Derivation of design equations

Stepoc Reinforced Block

Anderton Concrete

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Abstract

This report (Job No: 6057, Doc No: R01.1.0) gives the mathematical basis to the design equations for the bending and shear capacity of reinforced Stepoc and gives wall height limits for serviceability. The report uses a strain compatibility condition to link a plastic stress block in the masonry shell to a similar stress block in the concrete core to establish a moment capacity. The limitations on stress block imposed by BS EN 1996-1-1 are explored and used as limits within the revised model. The shear capacity of the section is considered by making comparisons between the reinforcement masonry strength as derived in BS EN 1996-1-1, inclusive of shear enhancement factors, and the shear strength calculated in accordance with BS EN 1992-1-1.

Contents

1 Introduction

This report assess a workable procedure for calculating the moment capacity and shear capacity of the section using recognised procedures relating to rectangular compressive stress blocks as adopted in both BS EN 1996-1-1[2, 3] and BS EN 1992-1-1[1, 4]. Axial capacity is not considered in this document; however, in typical applications for Stepoc the axial capacity of the section is rarely critical. The axial capacity of the section can be calculated in accordance with BS EN 1996-1-1.

Stepoc walls are traditionally designed by adopting the design procedures as outline in BS EN 1996-1-1. Such precedures take the f_k value for the block and assume that this is consistent throughout the section when considering the compressive stress block. However, it is recognised that the concrete core has a compressive strength higher than the masonry. Therefore, such assumptions are conservative.

The motivation throughout this report is to provide a procedure for calculations that, in part, utilises the increased compressive strength of the core whilst being compliant with the basal prescriptive rules of BS EN 1996-1-1 when considering bending.

It is commonly recognised, when dealing with reinforced masonry, and in particularly reinforced masonry retaining walls, that the shear capacity of the masonry section to BS EN 1996-1-1 can be a limiting factor. Two shear strength capacities are derived within this report. One is the reinforced masonry solution and the other treats the Stepoc as a shuttering block calculating the shear capacity of the net concrete section within the pockets of the Stepoc core in accordance with BS EN 1992-1-1. It is left to the structural engineer designing the wall to decide which is the most applicable in a given situation. The author of this report considers that both methods, used correctly, are valid proposals.

2 Moment Capacity

2.1 Theory for bending capacity

BS EN 1996-1-1 provides the following conditions relating to the moment capacity of the section:

$$M_{Rd} \leq 0.40 f_{md} b d^2 \tag{1}$$

$$M_{Rd} = A_s f_y z \tag{2}$$

$$z = d(1 - 0.50\frac{A_s f_y}{b d f_{md}}) \leq 0.95d \tag{3}$$

Where M_{Rd} is the design moment resistance, A_s is the area of steel reinforcement in tension, f_y is the design strength of the reinforcement, b is the breadth of the section, d is the effective depth and f_{dm} is the design strength of the masonry.

These equations allow the maximum depth of the compressive stress block (depth to neutral axis) x permitted in the standard to be calculated as a function of the depth assuming a rectangular stress block. Note for the purpose of these calculations x represents λx in BS EN 1996-1-1; it being acknowledge that the different $x - \lambda x$ permits the neutral axis to be formed.

$$z = d - \frac{x}{2} \tag{4}$$

Comparing equations 3 and 4 it is apparent that:

$$x = \frac{A_s f_y}{b f_{md}} \tag{5}$$

Which is expected to attain equilibrium of the forces in the reinforcement and masonry. Extracting $A_s f_y$ from 2 and apply to 5 gives:

$$x = \frac{M_{Rd}}{z b f_{md}} \tag{6}$$

We know from 1 that M_{Rd} has an upper limit which restricts the maximum depth of the stress block. Substitution into 6 yields:

$$x = \frac{0.40 d^2}{z} \tag{7}$$

Apply 4 for z in 7 gives:

$$x(d - \frac{x}{2}) = 0.40 d^2 \tag{8}$$

Which forms the quadratic:

$$x^2 - 2xd + 0.80 d^2 = 0 \tag{9}$$

This quadratic has the viable solution $x = d(1 - \sqrt{0.20})$. This is therefore the implied limit of the compressive stress block in BS EN 1996-1-1 (λx). This limit is maintained within these design calculations.

When analysing a section with a stress block in the outer shell and a stress block in the concrete core the combined stress block depth x shall not exceed $d(1 - \sqrt{0.20})$.

The moment capacity of the section is derived as a conditional function based on the depth of the compressive stress block. For stress block depth x and masonry shell thickness t_s:

$$0 < x \leq t_s \implies \text{Stress block in the shell} \tag{10}$$

$$t_s < x \leq d(1 - \sqrt{0.20}) \implies \text{Stress block in concrete core} \tag{11}$$

In deriving the depth of the stress block the following assumptions are made:

- The strain in the shell is constant across its width effectively forming axial compression (plastic distribution in bending).
- For compatibility of strains it will be assumed that the strain at the interface of the concrete and masonry is equal and that this forms a similar plastic distribution. This is to maintain consistency with discussion on BS EN 1996-1-1 above in the theoretical condition that the concrete and the masonry can have identical properties.
- Zero strain exists at a position x from the compressive face forming a quasi plastic neutral axis

Let ϵ_{mu} be the strain in the masonry shell at the outer face and ϵ_c be the strain at the core / shell interface. To convert strain to stress E_m, E_c are the Youngs modulus of elasticity for masonry and concrete respectively and f_{md}, f_{cd} are the design stresses in the masonry and concrete respectively.

For the strain assumptions to be compatible:

$$\epsilon_{mu} = \epsilon_c \tag{12}$$

Converting 12 to stress:

$$\frac{f_{md}}{E_m} = \frac{f_{cd}}{E_c} \tag{13}$$

Let α be the modular ratio E_c/E_m such that from 13:

$$f_{cd} = f_{md}\,\alpha \tag{14}$$

Let z_m be the lever arm to the masonry stress block from the reinforcement and z_c be that for the concrete.

The maximum forces in the concrete and masonry are:

$$F_m = \begin{cases} bxf_{md}, & 0 < x < t_s \\ bt_sf_{md}, & t_s < x \leq d(1-\sqrt{0.20}) \end{cases} \tag{15}$$

$$F_c = \begin{cases} b(x-x_s)f_{cd}, & t_s < x \leq d(1-\sqrt{0.20}) \\ 0, & 0 < x \leq t_s \end{cases} \tag{16}$$

substitute 14 into 16 to develop the concrete capacity in terms of masonry strength

$$F_c = \begin{cases} b(x-x_s)\alpha f_{md}, & t_s < x \leq d(1-\sqrt{0.20}) \\ 0, & 0 < x \leq t_s \end{cases} \tag{17}$$

The lever arms to the masonry and concrete compressive stress blocks are:

$$z_m = \begin{cases} d - \frac{x}{2} & 0 < x < t_s \\ d - \frac{t_s}{2} & t_s < x \leq d(1 - \sqrt{0.20}) \end{cases} \tag{18}$$

$$z_c = \begin{cases} d - \frac{(x + t_s)}{2}, & t_s < x \leq d(1 - \sqrt{0.20}) \\ 0, & 0 < x \leq t_s \end{cases} \tag{19}$$

The moment capacity of the section is calculated by taking the moment of the masonry and concrete compressive stress blocks about the plane of the reinforcement. The product of lever arm and force from the above equations yields the following two equations for the moment capacity of the section as a function of stress block depth x.

$$M_d = \begin{cases} bdf_m(x - \frac{x^2}{2}), & 0 < x < t_s \\ bdf_m(t_s - \frac{t_s^2}{2d} + \alpha x - \alpha t_s - \frac{\alpha(x^2 - t_s^2)}{2d}), & t_s < x \leq d(1 - \sqrt{0.20}) \end{cases} \tag{20}$$

Each value of x within the range noted can be assigned a moment capacity using 20. Also to each value of x, the force in the masonry F_m and the force in the concrete F_c can be calculated using 15 and 17. The below equation which forces equilibrium of the section can be used to establish the area of bending steel required.

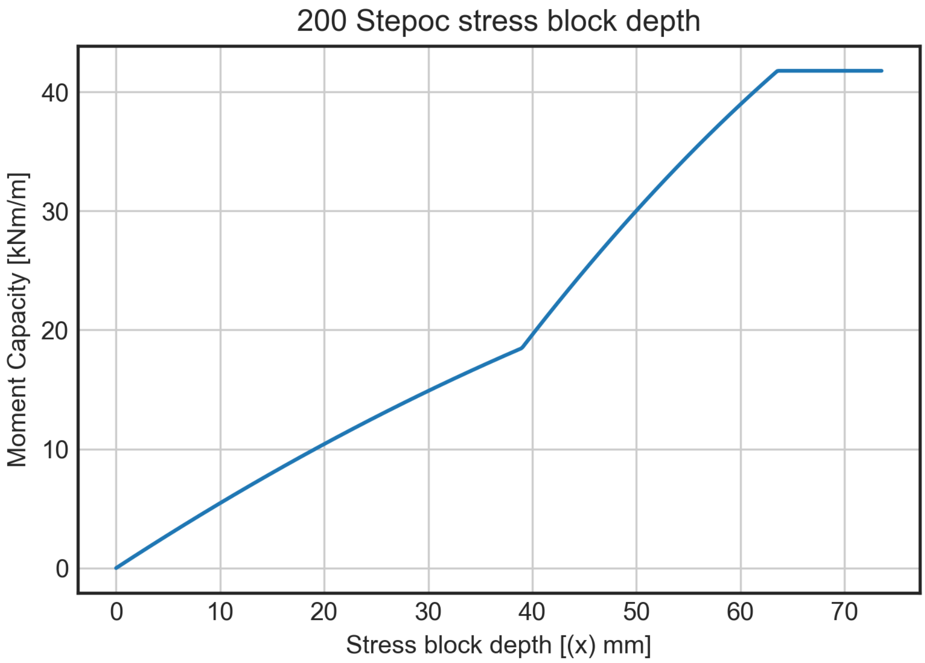
$$A_{st} = \frac{F_m + F_c}{f_{yd}} \tag{21}$$

Where f_{yd} is the design strength of the reinforcement (f_y/γ_M).

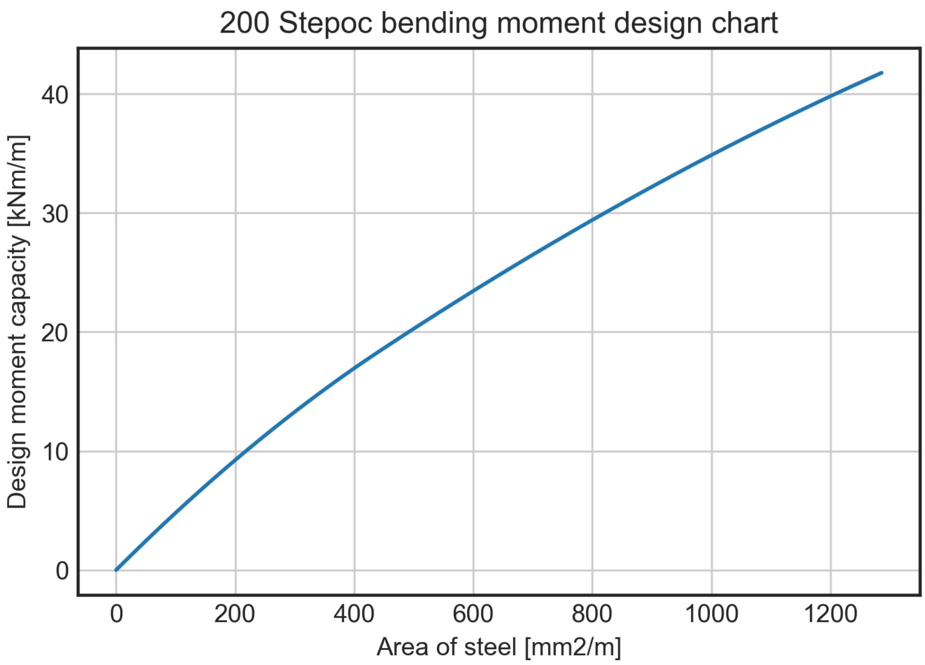
The graphs below plot the assigned values of moment capacity and force given x and provide the bending steel required for a given value of design moment. In each case the f_k for stepoc is assumed mapped to a design value by taking $\gamma_M = 2$. In addition, to be in keeping with historic design curves the vertical reinforcing bar is assumed to be 20 mm diameter and the horizontal bar is assumed to be 10 mm diameter. This factors in the calculation for the effective depth of the section d. A modular ratio α of 3 is assumed in these charts.

2.2 200 mm Stepoc

2.2.1 Stres block depth

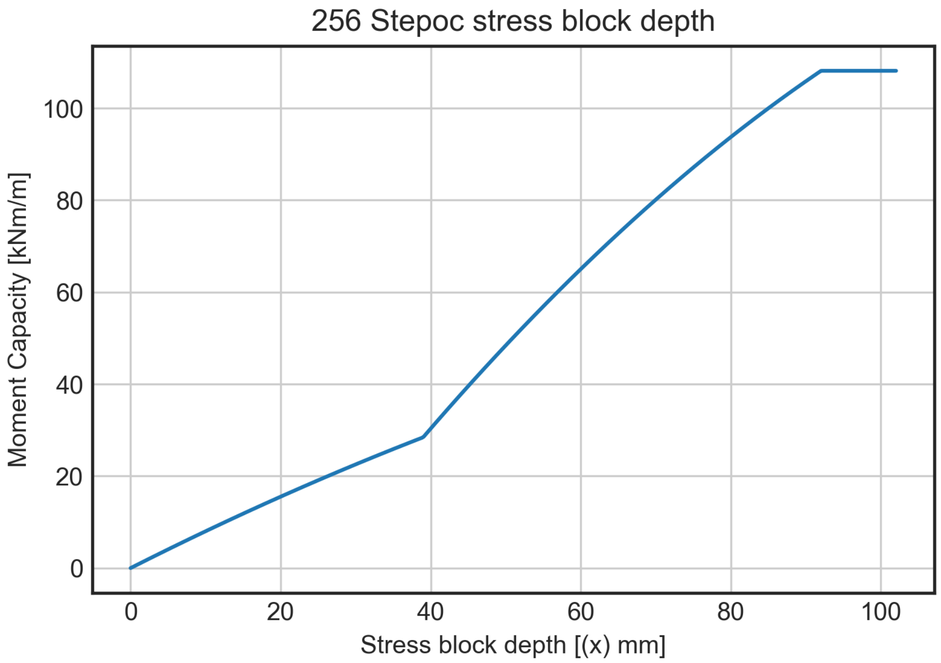


2.2.2 Design moment capacity

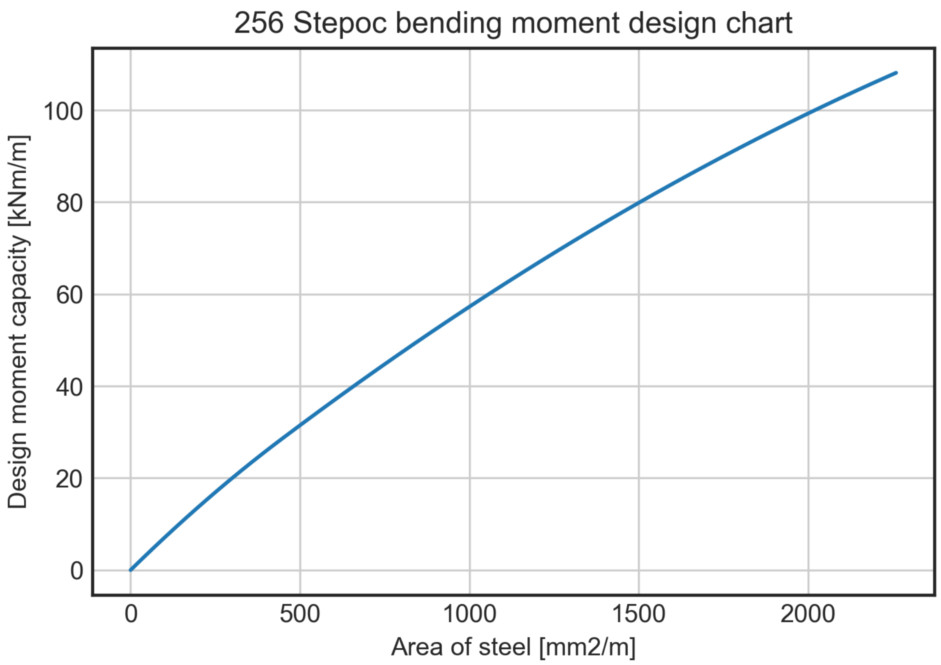


2.3 256 mm Stepoc

2.3.1 Stress block depth

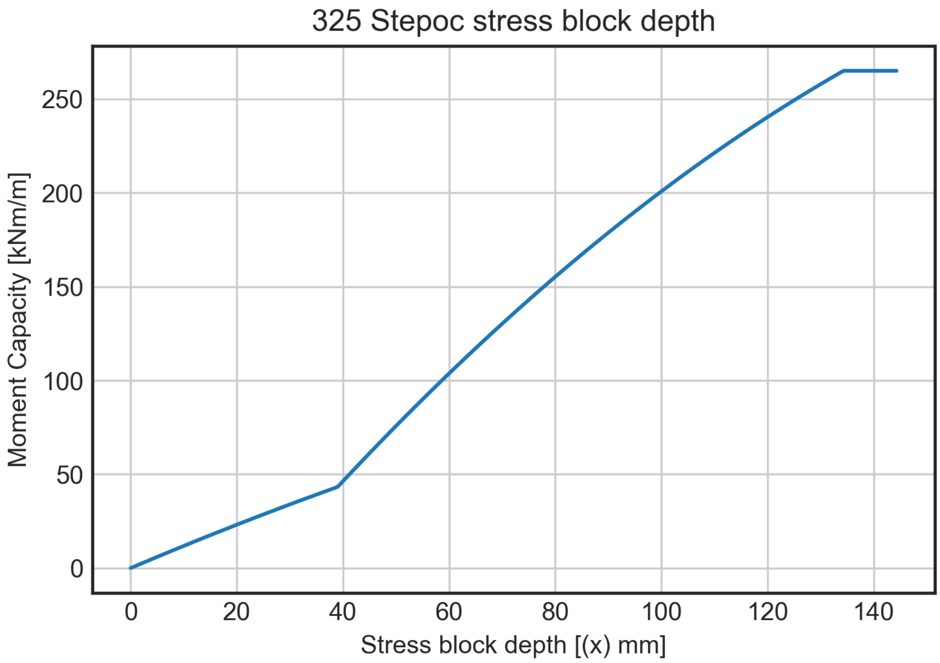


2.3.2 Design moment capacity

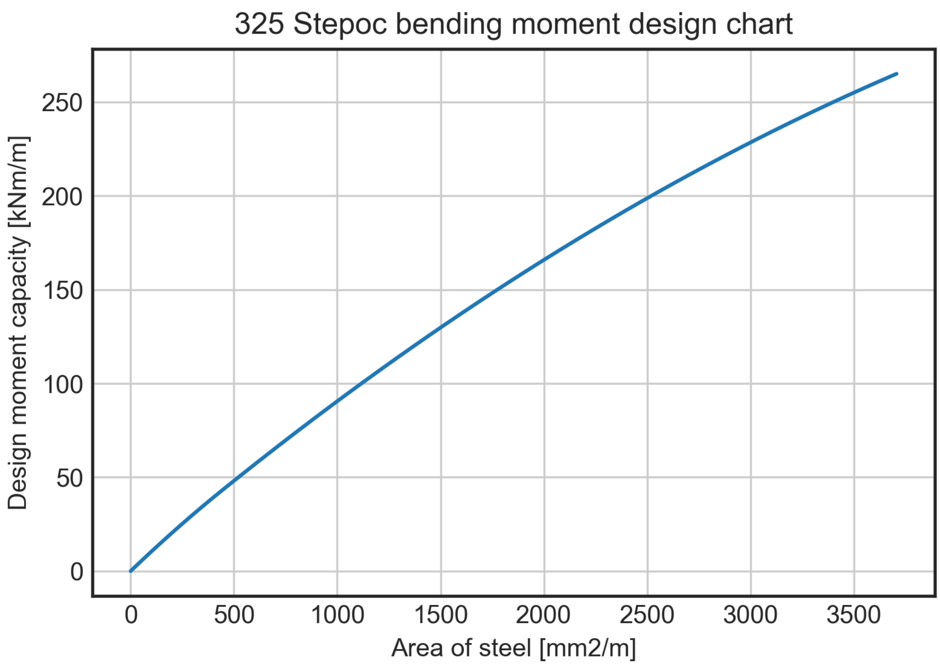


2.4 325 mm Stepoc

2.4.1 Stress block depth



2.4.2 Design moment capacity



3 Shear capacity

3.1 Theory used for shear capacity

The shear capacity of the section is primarily calculated treating the full section as a reinforced masonry section. This is a conservative assumption as the core of the units, once filled with concrete are more characteristic of a reinforced concrete wall. The structural engineer should decide which model they prefer to adopt within their design calculations.

3.1.1 Reinforced masonry method

Annex J of BS EN 1996-1-1 gives the following equation for the shear strength of reinforced masonry:

$$f_{vd} = \frac{0.35 + 17.5\rho}{\gamma_M} \leq \frac{0.7}{\gamma_M} \tag{22}$$

Where:

$$\rho = \frac{As}{bd} \tag{23}$$

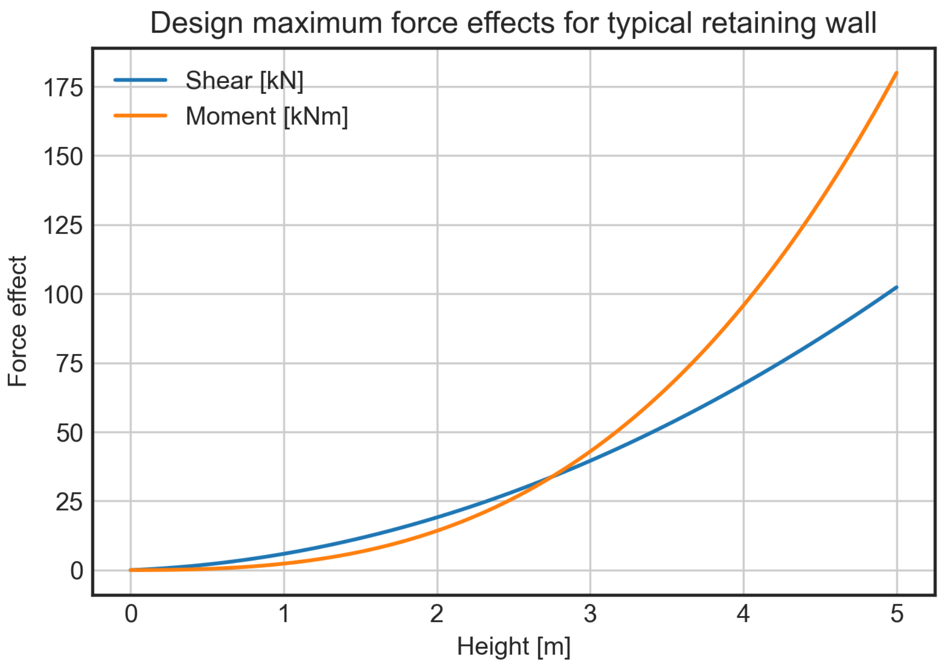
For cantilevered retaining walls the shear strength of the masonry given in equation 22 can be enhanced using the following factor:

$$\chi = \left(2.5 - 0.25\frac{a_v}{d}\right) \geq 1 \tag{24}$$

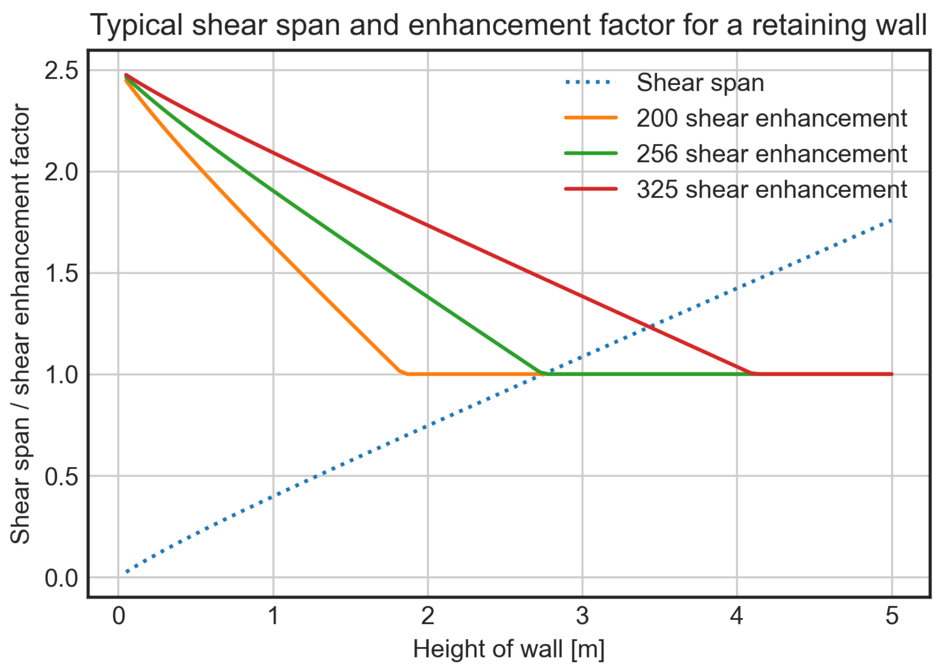
Providing that the final design shear strength f_{vd} does not exceed 1.75 / γ_m. For Stepoc units γ_M is taken to be 2.0. The parameter a_v is the shear span (maximum bending moment in the member divided by the maximum shear force in the member).

Plots of capacities are presented sections 3.2, 3.3 and 3.4.

To investigate the shear enhancement function in the typical condition of a typical wall under active soil conditions with a 5 kPa surcharge and no hydraulic pressure a plot of applied force effects is shown below.



For this plot, the shear span and shear enhancement factor, calculated at the base of the wall across a range of retaining wall heights is presented below:



This plot is not very sensitive to soil parameters. As an example, of its use, the shear enhancement factor for a 2.5 m tall 256 mm Stepoc retaining wall is approximately 1.1.

3.1.2 Reinforced concrete method

It would be to the discretion of the structural engineer as to whether the section is better modelled as a reinforced concrete element with no shear reinforcement. Such modelling will, in principle yield significantly higher shear loads than the reinforced masonry approach.

Within BS EN 1992-1-1 the following design equations govern the shear capacity of an unreinforced section.

$$V_{Rd,c} = 0.12kbd(100\rho f_{ck})^{1/3} \tag{25}$$

$$\rho = \frac{A_s}{bd} \leq 0.02 \tag{26}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \tag{27}$$

This value is subject to a minimum limit of:

$$V_{Rd,c} = 0.035bdk^{3/2}f_{ck}^{1/2} \tag{28}$$

In assessing the net section account must be taken of the size of the pockets. Whilst more rigerous calculations may be undertaken, this report assumes that half the area of a pocket multiplied by the number of pockets per meter of wall and divided by the effective depth of the section less the shell thickness provides a measure of effective breadth.

This gives the following approximate values for effective breadth:

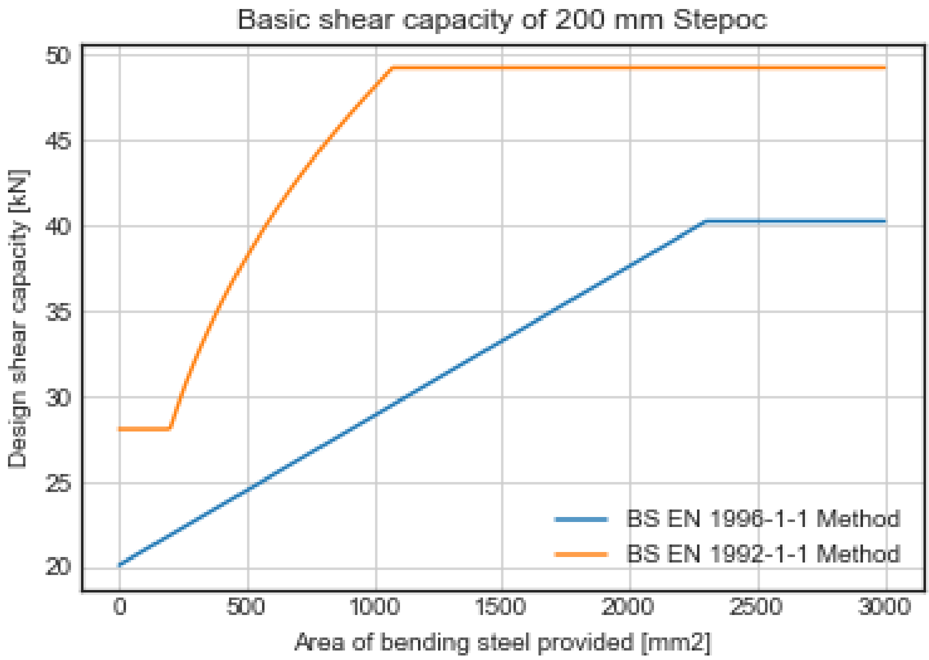
- 200 mm Stepoc = 715 mm
- 252 mm Stepoc = 592 mm
- 325 mm Stepoc = 543 mm

These reduced effective breadths account for the quasi discrete columns that are formed within the Stepoc section.

The concrete core shear capacities are plotted with the reinforced masonry capacities to faciliate the choice of method.

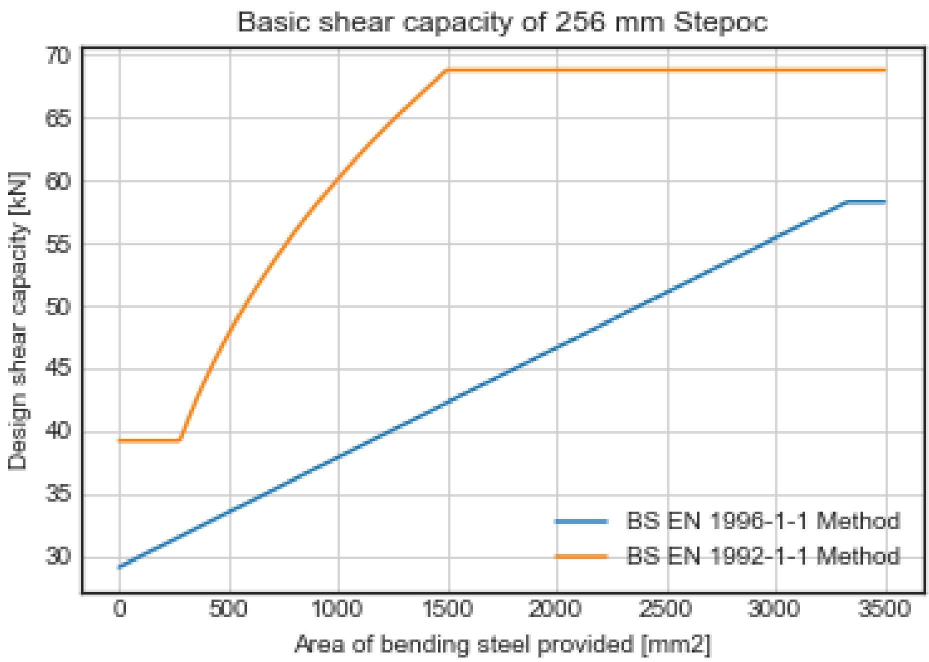
3.2 200 mm Stepoc

The masonry code derived shear capacities (BS EN 1996-1-1 Method) are not enhanced by the shear enhancement factor (see section 3.1.1)



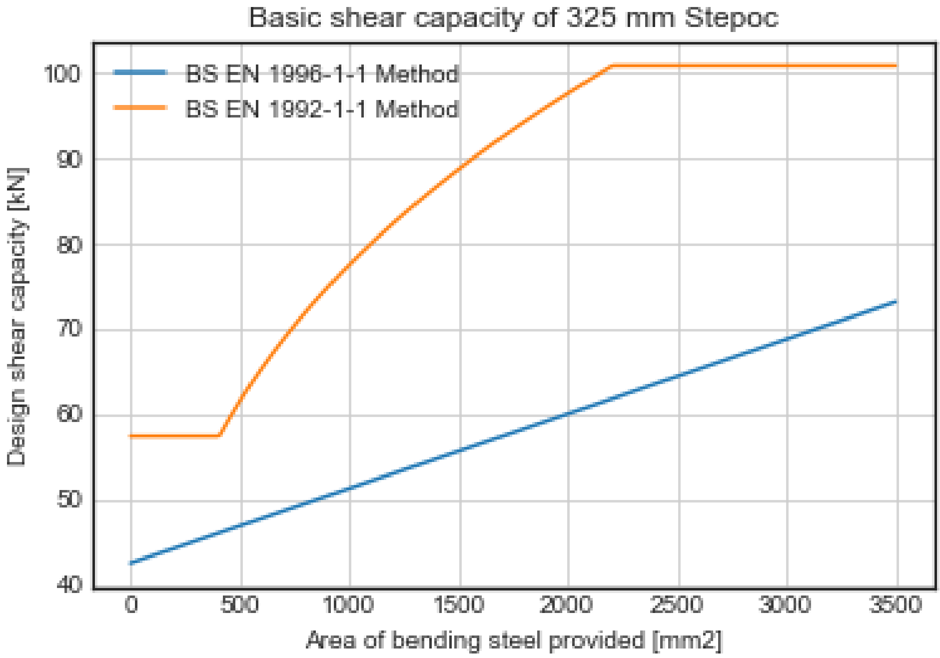
3.3 256 mm Stepoc

The masonry code derived shear capacities (BS EN 1996-1-1 Method) are not enhanced by the shear enhancement factor (see section 3.1.1)



3.4 325 mm Stepoc

The masonry code derived shear capacities (BS EN 1996-1-1 Method) are not enhanced by the shear enhancement factor (see section 3.1.1)



4 Servicability deflections

In accordance with Table 5.2 of BS EN 1996-1-1 the serviceability deflection of Stepoc, assuming a cantilevered wall, is restricted by limiting the maximum height of wall to 18d where d is the effective depth of the section.

Assuming 20 mm diameter vertical bar and a 10 mm horizontal bar these limits are:

- 200 mm Stepoc maximum height = 2070 mm
- 256 mm Stepoc maximum height = 3000 mm
- 325 mm Stepoc maximum height = 4300 mm

References

[1] British Standard Institution. *BS EN 1992-1-1:2004, Eurocode 2: Design of concrete structures. General rules for buildings*. British Standard Institution, London, 2004.

[2] British Standard Institution. *BS EN 1996-1-1:2005, Eurocode 6: Design of masonry structures. General rules for reinforced and unreinforced masonry structures*. British Standard Institution, London, 2005.

[3] British Standard Institution. *NA to BS EN 1996-1-1:2005, UK National Annex to Eurocode 6: Design of masonry structures. General rules for reinforced and unreinforced masonry structures*. British Standard Institution, London, 2005.

[4] British Standard Institution. *NA to BS EN 1992-1-1:2004, UK National Annex to Eurocode 2: Design of concrete structures. General rules for buildings*. British Standard Institution, London, 2014.