# Counting Polar Bears Using Probability

## 9th - 12th grade unit

Created by Dr. Jody Reimer and Linda Zhao, University of Utah, Department of Mathematics

## Math Core Curriculum:

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| **Grades 9-12**<br>**Making Inferences & Justifying Conclusions** | **CCSS.MATH.CONTENT.HSS.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population | **CCSS.MATH.CONTENT.HSS.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion |

## Learning Objectives:

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|---|---|
| To explore how an understanding of probability can help us count the number of polar bears in a population. | Students gain familiarity with the following concepts:<br>• Probability and random sampling<br>• Experimental design |

## Additional Reading Materials:

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|---|---|---|
| **Physical Mark-Recapture**<br>https://www.polarbearscanada.ca/en/research/population-inventories/physical-mark-recapture | **Capture-Mark-Recapture Science**<br>https://www.usgs.gov/centers/eesc/science/capture-mark-recapture-science?qt-science_center_objects=0# | **Academic Sources:**<br>Amstrup, S. C., McDonald, T. L., & Stirling, I. (2001). Polar bears in the Beaufort Sea: A 30-year mark-recapture case history. Journal of Agricultural, Biological, and Environmental Statistics, 6(2), 221-234. |

## Counting Polar Bears Using Probability

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| **Pre-lesson reading:** Pick one polar bear subpopulation to read about in this report (beginning on page 13):<br>https://www.iucn-pbsg.org/wp-content/uploads/2021/11/July-2021-Status-Report-Web.pdf | Write down any questions or new vocabulary words here: |

## Guided Reading Questions

**Using the pre-lesson reading, answer these questions:**

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| **1.** Which subpopulation did you pick to read about? | |
| **2.** Has the subpopulation of polar bears increased or decreased? | |
| **3.** Why do you think the sub population has either increased or decreased? | |
| **4.** What is one way you can come up with for scientists to count how many polar bears there are in a big area? What do you think might be some challenges to counting polar bears? | |

## How to count polar bears—using probability!

To get a good estimate for the number of polar bears in an area, we use a method called "mark-recapture." There are two steps: a "mark" step and a "recapture" step, which we will explore through the activity on the following pages.

## Hands on Activity

Materials you will need:

- Macaroni (or some other small pasta, beans, etc.)
- Marker in a different color than the macaroni (e.g. black)

**STEP 1: Count out 50 pieces of macaroni and place them in a pile. Each macaroni represents one polar bear, and this pile represents the full, true population (i.e., 50 polar bears). After making the pile, we will pretend that we don't know how many there are, because in the real world, we don't know how many there are!**

**STEP 2: Randomly pick out 20 pieces of macaroni from the population pile and mark them each clearly with the marker. This is the "mark" part of "mark-recapture."**

**STEP 3: Return and mix the marked macaroni back into the pile.**

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| **5.** If you now close your eyes and pick one macaroni from the pile, what is the probability that it has been marked? | |
| **6.** What information did you need to be able to answer that question? Could you have answered it without knowing how many macaroni you started with? | |

**STEP 4: *Randomly* select 20 new macaroni pieces from your pile; some will probably be marked, and some will not. This is the "recapture" part of "mark-recapture."**

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| **7.** Without looking, how many of these 20 pieces do you expect to be marked? | |

**STEP 5:** **Count how many of these 20 new macaroni are already marked (i.e., count how many you have "recaptured").**

**STEP 6:** **If x represents the number of macaroni already marked, then our estimate for the total number of bears in the pile can be calculated as:**

***Estimated number of bears =***

Using this equation check how close your estimate is to the true number, which is 50 (remember: you wouldn't know that it is 50 for a real population!) Did your estimate do a good job?

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| **8.** What do you think we could do to improve how well our estimate did? | |

**STEP 7:** **Take the average between your estimate and the estimates of a few friends or classmates.**

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| **9.** Is the average closer to the true number of bears or further? Why? | |
| **10.** Do you think this method will work well for polar bears in the wild? Why or why not? | |

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**STEP 2: Randomly pick out 20 pieces of macaroni from the population pile and mark them each clearly with the marker. This is the "mark" part of "mark-recapture."**

**STEP 3: Return and mix the marked macaroni back into the pile.**

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| **5.** If you now close your eyes and pick one macaroni from the pile, what is the probability that it has been marked? | Solution: since 20 pieces out of 50 are marked, the probability of any given macaroni being marked is 20/50 = 0.4. That means there is a 40% chance the macaroni is marked. |
| **6.** What information did you need to be able to answer that question? Could you have answered it without knowing how many macaroni you started with? | Solution: No, if you didn't know how many macaroni were in the pile, you could not answer the question since you wouldn't know the denominator of the fraction. |

**STEP 4: *Randomly* select 20 new macaroni pieces from your pile; some will probably be marked, and some will not. This is the "recapture" part of "mark-recapture."**

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| **7.** Without looking, how many of these 20 pieces do you expect to be marked? | Solution: Initially, students can answer with their gut feeling, maybe restricting the answer to a range such as "more than 5 but less than 15". The correct answer comes from multiplying each of the 20 pieces by the probability that piece is already marked, so (20)(0.4) = 8. So we should expect 8 pieces to be marked. (Note that this is the mean of the binomial distribution, where n=20 and p=0.4, the probability of "success" for each trial.) |

**STEP 5: Count how many of these 20 new macaroni are already marked (i.e., count how many you have "recaptured").**

**STEP 6: If x represents the number of macaroni already marked, then our estimate for the total number of bears in the pile can be calculated as:**

***Estimated number of bears =***

Using this equation check how close your estimate is to the true number, which is 50 (remember: you wouldn't know that it is 50 for a real population!) Did your estimate do a good job?

| | |
|---|---|
| **8.** What do you think we could do to improve how well our estimate did? | Solution: the answer we are looking for here is that sampling more macaroni would help improve our estimate. In general, more data is usually better. The more data we have, the less likely it is that we see a pattern by chance (e.g., think about flipping a coin 3 times to see if it is fair, versus flipping it 3000 times! You would learn a lot more by flipping it 3000 times.) |

**STEP 7: Take the average between your estimate and the estimates of a few friends or classmates.**

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| **9.** Is the average closer to the true number of bears or further? Why? | Solution: it should typically be closer to the true number. This gets at the idea of "replicates"—if you can repeat an experiment many times, you will get a better estimate! |
| **10.** Do you think this method will work well for polar bears in the wild? Why or why not? | Solution: there are lots of challenges to conducting mark-recapture studies in the wild. Bears are difficult to catch, since they are not all in a pile like the macaroni. Additionally, the true number of bears changes over time, as bears enter and leave the study area, as well as die and give birth. Students may come up with other ideas of why mark-recapture methods may be tricky to implement. Luckily, the statistical tools needed to deal with many of these challenges have been established so that we can still learn something about the size of polar bear populations! |