on the linguistic, left side of the brain, not the predominant mode of learning for young children.

Tally or craft sticks provide similar activities. Students represent one to four by placing the sticks vertically a little ways apart. To represent five, they place a stick horizontally across four sticks, much as adults do when they make tally marks. After each group of ten, students make a new row (Cotter, 2000). Because a number such as sixty requires six rows, which young children cannot subitize, this model eventually necessitates counting.

Rods of varying lengths and colors, sometimes marked in units, sometimes not, often represent quantities in mathematics instruction. According to Joan A. Cotter, writing for *Teaching Children Mathematics* (2000), these models have some drawbacks, for example, with rods of five or greater, only the color can be visualized. Combining two rods does not alone show the sum; the child must compare with a third rod or count. When the rods are marked in units, they compel counting because rods five through ten cannot be subitized.

Base ten blocks, another popular manipulative, are proportional, pre-grouped models for ones, tens, hundreds, and thousands. The ten-block is physically ten times longer than the one unit is, a hundred-block is ten times wider than the ten, and a thousand-block forms a cube, ten times thicker than a hundred block. Although base ten blocks are easy to use and can model large numbers, children cannot take them apart or put them together. This increases the potential that children will manipulate them without reflecting on the ten-to-one relationships (Van de Walle, 1998). Due to their configuration of ten units in a linear array, children cannot subitize base ten blocks and must resort to counting.

The AL abacus is a special, double-sided abacus with two groups of five beads in contrasting colors strung on each of ten wires. Children enter quantities by moving beads to the left and reading the quantities from left to right. Reversing the colors after five rows helps the children subitize the number of tens. The quantity 7-ten 4 (74), for example, is seven rows of ten beads and four beads in the next row. Children can enter and visualize any quantity from one to one hundred without counting. The children can also construct hundreds by stacking abacuses. For example to represent the quantity three hundred, three abacuses can be stacked (Cotter, 2000). Since research shows that children cannot subitize a linear array of four or more, the effectiveness of the AL abacus is questionable.

Finally, the models dice, dominoes, and playing cards frequently appear in early elementary mathematics classrooms. A search of the ERIC database found numerous references to the use of these models in mathematics education. However, no research revealed why the particular dot configurations used on these items are useful to math cognition. According to Cotter (2000), manipulatives should enhance young children's abilities to visualize number through appropriate groupings. Domino dot patterns have a serious limitation because they do not have an additive nature. For example, adding one dot to the five-dot pattern on a domino does not result in the six-dot pattern (Cotter, 2000). Obviously, the same applies to the dot patterns on dice and playing cards. These manipulative models encourage counting, because children cannot subitize their patterns.

In summary of the references to representation, there is sufficient research to indicate that models presenting collectible multi-units are helpful to many children in understanding numbers. Unfortunately, there is little research or agreement about just how these models should be used or which ones might be best for which children (see Baroody, 1990; Cobb, 1987b; Davis, 1984; Fuson, 1990a, 1990b; Hiebert, 1984; Kamii & Joseph, 1988; Labinowicz, 1985; and Ross, 1988, for discussions of some of the issues, as cited in Fuson, 1992). Much of the research examines only sequenced, multi-units (linear arrays) and, thus, may underestimate children's ability to solve the given problem if researchers studied patterned, multi-units instead (Fuson, 1992).

Mathematical Learning Disabilities

Regardless of how much experience they have with common manipulative models, many students are not learning the mathematics they need to learn (Kenney and Silver 1997; Mullis et al. 1997, 1998; Beaton et al. 1996 as cited in NCTM, 2000). Some students in grades 4 to 10 are still counting on their fingers, making marks to *count on*, or simply guessing at answers (Van de

Walle, 1998). The reason according to Van de Walle is simply that they have not developed efficient methods of producing a fast answer.

The learning of basic arithmetic facts appears to occur automatically for most children through counting (Siegler, 1986, as cited in Geary, 1994). Eventually direct retrieval of answers from long-term memory replaces counting (Fuson, 1992, as cited in Macaruso & Sokol, 1998). Most children typically make this shift to direct memory retrieval between grades 4 and 6. Unfortunately, the automatic recall of basic arithmetic facts is problematic for many students with mathematics learning disabilities (e.g. Garnett & Fleischner, 1983: Russell & Ginsburg, 1984, as cited in Macaruso & Sokol, 1998). Some children do not automatically develop memory representations for basic arithmetic facts, even after years of using counting or other types of strategies. These students have a semantic-memory form of mathematical disability (MD) (Geary et al., 1987, as cited in Geary, 1994). This condition is associated with the infrequent use of arithmetic-fact retrieval and a high rate of errors when retrieving facts from long-term memory (Geary, 1994). Research shows that MD second-graders count more slowly, produce more errors, and are more variable in their application of counting strategies than normal second-graders (Geary, Widaman, Little, & Cormier, 1987, as cited in Macaruso & Sokol, 1998).

Many children with this semantic-memory form of MD appear to have normal visual-spatial skills (Rourke & Finlayson, 1978, as cited in Geary, 1994). As brain research shows, the cognitive systems that underlie these skills and those that support arithmetic-fact retrieval are different. Fortunately for MD children, it is possible that teaching them a method for performing basic calculations that bypasses the apparently defective semantic-memory system might lead to long-term improvements in their basic ability to perform calculations (Rozin et al., 1971, as cited in Geary, 1994).

Conclusion

Research on number sense is a complex and vast realm. A goal of this literature review is to pull some of its various elements together. "Integration is about making connections between ideas, theories and experience…the intent is to make others think about and possibly re-evaluate what they have hitherto taken to be unquestionable knowledge" (Hart, 1998, p. 8). My research attempts to integrate the following:

1. Children must construct their own understanding of number relationships.
2. For learning to occur, new knowledge must connect to previous understanding.
3. Young children are tactual/kinesthetic learners.
4. Students learn best when teachers present information in a spatial sense, to the right side of their brains.
5. Children have an innate ability to subitize up to four objects (in a square pattern) without having to count.
6. Children cannot subitize linear arrays over three.
7. Manipulative models are important in mathematics education; however, most common models use linear arrays and necessitate counting.

With the integration of these ideas and theories, this literature review sheds light on the "unquestionable knowledge" that the patterns of manipulative models, dice, playing cards, and dominoes are beneficial to young students' math cognition. The goal of my research remains, could number patterns that work like a puzzle, benefit students in the development of number sense?

VISION STATEMENT

I will do an action research study of the students in a first-grade classroom that used an alternative, puzzle-like, number pattern system during the year. I will review students' work throughout the year, interview the teacher, and perform an assessment interview with each student. I hope to find support and enthusiasm for the use of an alternative system of number patterns in the teaching of first-grade mathematics.

METHODOLOGY

Design- Action Research

In my study, I wanted to know if I could teach the components of number sense—part-part-whole relationships, the representation of numbers, and place-value—in a more effective manner. According to Richard Sagor, an Assistant Professor of Education at Washington State University, research that is called *action research* "is conducted by people who want to do something to improve their own situation. Action researchers undertake a study because they want to know whether they can do something in a better way" (Sagor, 1992, p. 7). Action research includes three related stages of action: initiating action, monitoring and adjusting action, and evaluating action.

Stage One- Initiating Action

I began the first stage, initiating action, by developing a set of number patterns that I felt was compatible with current brain research and young children's learning styles (Appendix A). These patterns fit together like a puzzle, in a logical, consistent manner. Their design also incorporates children's innate ability to subitize four objects in a square pattern. I named this system *Number Puzzle Patterns* (hereafter referred to as *NPP*).

Stage Two- Monitoring and Adjusting Action

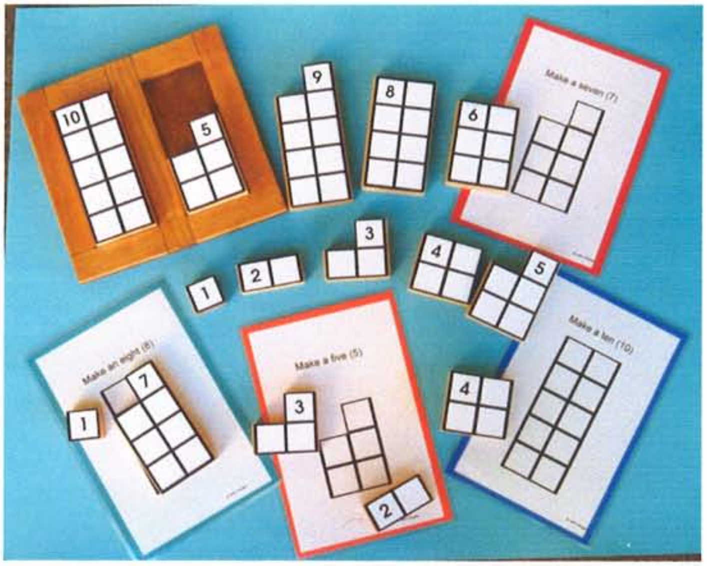
I began the second stage of my action research, monitoring and adjusting action, in September 2000 in a multicultural, socially and economically diverse public school, in which 48% of the students are ESL and 54% qualify for Free and Reduced Lunch. Subjects were from a first-grade math class that combined students from three different first-second grade split classes. The twenty-three subjects consisted of seventeen boys and six girls. One student was a child with special needs on an Individual Education Plan. Eleven of the students spoke another language at home. I did not select my sample, but used the class in which I was doing my student teaching.

The teacher had eleven years of experience teaching kindergarten and sixteen years teaching first grade. In the past, she was involved in a math support group that met every month during the school year. She participated in math workshops, learning from math educators and curriculum specialists Marilyn Burns and Kathy Richardson about different ways to incorporate mathematical thinking into the classroom. She also participated in an early childhood education group, which splintered into a collegial math group that met monthly for three years.

The teacher arranged the materials in the classroom to be easily accessible to the students. Manipulatives were stored in bins at floor level and included Unifix Cubes, Multilink cubes, one- inch cubed wooden blocks, small plastic bears, rabbits, dinosaurs, pattern blocks, geoboards, calculators, and geo blocks. *Number Puzzle Pattern* materials consisted of wooden *NPP* blocks, egg cartons cut in *NPP*, paper *NPP* ten frames, and magnetic *NPP* pieces.

Students met for 90 minutes four days a week. There were two teachers in the room with an occasional aide or parent helper. I created materials to adapt the TERC (Technical Education Research Centers) mathematics curriculum (Kliman & Russell, 1998) to incorporate *NPP*. I made dice, playing cards, dominoes, building blocks, and other manipulative models with *NPP* (see Figures 2 & 3).

The teacher and I carried out the activities in the TERC curriculum, substituting *NPP* materials for the canonical and manufactured models. TERC worksheets and activities were adapted to reinforce learning of *NPP* by placing counters in *NPP* instead of in random order.



- Figure 2

- Figure 3

We recorded number representations in the classroom in *NPP*, i.e. how many days in school and number of points earned (see Appendix A for examples of figural representations of numbers). For reference, we displayed a *NPP* frieze on the wall and *NPP* labels on students' desks. With these materials, students learned to subitize the four pattern as one "flower" because the shape of four elements in a square looks like a flower. From there, five becomes a flower and one extra, six a flower and two extra, seven a flower and three extra, eight two flowers, nine two flowers and one extra, and ten two flowers and two extra. The units in the tens place are displayed in the same manner, so students only had to learn the patterns to ten. (See Appendix A for examples). The incorporation of *NPP* into the curriculum required frequent monitoring of students' responses and adjusting of materials to enhance their learning.

Stage Three- Evaluating Action

In May 2001, I began the final stage of my research, evaluating action. I wanted to know if our inclusion of *NPP* in the curriculum had any effect on our students' development of number sense. Some critics believe that it is impossible for a teacher to obtain an outside-looking-in perspective. Sagor (1992), claims, "Not so! Action researchers have within themselves many of the means necessary to take a fresh look at the world they are immersed in" (p. 33-34).

Triangulation, the use of multiple methods to study the same subject, included student assessment interviews, student work, and an interview with the teacher. Triangulation is important in a qualitative study because of the uncontrolled variables in a single source of data. If these multiple sources of data showed the same picture, then that picture was likely a valid portrait of the effect of *NPP* (Sagor, 1992). By using these three data collection strategies, I hoped to find evidence that supported the use of *NPP* in first grade mathematics. As expressed by Sagor (1992), the main function of these data collection strategies is to "allay concerns or to at least create cognitive dissonance for the resisters" (p. 28) about the use of *NPP* in mathematics education.

I began my data collection by looking at student work (Bogdan & Biklen, 1998). The class kept portfolios with work collected throughout the year, consequently, I had access to students' math papers and assessments. I selected nine assignments from the portfolios to reflect development and use of *NPP* over the year. I included the three story problem assessments mandated by the curriculum. For various reasons such as absences, not all nine examples were available for every student. I evaluated the assignments based on correct answers and correct use of *NPP*.

I then conducted a semi-structured, open-ended interview with the classroom teacher. Qualitative interviews can vary in the degree to which they are structured (Bogdan & Biklen, 1998). Therefore, the conversation focused on the particular topics of number sense development and *NPP*. Some general questions guided the discussion (Appendix B). This technique was used to gather descriptive data in the teacher's own words so that I could develop insights on how she interprets the impact of *NPP* on student learning (Bogdan & Biklen, 1998). Cochran-Smith and Lytle (1999) emphasize "the importance of teachers as expert knowers about their own students" (p. 16). I taped the interview, transcribed the data, and then gave her the transcription to review and to make any further comments.

In April 2001, I interviewed each student on two individually administered, thirty minute, assessments designed to measure number sense and the effect of *NPP*. I received all twenty-three of the students' human consent forms and was able to assess all of the students. The assessment interviews modeled a study by Florence E. Fischer (Fischer, 1990), and included questions on number concepts (Appendix C) and place value (Appendix D). Problem-solving questions were included on both assessments. Because I wanted to control for reading difficulty, the language was purposefully simple and all the problems used names of students in the class. The types of problems included: Join Result Unknown (2), Join Change Unknown, Separate Result Unknown, Separate Change Unknown, and Separate Initial Unknown (Van de Walle, 1998). I administered the assessment interviews in the back of the classroom separated from the other students by a screen. The noise level was a distraction to the students, but I felt it important to keep them in a comfortable setting to relieve any anxiety. Surprisingly, I did not notice anxiety in any of the students. I attribute this to their familiarity with myself and to their enjoyment of math.

A computerized Power Point presentation tested subitizing of the *NPP*. The *NPP* consisting of yellow dots on a blue background flashed randomly on the screen with Power Point's minimum transition time of 0:00 seconds. I asked students to verbally respond with the number of dots they perceived and then recorded their responses.

Data Analysis

After sifting and sorting the data collected from the teacher interview, students' assessment interviews, and the students' work six main themes emerged.

1. Internalization- with the consistency that *NPP* provided to the curriculum, students internalized and retained the *NPP*, providing evidence that it made sense to their brains.
2. Student Confidence- students were confident in their mathematics abilities. Their dispositions and attitudes concerning math activities were highly positive.
3. Representation- students and the teacher used *NPP* as figural representations of numbers.
4. Pattern Recognition- students recognized the numerosity of sets of objects in patterned arrangements without counting.
5. Part-Part-Whole Understanding- students exhibited understanding of the part-part-whole relationships of numbers.
6. Place Value Understanding- students exhibited an understanding of place value.

The following section outlines the data supporting these themes. Italicized words are mine and I changed all the names.

Results and Interpretations

1. **Internalization- with the consistency and organization that *NPP* provided to the curriculum, students internalized and retained the *NPP*, providing evidence that it made sense to their brains.**

Evidence from the teacher interview:

> Some children come to school being able to do math really well and they have a great sense of pride and knowledge of what they think they can do. Others come to school with a great sense of knowledge about letters, sounds, words, and reading. However, I do not often see it come together in the same child at the same time and I have always wondered why. After working with *Number Puzzle Patterns*, I now see it is because we have never really provided something in math for more kids to know about numbers, about seeing them.

The teacher's comments coincide with the research of Rita and Kenneth Dunn (Dunn et al., 1994) that young children are tactual/kinesthetic processors. In addition, I believe that our culture has tried to provide children with a means to learn number sense through their strongest modality. However, something was missing. In our class, *NPP* seemed to provide this missing element of mathematics education.

The teacher's observations supported what the NCTM (2000) considers the importance of representations. She observed that *NPP* provided a means for organization and consistent reflection. In her words:

> It has definitely affected my teaching. It does not replace a curriculum, however, I think I have a clearer understanding of what I am trying to teach. It has cleared up the little things that make the lessons flow. Because there is something that is

> consistent for me, it is consistent for the students. I am not changing the rules. I am not changing the dynamics, even though the activities and the goals are changed. There is no variation in the ultimate perception of how you view a number. It does not change. It is going to be there in whatever materials you use, in some way, for reflection. Because we are working with very young, concrete, children we need to have something that is always going to be the same as their base of reference. If we change it every time we give them a new problem, some of the students are going to get it and others are going to wonder what world they live in. Out of confusion comes a way to organize!

Besides the consistency and organization that *NPP* provided to the math curriculum, the teacher felt that *NPP* enabled students to see numbers in a way that made sense to their brains. When I asked if she had found a way of instruction that made sense to young children, her response was:

> Your pattern does Lynn! It truly does. It took me three years. And I said, 'oh' and then I'd say, 'now why?' and then I'd think, 'hmm' and then I'd see it and then we'd work with it. This year it really took its firmest hold because of the consistency of making sure that we reflectively planned every lesson based on the *NPP* system. When you take the time, you can turn any lesson into a gem. I can't think of life without it now.

With the amount of math education, expertise, and experience, this teacher accumulated over 27 years; it was surprising that *NPP* had such a huge impact on her teaching and her students. Because *NPP* is a spatial based system, her feeling that *NPP* made sense to students supported the literature suggesting that programs that employ the right hemisphere may enhance students' mathematical understanding (Wheatley et al., 1978). Her comments also demonstrated that accommodating the students' learning strengths and attending to ways the brain absorbs and processes information result in more effective learning (Guild, 1997).

In my literature review, I cited Rozin (Rozin et al., 1971, as cited in Geary, 1994) concerning students with the semantic-memory form of mathematical disability (MD). He thought it possible that teaching MD students a method for performing basic calculations that bypasses the semantic-memory system might lead to long-term improvements in their basic ability to perform calculations (Rozin et al., 1971, as cited in Geary, 1994). Further comments by the teacher suggested that *NPP* might be this method:

> I always wondered why students did not understand when we threw dice that were typically dotted. Why did they not comprehend place value? Why did they not get the shifting of numbers? What was so difficult? It was not so much that they could not do it, but that they did not retain it. This system enables children to retain their learning over a long period. It is something that they can internalize and come back to repeatedly. I hear kids talking about it and say, 'Oh, I get it. I can put this together. I can visualize it in my head.' They are not even looking at the pattern. They have it in their head. They have it inside of themselves.

The following section, in the students' words, confirms the teacher's observations that *NPP* made sense to students.

Evidence from student responses during the assessment interviews:

Can you count these tiles?

"There's six already! There is a three and a three. I know what a six looks like."

How do you know this is a five?

"Because I know how five looks."

How do you know there are seven tiles?

"Because I think in my head." W*hat are you thinking in your head?* "About the pattern."

How do you know you have fourteen?

"I know what the pattern looks like for fourteen."

Do these two groups have the same number of tiles?

"Yes, it's in the number pattern just farther out."

There are seven squares on this paper but some of them are hiding. Can you tell me how many are hiding?

"I saw 4 at the bottom so it must be three at the top."

"Three, because I remember the pattern."

"Three. Because here there's a five, here there's a six and here there's a seven" (he says as he points to the spots where the missing squares would be in the *NPP*).

During the assessment interviews, the students frequently spoke of "the pattern." They said they saw the pattern in their mind. Perhaps because *NPP* made sense to them and gave them a tool to understand mathematics, our students' attitudes in math class were very upbeat, as seen in the following section.

2. Student Confidence- students were confident in their mathematical abilities. Their dispositions and attitudes in the classroom were highly positive.

Evidence from the teacher interview:

"Aside from some individual differences, levels of immaturity, and daily fluctuations of social and emotional needs, I would say that most of the kids are highly successful. Probably more than I've ever seen first graders able to do." The teacher spoke of two students as prime examples of this success:

> Jennifer came in with very confused thoughts on math. She was a premature baby and is developmentally delayed. This is just amazing. She has the math now, she can do it, and she feels highly successful. When Brad came in, he could not write. He still cannot do a great job. However, he has all the math skills in his head. We have kids coming from very different directions and they can all be successful.

To highlight the teacher's observations, the following is a transcription of Jennifer's comments during her assessment interview:

Can you count these tiles?

"Six!"

How do you know there are six?

"Because, I didn't count them. I looked with my eyes and then I said, hmm, it's six."

Can you show me a group of seven tiles? (She makes a seven *NPP*) *How do you know it is seven?*

"Because this is a five and then two more makes seven."

Can you show me a group of twelve tiles? (She makes a display of twelve in NPP)

Are you sure there are twelve?

"Yeah. Because I counted them in my head."

Which one of these cards has the most squares?

(Pointing to the cards) "Because that one is a seven and that one is a five."

How do you know that is a seven and that is a five?

"Because I didn't count any of them I just said (sigh) this is a five, that's a seven so that ones more."

Can you put these four cards in order from the smallest number to the biggest number?

"Of course!"

How do you know that is the right order?

"Because that's a four and a five and a six and a seven."

How do you know that is a four, a five, a six, and a seven? Did you count them?

"No."

Do you know how many little cubes there are?

"Seven."

How do you know there are seven?

"Because there's four and three left over." (She hums while she waits!)

Does this one have the same number of cubes?

Immediately upon seeing it, she answers, "Yep!"

How do you know?

"Because," then she says, "Uh oh, no." (Long pause) "Yeah!"

How do you know?

"Cuz, that's a five and that's a two."

How many dots do you see?

"Eight."

How do you know that is eight?

"Because I put them all together, I said, you put them like an eight."

Now watch what I do. Is there still the same number?

"Yes. Because you changed five of these and three left over. Hee, hee, hee, I'm smart! I want to do it again!"

This card has seven black squares on it but some of them are hiding. Can you tell me how many are hiding?

"Hmm, four and to make a seven there's three."

I have to say that I was close to tears watching this child exude confidence in her math abilities. She was obviously so proud of herself. This was our student on an IEP for her developmental delay. *NPP* opened the world of math for her to understand. Most students in the class reflected Jennifer's attitude. This is evident in the following three sections.

Evidence from students' responses during the assessment interviews:

How do you know three are missing?

"I saw four at the bottom so it must be three at the top. This is so fun!"

"Patterns! I'm so glad you're teaching us the patterns."

How do you know there are twelve?

"See I knew this was eight and then I counted these (he first made three groups of four) and then I put them together and knew that was eight and then I counted the rest. This is fun!"

Do you know how many tiles there are?

(This student counted but did not group the tiles into *NPP*) "I think there's fifteen." ***Are you sure there are fifteen?*** He put the tiles into *NPP* and said, "There's thirteen." ***Are you sure?*** "Yes, because this time I made the pattern."

Evidence from my observations during student assessment interviews:

Every student in the class returned their Human Subject Consent Form. I heard many comments from the students such as, "When do I get to do math? Can I do some more?" and, "This is fun," which indicated to me a positive attitude towards math activities. I thought students might complain about having to do four story problems in one sitting because in class we usually only did one on a single day. Instead, I got comments like "This is fun. I'm doing some hard math thinking."

When I asked them to show me seven tiles, every child placed the tiles into the seven *NPP*. When I asked how they knew it was seven, they just looked at me as if to say, "Well, Mrs. Kuske, who would not know it is seven? I see the six and the one, or the five and the two, or I just know it is seven. It is the pattern!"

When students arranged tiles into *NPP* they often did not appear to be counting. When they had built the *NPP,* they knew how many they had by looking at the pattern. If they were indeed counting, I did not observe any of them having to recount once they had built the *NPP*.

When I knew a student had given a correct answer, but had not yet told the student it was correct, I asked how he or she knew the answer was right. I observed that students immediately began to explain their reasoning. They did not seem to doubt their answer.

Evidence from student work:

Beginning with the worksheets of Making Ten using *NPP* all students experienced success in the early stages of first-grade math. On the nine assignments reviewed, students got the right answer 98% of the time. According to the teacher, this high percentage was uncommon for first-graders.

The early and frequent success our students experienced in first-grade mathematics most likely led to their positive attitudes. I have a saying that confidence comes from competence.

Our students certainly felt competent in their mathematical abilities. In my years of tutoring, I observed students solving math problems by counting on their fingers. Doing so, they made errors in their counting and became discouraged. Our students rarely used counting as a strategy, therefore their error rate was very low. I believe this contributed to the fact that we had 23 students, not one that hated math. With their positive attitudes and confidence, students used *NPP* to show pictures of their work (figural representation), the next theme in this data. The teacher interview shows that *NPP* also became the method of number representation in the classroom.

3. Representation- students and the teacher used *NPP* as figural representations of numbers.

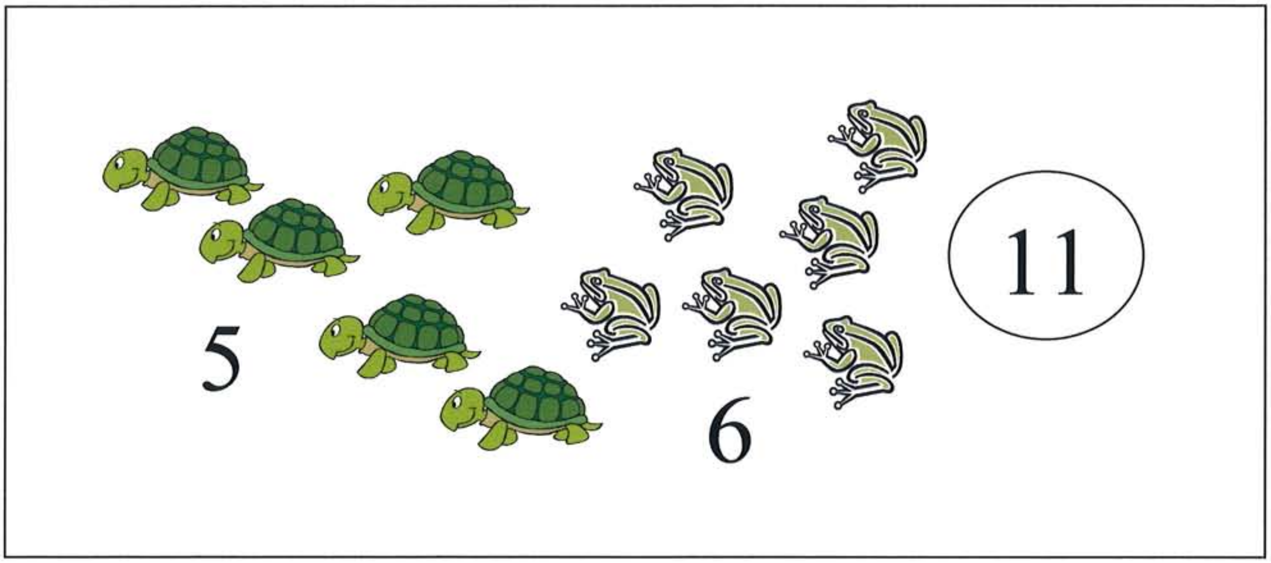
Evidence from the teacher interview:

> A kindergarten teacher that had taught 27 years was in a math study group with me. She said that really what learning is, is figuring out the patterns that we need to know in order to survive in our life. This is what language is about. We learn the patterns of language. I have always felt that is true. Now we are learning the patterns of math and how it makes sense. This is excellent.
>
> The other thing is it has changed me! I am not adding and subtracting quite the same way. I am beginning to think of numbers in the patterns, especially putting the hundreds in a column. It has been a big shift for me, how you count along the patterns and how you move through the system.

The teacher experienced that *NPP* invaded her old ways of thinking about number.

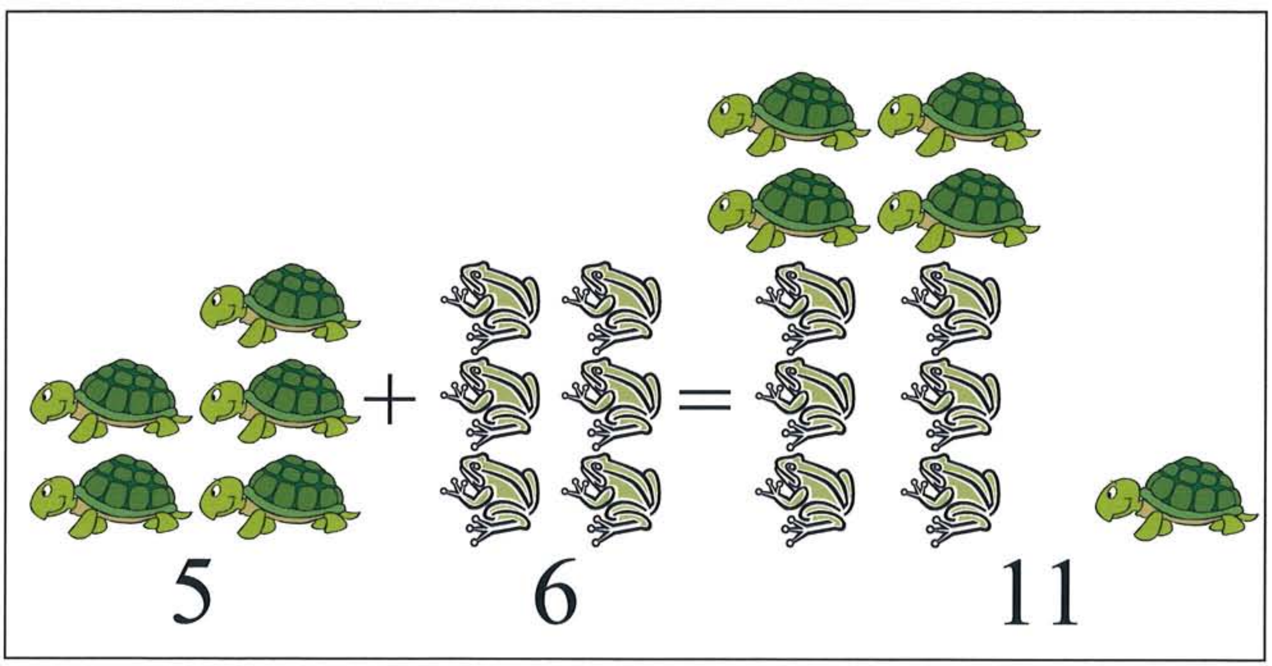
Evidence from student work:

In September, students were representing their numbers by drawing their figures all over the page. For example, when asked to draw their thinking of five frogs plus six turtles equals eleven, their picture might have looked like Figure 4.



- Figure 4

By May, most students' figural representations looked like Figure 5.



- Figure 5

For further examples of student work with *NPP*, see Appendix E.

Evidence from student responses during assessment interviews:

How do you know that is twelve?

"I made the pattern. Four plus six equals ten and two equals twelve."

"I made a flower, a flower and two extra and then went one, two up to twelve."

(3 + 4 + 8 =15) "I took eight cause that was the biggest number. Then I got a three but that wouldn't fit so I got a two and a one. Then I got the four. And so I realized it was fifteen because it's in the number pattern."

On the assessment question, "Can you show me a group of seven tiles?" twenty-two of the twenty-three students placed the tiles into the seven *NPP* to represent their group of seven. On the assessment question, "Can you show me a group of twelve tiles?" eighteen of the twenty-three students placed the tiles into the twelve *NPP* to represent their group of twelve.

As discussed in my literature review, the NCTM feels that "instructional programs …should enable all students to – use representations to model and interpret physical, social, and mathematical phenomena" (NCTM, 2000, p. 67). *NPP* gave our students a tool for understanding and representing their mathematical knowledge.

According to Karen Fuson, most studies examine representation models in linear arrays (Fuson, 1992). Instead, integrating *NPP* into our instructional method enabled students to use patterned multi-units to solve problems. The students easily assimilated *NPP* as their representational model, using them to show their thinking.

Besides being able to use *NPP* to represent numbers, students were able to recognize numbers from *NPP* arrangements without counting. This ability is recognized by John Van de Walle as important to students' thinking of a number as a single unit rather than a count. This leads eventually to an enhanced understanding of the part-part-whole relationship of numbers. (Van de Walle, 1998). The data revealed the following evidence that our students identified numbers as a unit.

4. **Pattern Recognition- students recognized the numerosity of sets of objects in patterned arrangements without counting.**

Evidence from student work:

We spent September and October learning the *NPP*. We colored the *NPP* on worksheets, played with *NPP* blocks, and counted objects into *NPP* egg cartons. We played math games with dice, dominoes, and playing cards that displayed *NPP* instead of the more common number patterns. By May, students were no longer counting the dots on these manipulatives. They could recognize the number from its pattern.

Evidence from student responses during assessment interviews:

How do you know there are six?

"I see it's in the six pattern."

How do you know there are seven tiles?

"Because there's a flower and three extra. We did it yesterday too. We did seven."

"Cause it's in the seven pattern."

How do you know that is twelve?

"It's in the twelve pattern."

"The tens pattern and two extra."

"Because there's an eight and a four." *How do you know there is an eight and a four? Did you count them?* "No. There's the eight and there's the four."

Do you know which one of these cards has the biggest number on it?

"Because it's a seven, and seven is more than five."

How do you know it is four, five, six, and seven?

"A flower is 4, a flower and one extra is 5, a flower and 2 extra is 6, a flower and 3 extra is seven."

There were seven opportunities during the assessment interviews for me to observe the strategies students used to recognize the numerosity of a set of tiles displayed in *NPP*. I noted that 86% of the time students appeared to recognize the number without having to count.

Further evidence was the students' abilities on the subitizing test (Appendix F). When a *NPP* flashed on the screen students gave the correct number almost 100% of the time up through the number six. The four students, who gave an incorrect response, gave the correct response on the other trial of the same number. Ninety-one percent of the students subitized the number seven correctly: 65% the number eight, 83% the number nine, and 100% the number ten. (Perhaps this last one was due to guessing!).

Mandler and Shebo said that patterns can be learned to improve the accuracy of subitizing and that it should be possible to teach the use of larger canonical patterns for efficient approaches to the numerosity task (Mandler & Shebo, 1982). Pinpointing the easiest patterns to learn remains an area of investigation for educators (Fischer, 1992). Our investigation of *NPP* showed that students easily learned and used the *NPP* and that their resulting accuracy in subitizing them was very high.

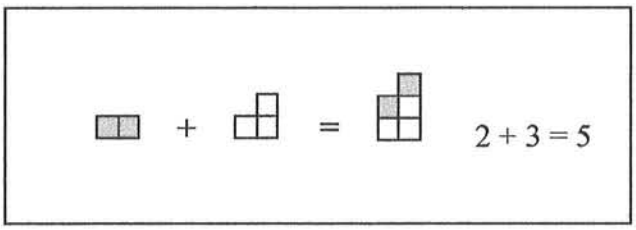
Douglas H. Clements said that students could use pattern recognition to discover essential properties of number, such as unitizing, composing and decomposing numbers, as well as develop their understanding of arithmetic and place value (Clements, 1999). This study supports

Clements claim, as can be seen in the next two sections. Our students developed a good understanding of the part-part-whole relationships of numbers and place-value.

5. **Part-Part-Whole Understanding- students exhibited understanding of the part-part-whole relationships of numbers.**

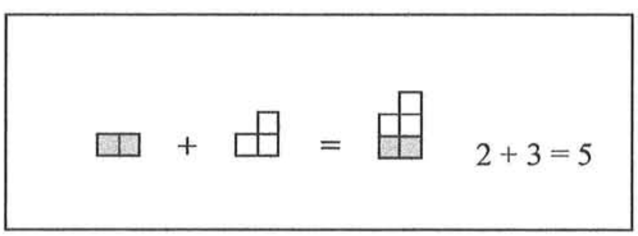
Evidence from student work:

In May, some students' work showed a splitting of the parts to form the whole as shown in Figure 6. Students split apart the two to put together with the three and form the five.



- Figure 6

We modified our instruction to help students keep the parts together to form the whole as shown in Figure 7.



- Figure 7

At the end of this study, students were working on keeping their pieces together. Even though their work showed a splitting of the number, the following students' comments indicated that they saw each number as a unit.

Evidence from student responses during assessment interviews:

On the five part-part-whole questions on the assessment interviews students answered correctly 96% of the time.

How do you know there are six?

"Because I see three and three."

How do you know there are seven tiles?

"Because there is a four and a three and I know 4 + 3 = 7."

"Because there's a six and a one. Or a four and a three."

"I took the one out, I saw the six, and with the one it's seven."

How many dots?

"Eight." *How do you know?* "Because it's four plus four."

How do you know that is twelve?

"Because it's ten plus two."

"Because four plus four plus two plus two equals twelve."

This card has seven squares but some of them are hiding?

"Three, because four and three make seven."

As reported in my literature review, Van de Walle (1998) said that to conceptualize a number as formed of two or more parts is the most important relationship a child can develop about numbers. He advocated activities that focus on this relationship as crucial to children's development of number sense. *NPP* provided consistent opportunities for our students to work with this concept. *NPP* seemed to help our students develop number sense because their understanding of numbers was from patterns and not from counting.

The final theme from the data was students' development of an understanding of place-value. Clements (1999) said that patterns could facilitate this awareness and NPP seemed to do just this with our students.

6. Place Value Understanding- students exhibited an understanding of place value.

Evidence from student responses during assessment interviews:

On the four questions concerning place value in the assessment interview, students answered correctly 95% of the time.

Can you tell me how you figured that out that it was four? (Story problem, 8 + __ = 12)

> "I got the eight already and I had a four." (She tries to put the four on the top of the eight and says) "It won't fit. So, I took a two and a two to make four. So once I put this up here (she puts a two on the top of the eight to make a ten) the other two would go down here (to the right of the ten in the ones column) and ten and two makes twelve, so it's four."

How do you know you have fourteen?

> "Because I know a four pattern is a flower and I have one ten here so if I have one ten that means there must be one number ten. So and I have a four over here and then I notice one group of ten and if there's four more it must be fourteen. And I know when I look at the number fourteen wrotten (sic) the one means there's one group of ten already and I can see the four so it's fourteen."

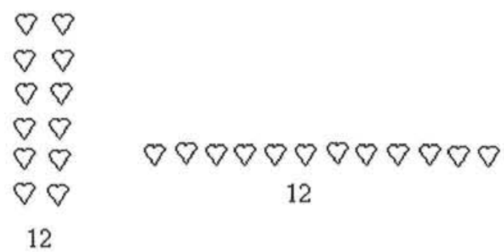
> "Because it's one group of ten and four extra."

"Because I counted them. But not with my fingers, I just looked at them and then I said that's ten and that's four, fourteen! One group of ten, and two groups of ten is twenty!"

In the story problem $8 + 5 = 13$, nineteen students showed their answer with a figural representation in *NPP*. All nineteen displayed an understanding of place value by making one *NPP* ten on the left and the *NPP* three on the right. The other four students made no figural representation of the answer because they were the last students tested and I determined I had enough evidence. In the interest of time, I did not ask them to record their work.

Evidence from student work:

Student work from the fall showed little understanding of place value. For example, when representing their work in pictures for the number twelve, most students placed the two ones vertically or horizontally after the ten.



Later in the year, when students began to understand to group the extra ones separately from the ten many students put the ones to the left of the tens. By May, most students were representing their numbers with correct place value form similar to the example in Figure 5, page 44. Along with the assessment interviews, this made it evident that students had developed a good understanding of this last theme of place value.

The overall analysis of the data collected in this study from the teacher interview, students' assessment interviews, and the students' work revealed six themes. I expected to find four of the themes after my review of the literature on number sense development, however the additional two themes were unforeseen. The literature suggests that children's development of number sense includes the themes of representation of numbers, pattern recognition without counting, part-part-whole understanding, and place-value understanding. The surprises from this study were the themes of internalization and student confidence.

Conclusions

The purpose of this study was to discover if I could improve my teaching of number sense to first-grade students. Mathematics activities in elementary school that use dice, playing cards, dominoes and other manipulatives employ number patterns that are incompatible with current brain research and young children's learning styles. I have questioned the "unquestionable knowledge" (Hart, 1998) that the canonical patterns of manipulative models, dice, playing cards, and dominoes are beneficial to young students' math cognition.

The goal of my action research was to explore whether a set of logical, consistent number patterns more in line with how young children's brains function i.e. that work like a puzzle, could benefit students in developing good number sense. I sought to reveal the effects, on first-grade students' development of number sense, of the integration of *Number Puzzle Patterns* into their mathematics curriculum.

The results showed that after a year of exposure to *NPP* students exhibited an understanding of place value and the part-part-whole relationships of numbers, fulfilling the definition of number sense as defined by Sharon H. Ross. It seems to follow that her additional requisite, the frequent and flexible use of this understanding, will develop with experience. An overall analysis of the data indicated that number patterns that work like a puzzle facilitated the direction in which students made meaning for numbers. Being tactile/kinesthetic learners, the first graders were able to manipulate the math materials in a way that made sense to their spatially dominated brains. This point, added to the consistency that *NPP* provided to the curriculum, facilitated students' internalization and retention of the *NPP*. Evidence came from the students' figural representations of numbers in *NPP* and the students' abilities to recognize the numerosity of sets of objects in patterned arrangements without counting.

Besides the six themes that emerged from the data, this study found that *NPP* addressed the learning and equity principles for school mathematics advocated by the NCTM. Concurring with Piaget's theory of accommodation and assimilation, the NCTM's principle on learning says that students must actively build new knowledge from experience and prior knowledge (NCTM, 2000). John Van de Walle concurred that the more connections made with ones existing network of schemas the better one understands new ideas. Being innate, subitizing qualifies as prior knowledge. *NPP* provided our students with a connection of the new concepts of primary mathematics to their existing knowledge of subitizing, and learning occurred.

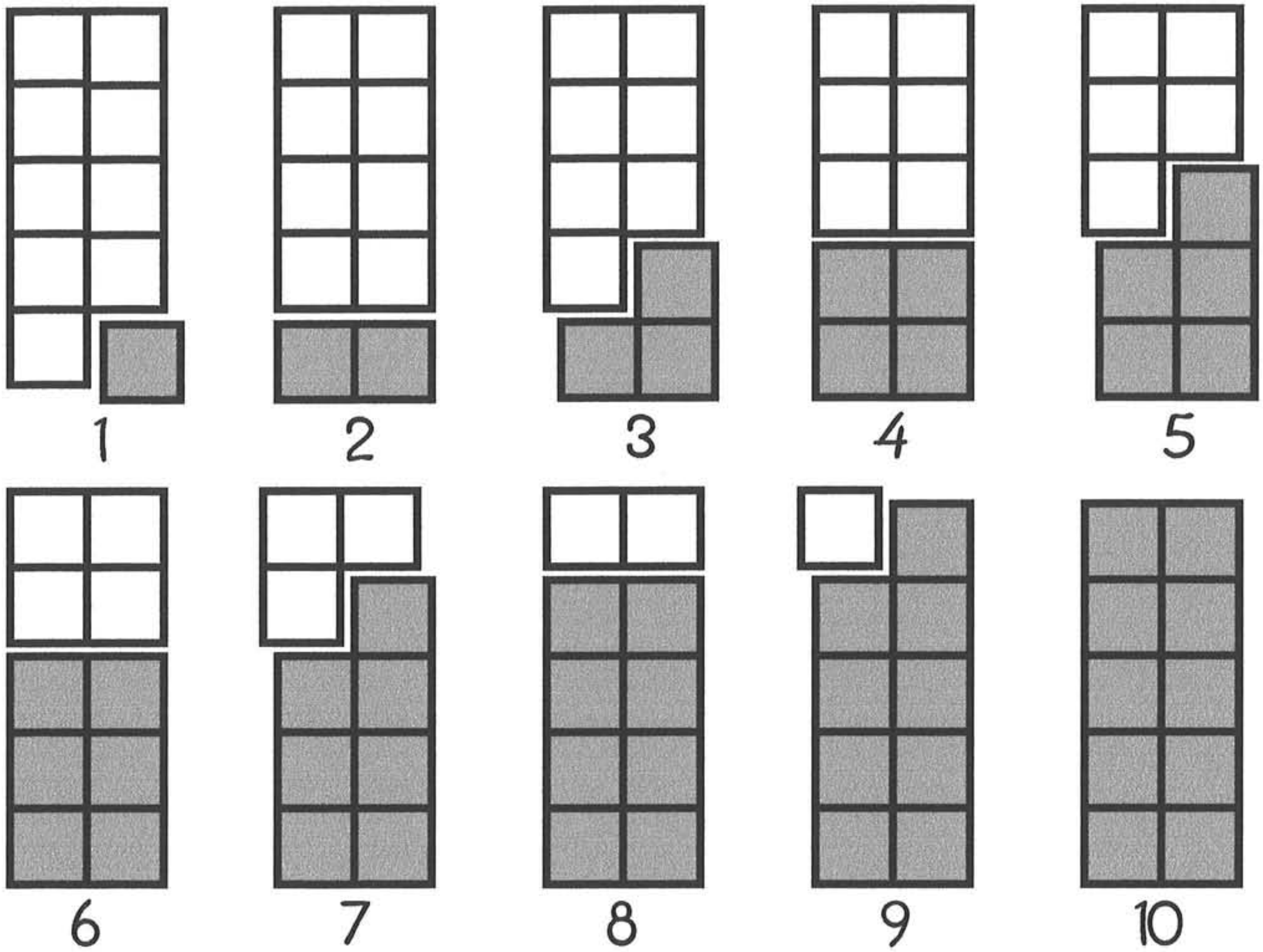
Regarding the equity principle, the innate ability to subitize four objects in a square pattern without counting was equally present in all children. Regardless of what they brought to school in the way of previous math experience, or difficulties they faced with language barriers, our students received math instruction based on this equal foundation. Perhaps because we built upon this equity and they experienced early success in first grade math, our students were confident in their mathematical abilities. Their dispositions and attitudes concerning math activities were highly positive. If you asked students who had not experienced *NPP* to show you a group of seven they would most likely count out seven and leave them in a pile. When you asked how they knew there were seven they would not know for sure. They would just say that they counted. To be sure they would have to count again, whereas, using *NPP* our students were confident, they knew it was seven.

The findings of this project are applicable to teachers of K-2 math, specifically those using the TERC curriculum. However, the concepts could be adapted to other K-2 curriculums. The results of this study have informed my practice as a teacher of K-2 mathematics and answered yes to the question, "Can I teach number sense in a more effective manner?" By incorporating *NPP* into the curriculum, I expect to be able to reach more of my students with an understanding of number sense. A line in one of my favorite songs goes, "When you know the notes to sing, you can sing most anything." I feel this same way about *NPP* for children. We need to give children the notes of mathematics as we do in music. Then we will sing, "When you know what numbers are, math will take you very far."

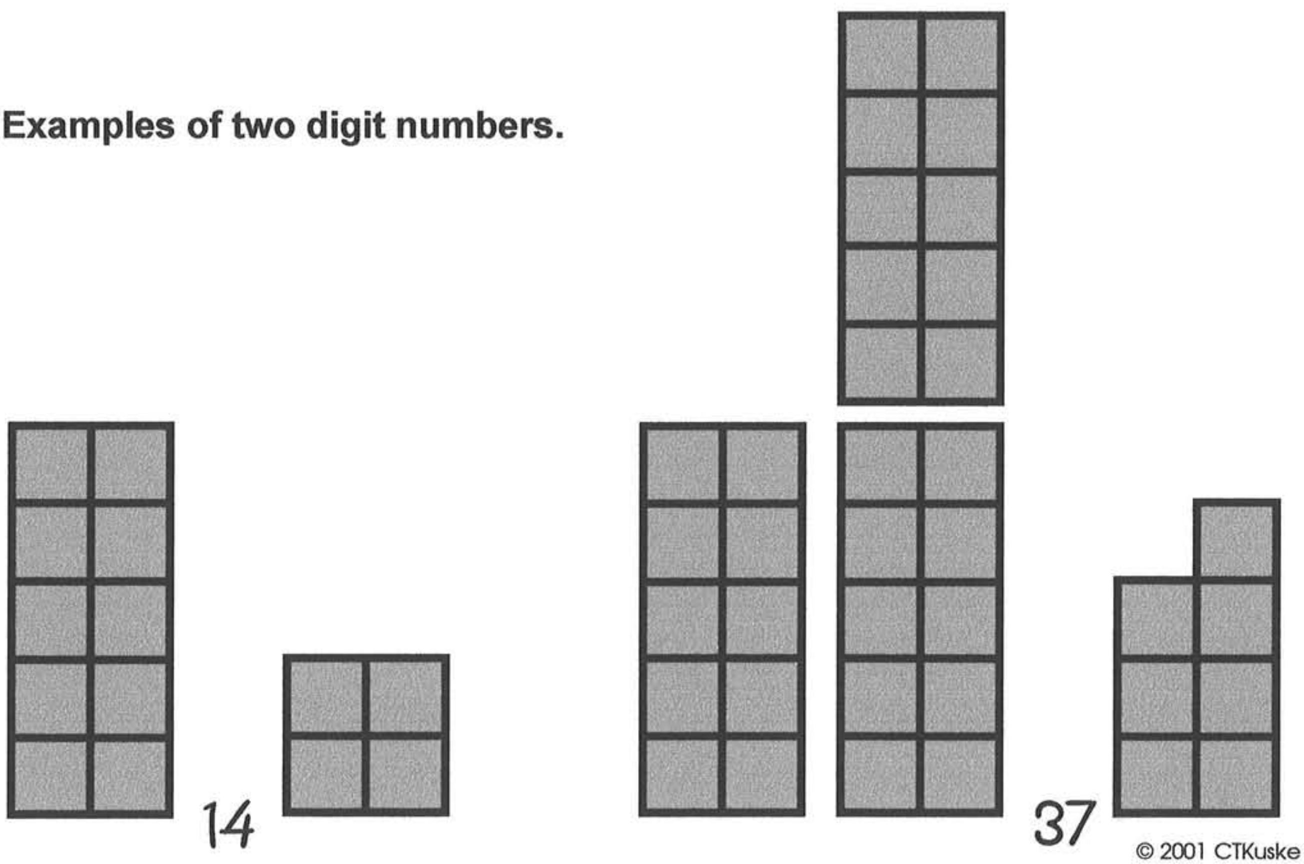
This research project was so successful that the school district in which I conducted my study plans to help me implement a three-year study of the effect of *NPP*, beginning in September 2001. I will be integrating *NPP* into the kindergarten TERC curriculum. The goal is to teach the patterns to the students so they will know them when they enter first-grade. I will follow these students to first-grade, continuing to use *NPP* in their mathematics curriculum. Then, I will continue with them into second-grade, at which point we will evaluate the need for further support. The school district plans to pre-test and post-test the students each year. They will keep data on a control group in a school using the same curriculum without *NPP*. This study will give us a better understanding of the impact of *NPP*, however, I hope those who read my current study and find *NPP* intriguing will not wait three years to make this tool available to students and teachers of mathematics. My recommendations for further research include using *NPP* to teach multiplication and fractions to students who have already learned addition and subtraction with *NPP* and replicating previous mathematics studies replacing canonical number patterns and representations with *Number Puzzle Patterns*.

APPENDIX A

Number Puzzle Patterns



Examples of two digit numbers.



APPENDIX B

Teacher Interview

Name________________________ School________________ Date________________

1. Describe your math program for first grade. (What is most important for your students to learn?) (How do you teach single digit addition and subtraction?)

2. Do you feel that available methods for teaching an understanding* of number and simple addition and subtraction are adequate? Why or why not?

3. What materials, if any, do you find are most useful in teaching math to first graders?

4. Why do you think some students struggle to learn math?

5. What do you notice about the learning styles of your students who struggle with math?

6. How do you address different learning styles during math class?

7. What do you like about the TERC curriculum for first grade?

8. What do you not like about TERC for first grade?

9. What do you enjoy about teaching math?

10. What do you not like about teaching math?

*Number Sense- Rote counting, rational counting (objects), cardinal number (how many?), subitizing (identifying the numerosity of 2,4,6 objects), one-to-one correspondence, conservation of number, inequality, ordinal numbers, part-part-whole, place value.

APPENDIX C
Number Concepts Assessment Interview

1. "Count for me. I'll tell you when to stop."
   (Count like this for me. One, two, three,... Now you do it.)

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30						

2. Present the child with a set of six tiles in *NPP* arrangement and ask,

   "Can you count these tiles?"

   1 2 3 4 5 6

3. Cover the six tiles used in #2 and ask, "How many tiles are on the table?"

   Response:_____________

4. From a set of about twenty tiles, ask,

   "Can you show me a group of 7 tiles?"

   Number grouped:________________ Displayed in *NPP*: YES NO

   Ask, "How do you know there are 7 tiles?"

   Response:____

5. From a set of about twenty tiles, ask,

   "Can you show me a group of 12 tiles?"

   Number grouped:________________ Displayed in *NPP*: YES NO

   Ask, "How do you know there are 12 tiles?"

   Response:____

6. Display a row of 7 tiles. Ask, "Can you make another row that has the same number of tiles?"

   Number of tiles displayed:_______________

7. Count out two horizontal rows of 7 tiles arranged in one-to-one correspondence. Then spread out the tiles in one row and ask,

   "Do the two rows have the same number of tiles?"

8. Count out a row of 9 tiles. Then move the tiles into a circle. Ask,

   "Can you tell me without counting how many tiles are in the circle?"

9. Show the child two cards, one with a *NPP* for 5 and one with a *NPP* for 7. Ask,

   "Which card has more dots?"

   Response: _____The card with 5. _____The card with 7.

   Strategy used: Counts all Adds/Knows "Sees" *NPP*

10. Show the child four cards showing the *NPP* for 4, 5, 6, & 7. Ask,

    "Can you put these cards in order from the smallest number to the biggest?"

    Order: ____ ____ ____ ____

    Strategy used: Counts all Adds/Knows "Sees" *NPP*

11. Take the multi-link cube *NPP* 7 made out of the *NPP* 3 and 4 (Exhibit A). Place it in front of the child and ask,

    "How many cubes are there all together?" Response__________

    Strategy used: Counts all Adds/Knows "Sees" *NPP*

    Make a multi-link cube *NPP* 7 out of *NPP* 2 and *NPP* 5, while the child watches. With the two *NPP* 7s in front of the child, ask,

    "Do these two groups have the same number of cubes? YES NO

    Strategy used: Counts all Adds/Knows "Sees" *NPP*

12. Make the *NPP* for 8 out of tiles that all have the same color on one side and a different color on the other side. Put the same color facing up. Ask,

"Now many tiles are there in all?" Response__________

Then turn over the *NPP* 5 within the 8 and ask,

"Is there still the same number of tiles in all?" Response YES NO

13. Lay a paper with the *NPP* 7 on the table in front of the child with the top 3 *NPP* covered. Tell the child,

"There are 7 squares on this paper but some of them are hiding. Can you tell me how many are hiding?

Response:_____________

"How do you know _(child's response from above)_ are hiding?"

Response:

Problem Solving Questions

1. Marcus has 8 cookies. Alisa gives him 5 more cookies. How many cookies does Marcus have now?

2. Shing has 13 blocks. He gives 9 blocks to Brittney. How many blocks does Shing have now?

5. Place two *NPP* 7 made out of 1" tiles on the table in front of the child. Ask,

   "Do these two groups have the same number of tiles?" Response YES NO

   Spread out one of the *NPP* 7 but keep it in the *NPP* form. Ask,

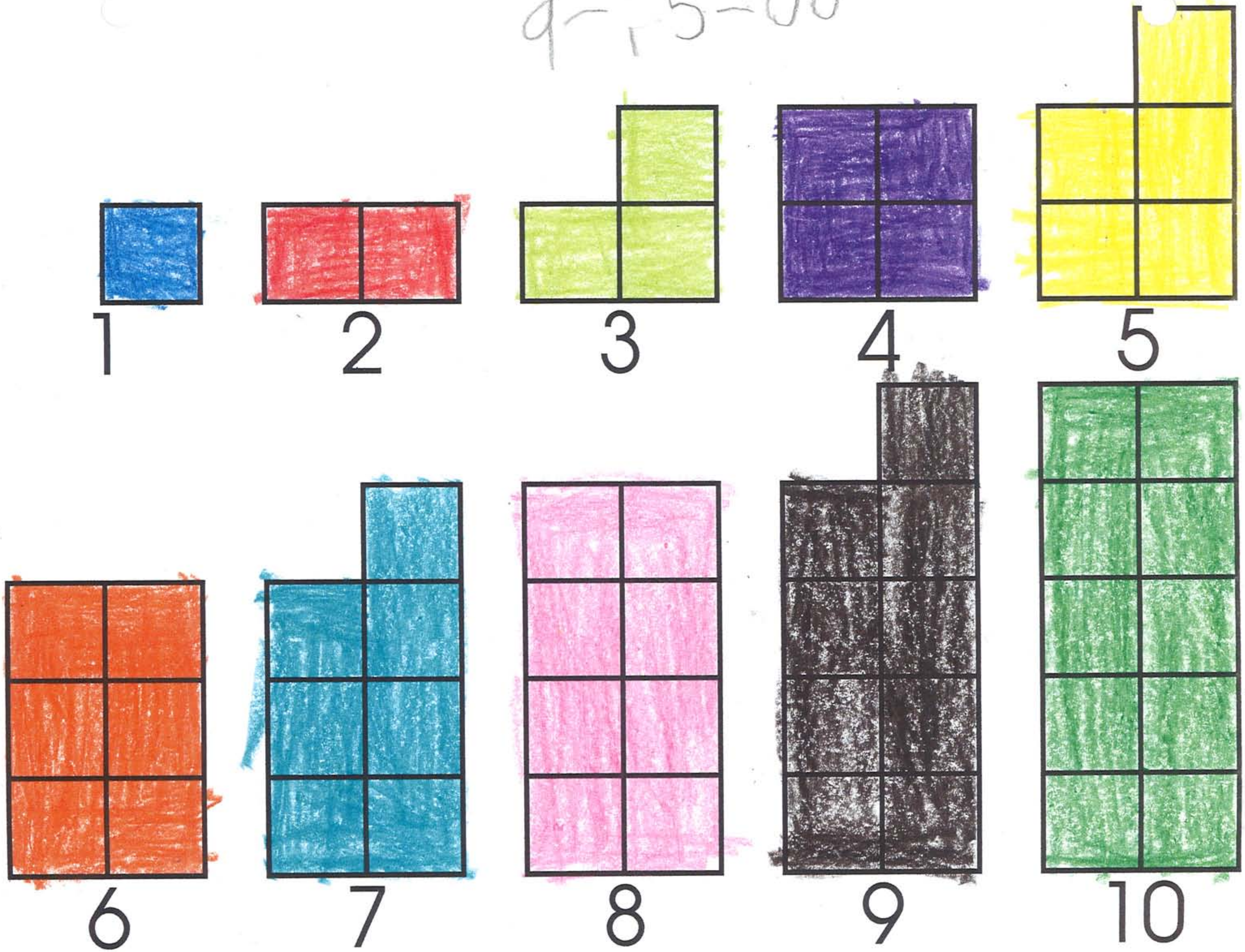
   "Do these two groups still have the same number of tiles?" Response YES NO

Problem Solving Questions

1. William found 3 soccer balls. Jamie found 8 soccer balls. Jason found 4 soccer balls. How many soccer balls did they find?
2. Sammy has 8 pennies. Vedant gives him some more. Now Sammy has 12 pennies. How many pennies did Vedant give to Sammy?
3. Paige has 14 books. She gives some of the books to Eric. Now Paige has 6 books left. How many books did Paige give to Eric?
4. Arlan has some pencils. He gives 4 to Zack. Now Arlan has 8 pencils. How many pencils did Arlan have before he gave some to Zack?

APPENDIX E
Examples of Student Work

4-15-00

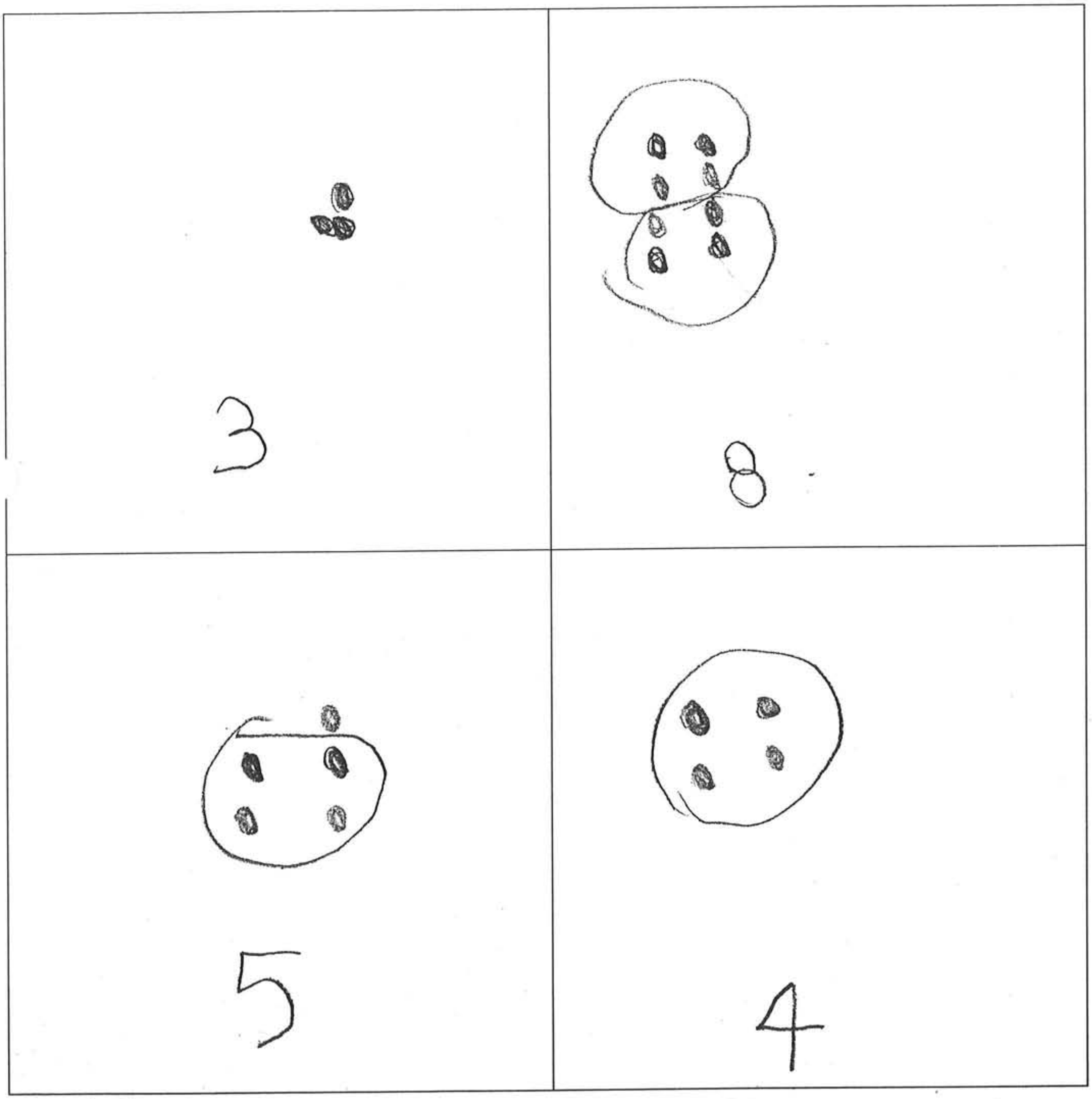


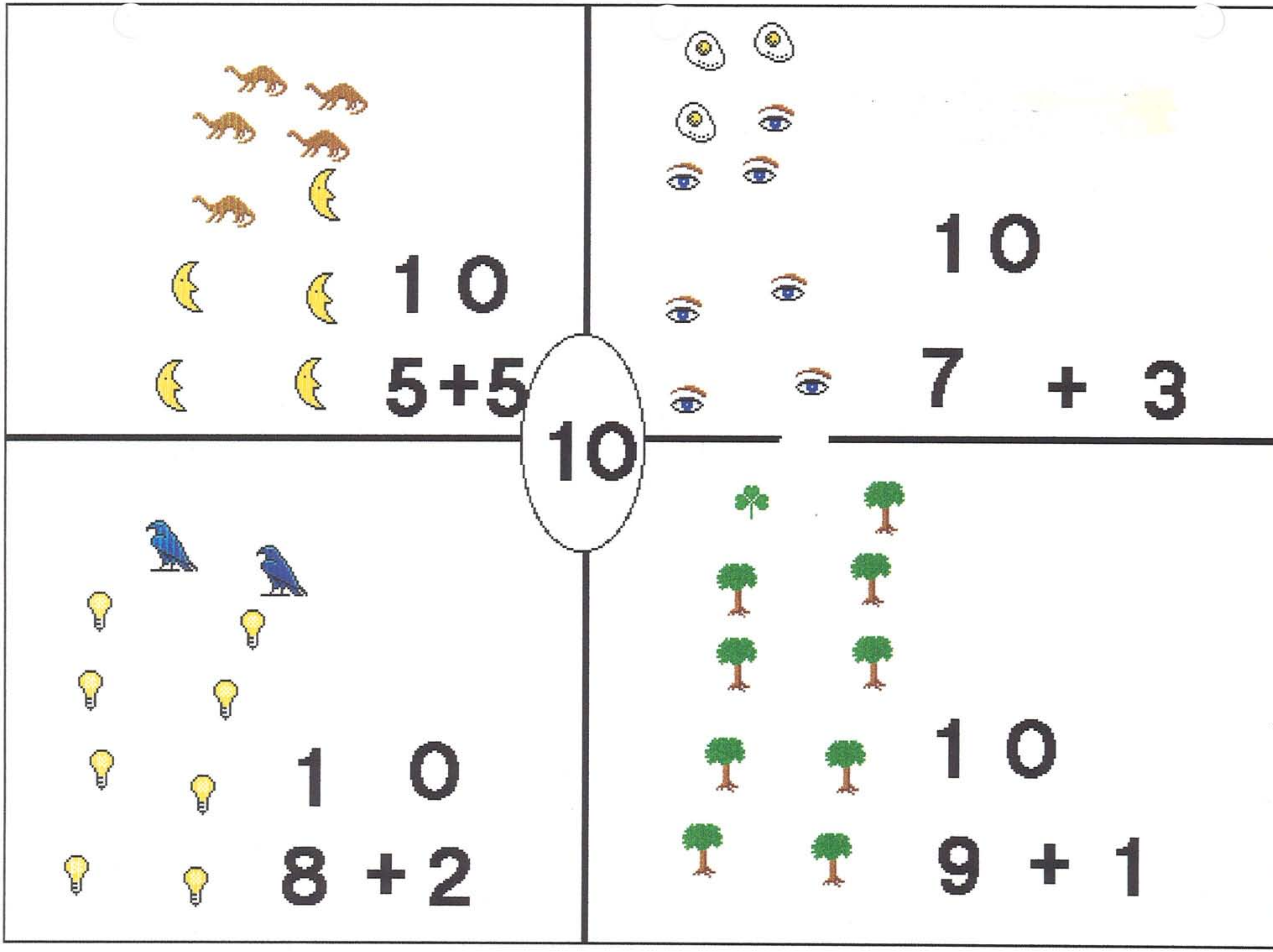
Name ______________________________ Date 9-19-00

Student Sheet 4

Making Quick Images

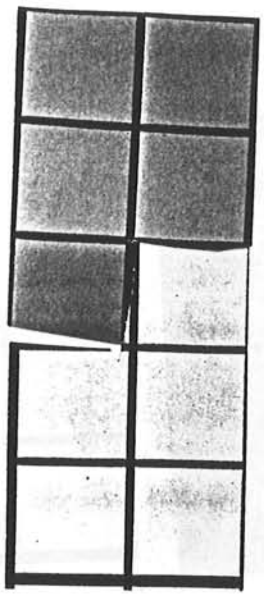
Number of dots: ____________



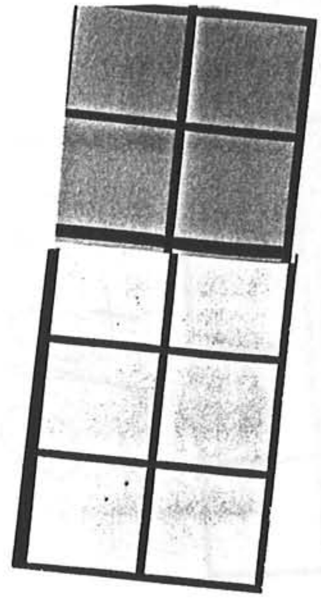


Sept. 2000

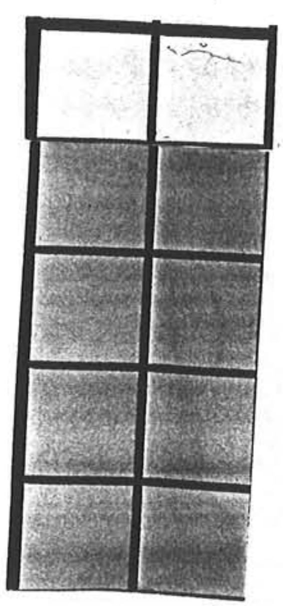
4-27-01



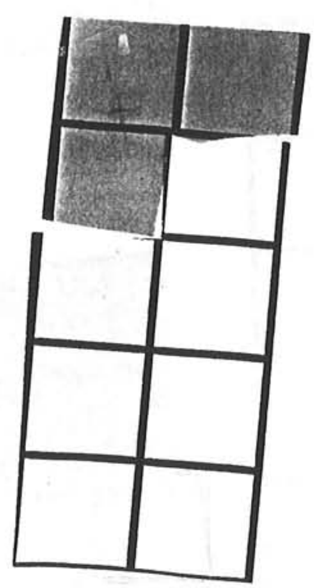
$5 + 5 = 10$



$6 + 4 = 10$

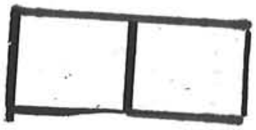


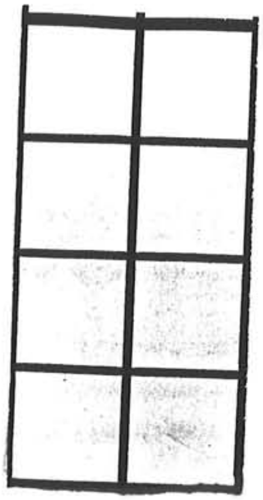
$8 + 2 = 10$

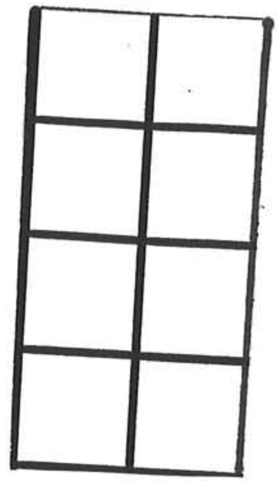


$7 + 3 = 10$

5-1-01

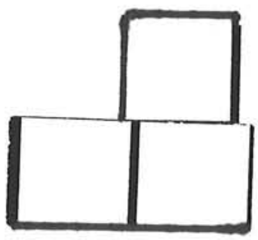


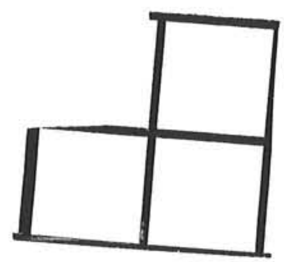


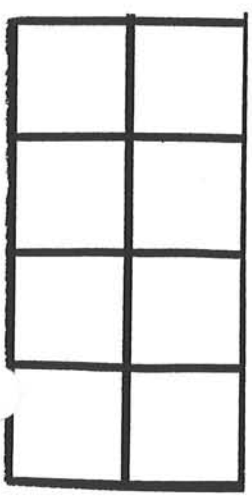


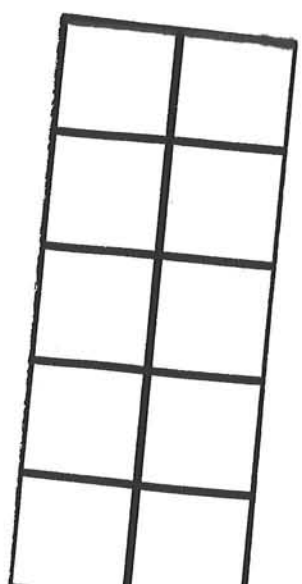
10 - 2 = 8

9 - 1 = 8









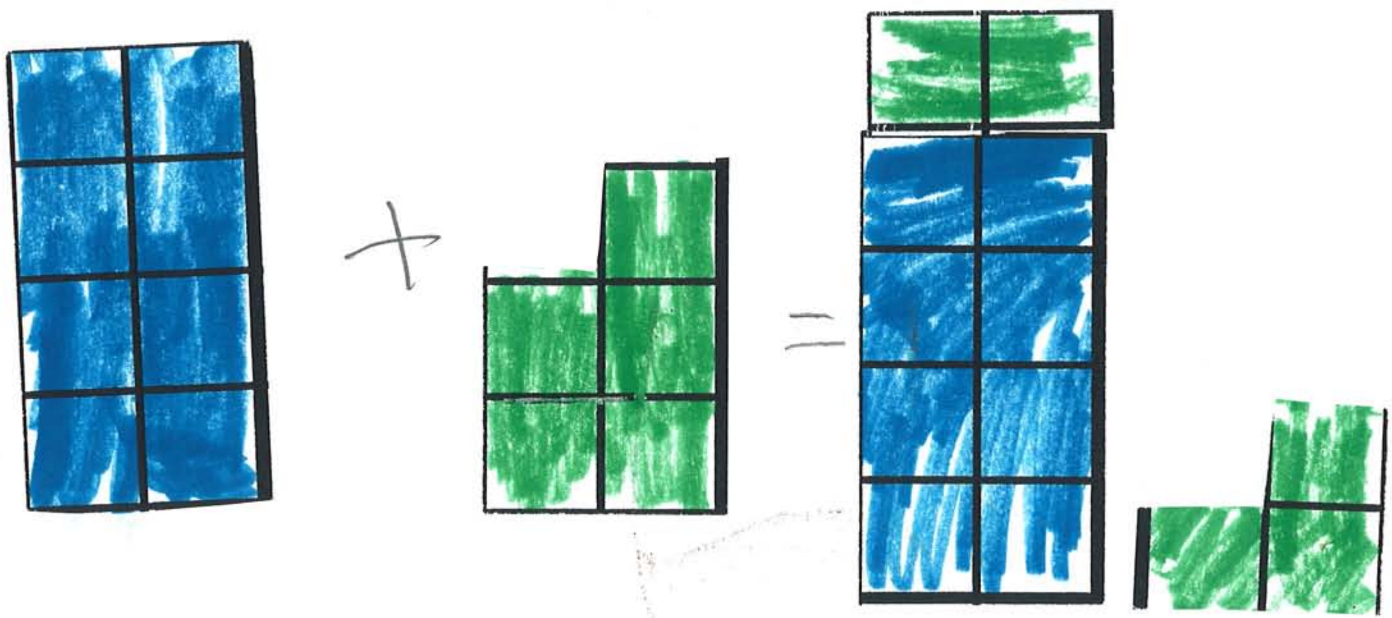
11 - 3 = 8

13 - 3 = 10

5/1

Marcus has 8 cookies. Samuel gives him 5 more cookies. How many cookies does Marcus have now?

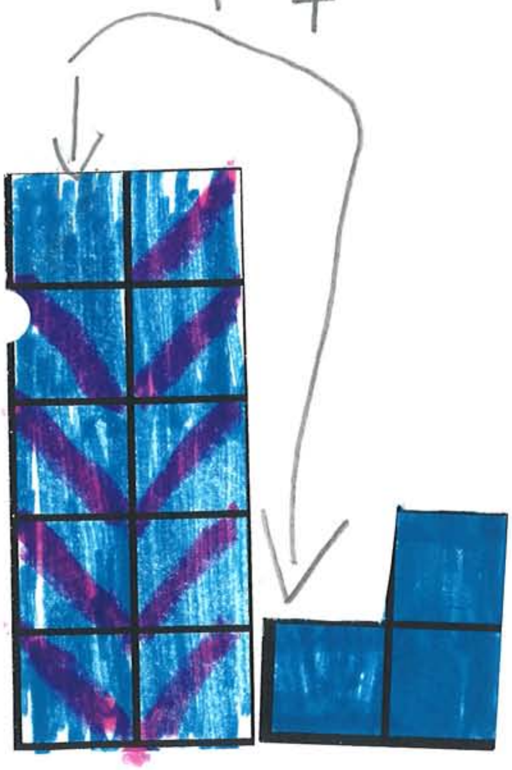
8 + 5 = 13



5/01

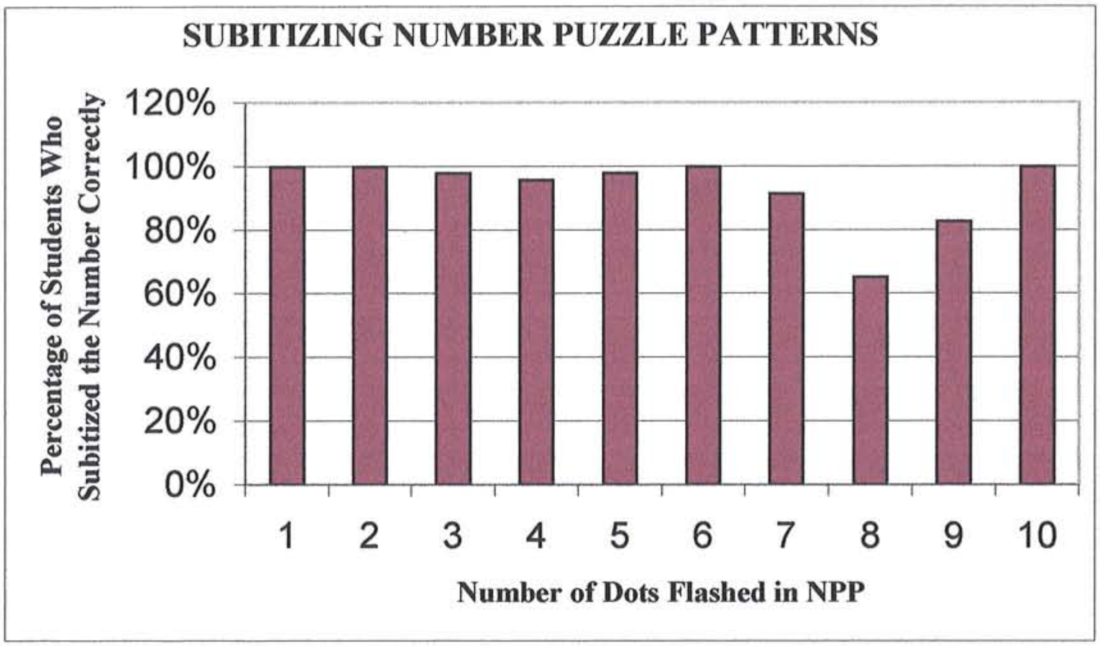
Shing has 13 blocks. He gives 9 blocks to Brittney. How many blocks does Shing have now?

$13 - 9 = 4$



APPENDIX F

Results of Subitizing Assessment



NUMBER FLASHED ON SCREEN
(1 = CORRECT, 0 = INCORRECT)

Data Table

ID#	1	1	1	2	2	3	3	4	4	5	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
9	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1
10	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
18	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
	100%	100%	100%	100%	100%	100%	96%	100%	91%	96%	100%	100%	91%	65%	83%	100%
		100%		100%		98%		96%		98%		100%	91%	65%	83%	100%

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ANTIOCH UNIVERSITY
CENTER FOR PROGRAMS IN EDUCATION
2001
PROPOSAL FOR RESEARCH WITH HUMAN SUBJECTS
(Must be approved by your advisor prior to submission to Research Review Committee)

Name of researcher: Carolyn T. Kuske

Advisor: Judith Gray

Date proposal submitted: February 22, 2001

Title of Research Project: IT ALL FITS TOGETHER: Number Patterns that Foster Number Sense in K-2 Students—A Brain Based Model

1. Purpose of Research:

My interest in this research stems from twenty years of working in classrooms, tutoring, and watching students struggle to understand math. The students I assisted did not exhibit a strong number sense. The National Council of Teachers of Mathematics says that good number sense is vital to children's mathematical development. Students, who struggle with basic number sense will, in turn, struggle with higher mathematics learning.

The purpose of this research is twofold. Because early elementary mathematics classes play numerous games with dice, playing cards, and dominoes I want to investigate, through a literature review, what is the value of the patterns on these items in the development of children's mathematical understanding.

Most young children learn through touching and moving. Such students benefit from "resources…that include writing, manipulative games, and puzzles"(Dunn, 1994, p.20). The familiar number patterns that represent one to ten on dice, playing cards and dominoes are impractical to manipulate like a puzzle to do math. I want to investigate whether or not a different set of patterns that work like a puzzle might benefit young students in developing good number sense.

2. Design of Study

In the spring of 1999, I developed a set of number patterns that can be put together like a puzzle, in a logical, consistent manner. I named this system *Number Puzzle Patterns (NPP)*. I created materials to adapt our class math curriculum to incorporate *Number Puzzle Patterns*. I made dice, playing cards, and dominoes with *NPP*.

In the fall of 2000, I did my student teaching in a first-grade classroom. During this time, we incorporated *NPP* into the math curriculum. My host teacher, who has been teaching for over 20 years, felt that *NPP* helped the students understand and enjoy math.

For the qualitative portion of my study, I will conduct a semi-structured, open-ended interview (Appendix D) with the classroom teacher. I will ask her permission to tape the interviews and then transcribe the data. The teacher will review the transcript for approval. This method will provide insight into the teacher's view of the math curriculum and teaching first-grade math. I will spend time as a participant observer in the classroom, taking notes (Appendix E) to describe the students' social dynamics and the teacher's instructional strategies. I will participate as a teacher's aide to observe students' thinking. The school district uses portfolios so, I will look at student work on previous assessments from this year.

The quantitative portion of my study will involve assessing the students in the class as permitted by their parents. The assessment design models a study by Florence E. Fischer (Fischer, 1990), and includes questions on number concepts (Appendix A), problem solving (Appendix B), and place value (Appendix C).

The findings for this project will be applicable to teachers of K-2 math, specifically those using the same curriculum. However, the concepts could be adapted to other K-2 curriculums.

3. **Impact on Research Participants:**

a. Name of participants:
Teacher- Isabelle Phipps
Students in Mrs. Phipps class for observation, student work, and assessment.

b. Age group: 6-7 years old

c. Gender group: male and female

d. Method of selection: Teacher by permission, students by parent permission.

e. Describe questionnaires and/or task given to research participants: See Appendixes A-E

f. State what each research participant is expected to do: The teacher is expected to participate in the interview. Students who are tested are expected to sit with the examiner (myself) and manipulate objects to verbally answer the questions. On the problem-solving test, they are also expected to show their work with pencil and paper.

4. **Risk:**

a. <u>Current Risk</u>- the student testing takes place in a relaxed atmosphere to lessen any "test anxiety." The testing consists of three different sessions so students will not feel overworked. Students will receive a treat for participating.

b. <u>Future Risk</u>- To insure anonymity, only numbers will identify subjects. At no time will a minor's name be used. The use of the teacher's name will require written permission. Future grant applications and curriculum discussions by the author may require the data.

5. **Benefit:**

a. Research participants: The teacher will receive the results of the study. She can then choose to incorporate the methods into her own practice. The students will receive extra attention and practice during the testing sessions.

b. Humankind: If the data from this study indicate *NPP* benefit the development of number sense, I will use the data to seek grant support for further research into the effectiveness of *NPP*. My goal for the Antioch Masters Symposium is to develop a presentation that illustrates how to adapt a math curriculum to incorporate *NPP*.

6. **Consent Form:** See the attached permission slip. Children will take one home to their parent/guardian. I will make every effort to help parents understand the form in their native language.

7. **Minors and others:** I will rely on the teachers' influence to obtain the students' agreement to participate.

8. **Illegal activities:** None

9. **Recommendations from the reviewer:**

10. **For research conducted by Students:** This research involving human participants, if approved, will be under my supervision.

Faculty Advisor's Signature____

Research Reviewer's Signature____

Research in the Center for Programs in Education at Antioch University which involves human participants is carried out under the oversight of a Research Review Committee. Questions or problems regarding these activities should be addressed to the Wendy Rosen, Coordinator of TCMA, Center for Programs in Education, Antioch University, 2326 Sixth Avenue, Seattle, WA 98121-1814. Phone (206) 441-5352 ex 5601.

March 14, 2001

Carolyn T. Kuske
Antioch University Seattle
Masters of Arts in Education Program
2326 Sixth Avenue
Seattle, WA 98121-1814
(206) 441-5352

Dear Parents/Guardians,

This past fall, I did my student teaching in your child's math class at Lake Hills Elementary. To complete my master's degree, this spring I am evaluating how well our students have done. (Mrs. Phipps and I think they have done extremely well! But, I need some data or proof!) I will be observing in the classroom to see what lessons are covered, what materials are used and what teaching strategies are employed. I will be observing the students to see how they interact with each other, what kinds of questions they ask, and how they respond to the math activities. I will observe the class as a whole, not individual students.

I will give an assessment of number sense to all students who have permission. It will be done in a relaxed way, not as a "test". There will be three sessions of about 15 minutes each. The students will receive a "Thank You" pencil and stickers for their participation. To insure confidentiality, no portion of this study will contain your child's name.

Future grant applications and curriculum discussions, involving myself, may use the results of this research. By approving your child's participation in this study, you can contribute to a project that hopes to improve the math abilities of all students.

Thank you for your cooperation.

Most Sincerely,

Carolyn T. Kuske

HUMAN SUBJECTS CONSENT FORM

I agree (AND, where applicable, I give my consent for ____)
to participate in the research study entitled: **IT ALL FITS TOGETHER: Number patterns that Foster Number Sense in K-2 Students—A Brain Based Model** conducted by **Carolyn T. Kuske**.

I understand that this participation is entirely voluntary. I can withdraw my consent at any time without penalty and have the results of the participation, to the extent that it can be identified as referring to me or others in whose behalf I have given consent, returned to me, removed from the research records, or destroyed.

The following points have been explained to me:

1) The purpose of the research. (See reverse.)

2) The design of the study. (See reverse.)

3) No unusual discomforts, stresses, or risks are foreseen to result from participating in this research.

4) The results of this participation will be confidential, and will not be released in any individually identifiable form without my prior consent, unless otherwise required by law.

5) The investigator will answer any further questions about the research, now or during the course of the project.

6) A copy of the results of the study will be made available to me if I wish.

Signature of Investigator	**Date**	**Signature of Participant [or parent(s) or guardian(s)]**	**Date**

PLEASE SIGN BOTH COPIES OF THIS FORM.
KEEP ONE AND RETURN THE OTHER TO THE INVESTIGATOR.

Research in the Center for Programs in Education at Antioch University Seattle which involves human participants is carried out under the oversight of the Research Review Committee. Questions or problems regarding these activities should be addressed to the Wendy Rosen, Coordinator of TCMA, Center for Programs in Education, Antioch University Seattle, 2326 Sixth Avenue, Seattle, WA 98121-1814, phone (206) 441-5352 ex 5601.