POLAR BEARS INTERNATIONAL

Geometry of Polar Bear Movement Lesson Information

Created by Dr. Jody Reimer and Linda Zhao, University of Utah, Department of Mathematics

Math Core Curriculum

Gr. 6 – Geometry (CCSS. MATH.CONTENT.6.G.A.1: Solve real-world and mathematical problems involving area, surface area, and volume)	**Gr. 7 – Geometry (CCSS. MATH.CONTENT.7.G.B.6:** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume)	**Gr. 8 – Geometry (CCSS. MATH.CONTENT.8.G.B.7:** Understand and apply the Pythagorean Theorem)

Learning Objectives

To use concepts from geometry such as perimeter, area, and the Pythagorean theorem to explore how polar bears move through their environment.

Students will need to be familiar with the following techniques to solve real life problems:

- Standard operations including addition, subtraction, multiplication, division.
- Movement rates in units of km/hr; ratios and proportional units
- Pythagorean Theorem

Academic Sources

Johnson, A. C., Pongracz, J. D., & Derocher, A. E. (2017). Long-distance movement of a female polar bear from Canada to Russia. Arctic, 121-128.

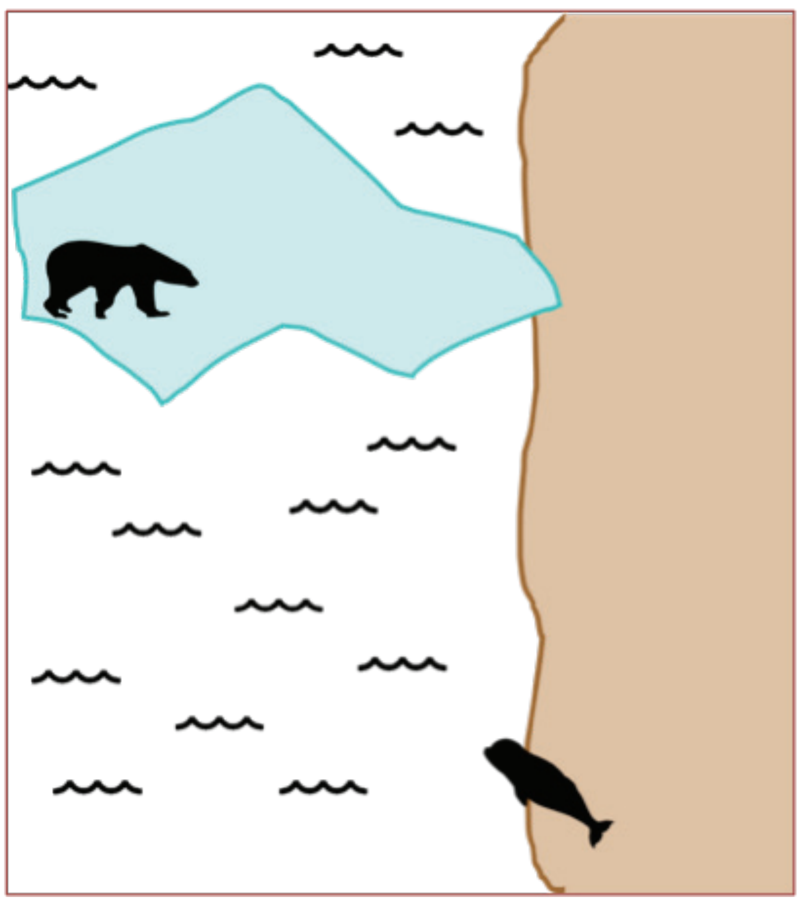
Griffen, B. D. (2018). Modeling the metabolic costs of swimming in polar bears (Ursus maritimus). Polar Biology, 41(3), 491-503.

Geometry of Polar Bear Movement

Before the lesson *(optional)*:

Open this link: https://polarbearsinternational.org/polar-bears-changing-arctic/polar-bear-facts/habitat/ and read the sections on habitat, especially “Getting Around the Arctic”. While reading, write down any vocabulary words you do not understand in the box.

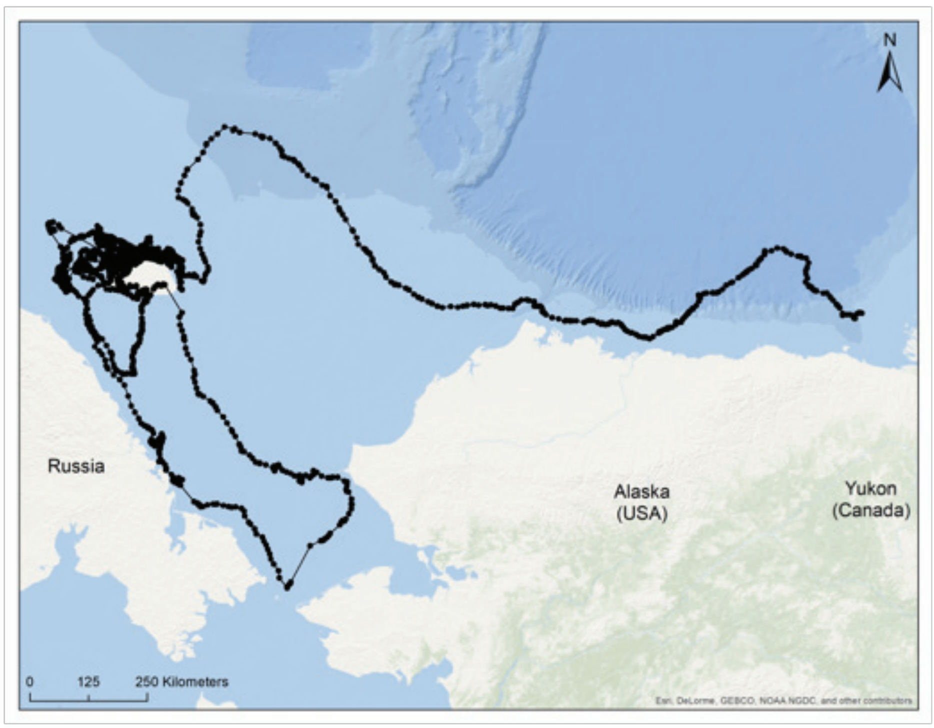
New words:

1. Imagine a polar bear walks around the edge of an ice floe that has five sides, each measuring approximately 1.5 km, hoping that a seal pokes its head up. Draw a diagram. What shape is this ice floe?	
2. What is the total distance the polar bear has travelled while walking all the way around the ice floe?	
3. If the bear is walking at a rate of 5 km/hr, how long does it take for them to walk all the way around this ice floe? *Bonus*: What is the area of this ice floe?	
4. Now imagine there is a polar bear on the far side of an ice floe that has bumped up against the shore and it smells a whale carcass on the shore (see the picture). The polar bear can either walk east for 3km until it hits land and then walk south along the shore for 4km, or it can swim directly to the whale carcass. The polar bear wants to be the first bear to this feast! Without doing any math, what is your guess about whether they should swim or take the long way walking?	

5. Calculate which path would be faster if the path leading to land is 3 km and the whale carcass is 4 km away once land is reached. Assume the bear can walk at a speed of 5 km/hr or swim at a speed of 2 km/hr. The following table might be helpful to organize your calculations.

	Walking Route	Swimming Route
Distance Travelled		
Time Required to Reach Whale		

Polar bears can travel surprisingly long distances. A female polar bear travelled a total of 11,686 km from Canada to Russia. The bear began traveling in spring of 2009 and her recorded journey lasted 798 days, more than 2 years!



6. On average, how far did she travel each day?

7. Do you think you could walk that far in a day? Could you swim that far in a day?

Geometry of Polar Bear Movement

Lesson Solutions

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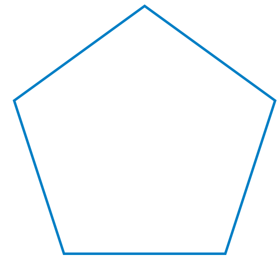
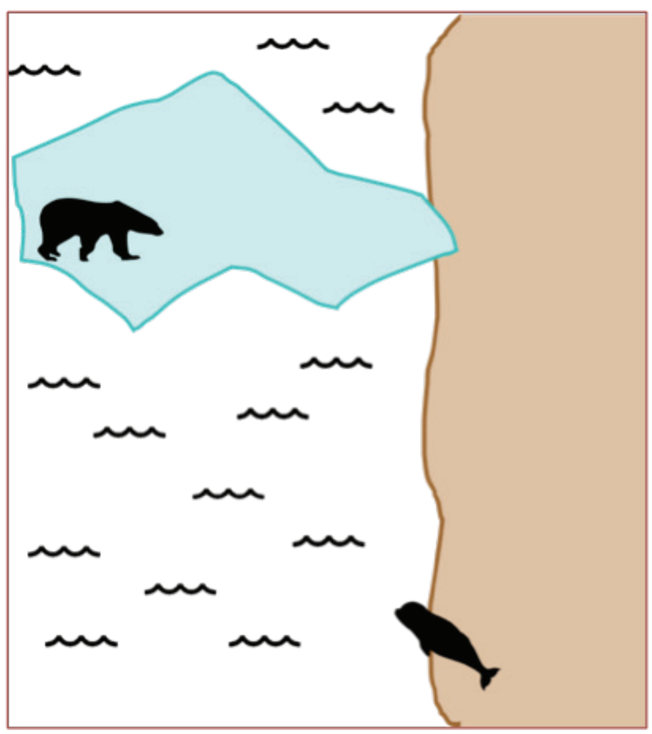
New words:

1. Imagine a polar bear walks around the edge of an ice floe that has five sides, each measuring approximately 1.5 km, hoping that a seal pokes its head up. Draw a diagram. What shape is this ice floe?	***Solution:*** *This is a regular pentagon. It would be good to discuss with the students whether or not they think there are other ways to draw the ice floe such that each of the 5 sides are 1.5 km in length, i.e., is the solution unique?* 
2. What is the total distance the polar bear has travelled while walking all the way around the ice floe?	***Solution:*** *1.5+1.5+1.5+1.5+1.5 = 5 x 1.5 = 7.5 km*
3. If the bear is walking at a rate of 5 km/hr, how long does it take for them to walk all the way around this ice floe? *Bonus*: What is the area of this ice floe?	***Solution:*** *7.5 ÷ 5 = 1.5 hr. This could also be accompanied by a discussion of how the units work for this problem, i.e., [km]/[km/hr] = [hr].* ***Bonus Solution:*** *The formula for the area of a regular pentagon, A, given a side length of x, is* A=x245(5+25) Here x=1.5 km, so A = 3.87 km2.
4. Now imagine there is a polar bear on the far side of an ice floe that has bumped up against the shore and it smells a whale carcass on the shore (see the picture). The polar bear can either walk east for 3km until it hits land and then walk south along the shore for 4km, or it can swim directly to the whale carcass. The polar bear wants to be the first bear to this feast! Without doing any math, what is your guess about whether they should swim or take the long way walking?	 ***Solution:*** *Encourage discussion about variable speeds of travel (e.g., do the students think swimming is faster or slower than walking?)*

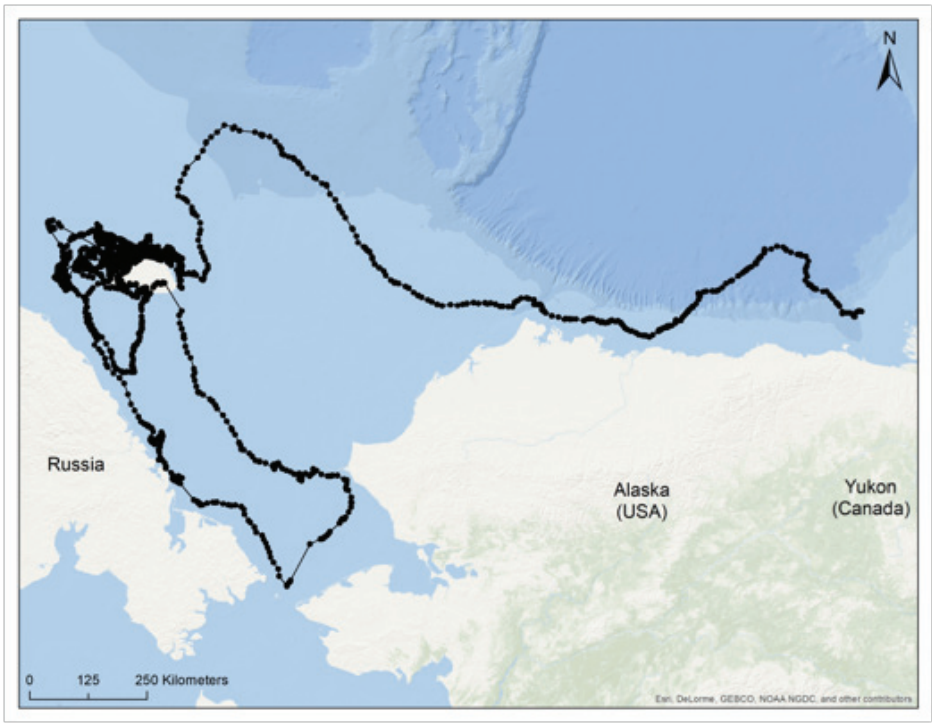
5. Calculate which path would be faster if the path leading to land is 3 km and the whale carcass is 4 km away once land is reached. Assume the bear can walk at a speed of 5 km/hr or swim at a speed of 2 km/hr. The following table might be helpful to organize your calculations.

Solution: *To solve this problem, you will need to discuss how the walking route along the ice and then the shore create two sides of a right angle triangle, with the swimming route as the hypoteneuse. This can then set up the problem to use the Pythagorean Theorem.*

	Walking Route	Swimming Route
Distance Travelled	3 km + 4 km = 7 km	5 km (by Pythagorean Theorem)
Time Required to Reach Whale	7 km ÷ 5 km/hr = 1.4 hr (= 1 hr and 24 min.)	5 km ÷ 2 km/hr = 2.5 hr (= 2 hr and 30 min.)

So it is faster for the bear to walk, even though that requires the bear to travel a longer distance.

Polar bears can travel surprisingly long distances. A female polar bear travelled a total of 11,686 km from Canada to Russia. The bear began traveling in spring of 2009 and her recorded journey lasted 798 days, more than 2 years!



6. On average, how far did she travel each day?

Solution: *11686 km ÷ 798 days = 14 km per day.*

7. Do you think you could walk that far in a day? Could you swim that far in a day?

Solution: *Students may be more comfortable thinking in miles, so it may be helpful to convert 14 km to miles by dividing by 1.609 to get distance of 8.7 mi.*

Polar Bears and Energy Lesson Information

POLAR BEARS INTERNATIONAL

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Math Core Curriculum

Gr. 6 – The Number System	Gr. 7 – Ratios and Proportional Relationships
CCSS.MATH.CONTENT.6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm	**CCSS.MATH.CONTENT.7.RP.A.2:** Recognize and represent proportional relationships between quantities.

Learning Objectives

To understand conversions between calories and MJ (the standard scientific units of energy), in the context of polar bear energy use.

Students will need to be familiar with the following techniques to solve real life problems:

- Standard operations including addition, subtraction, multiplication, division.
- Unit conversions

Additional Reading Material

Polar Bears Diet & Prey

https://polarbearsinternational.org/polar-bears-changing-arctic/polar-bear-facts/diet-prey/

A demanding lifestyle

https://www.science.org/doi/full/10.1126/science.aan8677

Academic Sources

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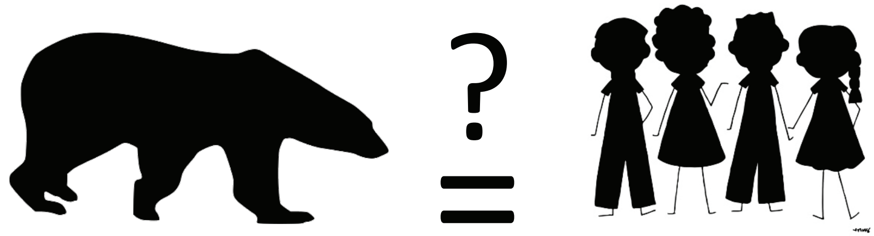
Pagano, A. M., & Williams, T. M. (2019). Estimating the energy expenditure of free-ranging polar bears using tri-axial accelerometers: A validation with doubly labeled water. Ecology and evolution, 9(7), 4210-4219.

Unit Conversion

Most of us measure our weight in pounds (lb), but scientists prefer to measure mass in kilograms (kg) instead. To go between the two measurements, we just need to know that **1 kg = 2.205 lb**.

1. Using this information, can you convert your weight from pounds (lb) to kilograms (kg)?

2. Now that you know your weight in kgs, we can compare you to a polar bear! If an average polar bear weighs 450 kg, how many of you would make up a polar bear?



Nutrition Facts

Serv. size 1 bar (68g)

Calories per serving 240

Amount/serving	% DV	Amount/serving	% DV	Amount/serving	% DV
Total Fat 3.5g	**5%**	**Cholesterol** 0mg	**0%**	Total Sugars 23g	
Sat. Fat 0.5g	**3%**	**Sodium** 115mg	**5%**	Incl. 19g Added Sugars	**38%**
Trans Fat 0g		**Total Carb.** 46g	**17%**	**Protein** 9g	**17%**
Polyunsat. Fat 1g		Dietary Fiber 4g	**14%**		
Monounsat. Fat 1.5g		Insoluble Fiber 3g			

Vit. D 2mcg 8% • Calcium 201mg 15% • Iron 2mg 10% • Potas. 233mg 4% • Vit. A 2% • Vit. C 6% • Vit. E 10% • Thiamin (Vit. B_1) 15% • Riboflavin (Vit. B_2) 15% • Niacin 25% • Vit. B_6 10% • Vit. B_{12} 25% • Phosphorus 20% • Magnesium 20%

Polar bears really like to eat seals – they are full of yummy fat! One bearded seal provides approximately 3700 MJ (megajoules) of energy. It's easier to understand just how much energy that is if we change the units to something more familiar. In fact, 1 mJ = 239 calories (the unit that is used to measure energy on food packaging), roughly the number of calories in one Clif Bar.

3. Approximately how many Clif Bars are equivalent to one bearded seal?

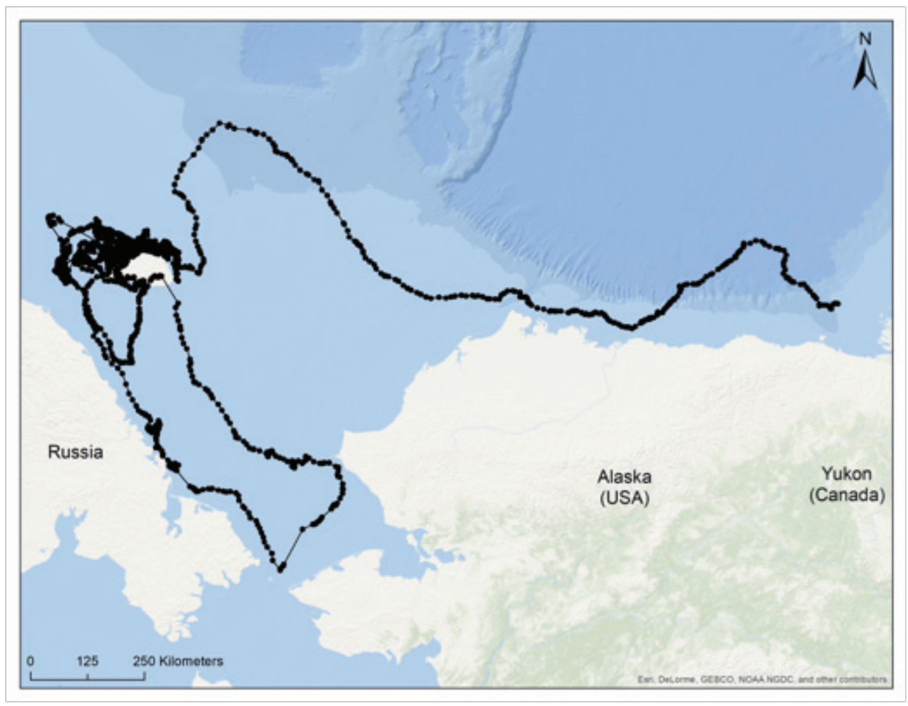
Ringed seals are smaller, and each ringed seal provides approximately 600 MJ.

4. How much energy will a polar bear gain if it hunts and eats 10 ringed seals? Do you have a guess for how long it might take a polar bear to catch 10 ringed seals?

? =

Bearded Seal on ice, Svalbard, Hornsund, 2001. Photo by Michael Haferkamp

Polar bears can travel really long distances. A female polar bear travelled a total of 11,686 km from Canada to Russia. The bear began traveling in the spring of 2009 and her recorded journey lasted 798 days, more than 2 years! Her path can be seen in the map on the right.



5. How much energy would she have used on this long journey if she uses 55 MJ per day?

6. How many bearded seals would she need to eat to fuel her trip?

Polar Bears and Energy
Lesson Solutions

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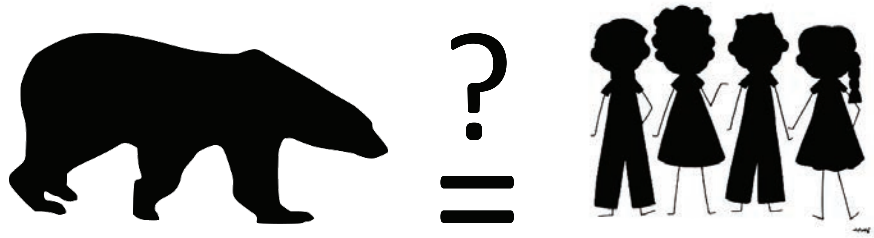
Unit Conversion

Most of us measure our weight in pounds (lb), but scientists prefer to measure mass in kilograms (kg) instead. To go between the two measurements, we just need to know that **1 kg = 2.205 lb**.

1. Using this information, can you convert your weight from pounds (lb) to kilograms (kg)?

Solution: *for example, if your weight is 100 lbs, then your weight in kgs can be calculated as 100/2.205 = 45.35 kg.*

2. Now that you know your weight in kgs, we can compare you to a polar bear! If an average polar bear weighs 450 kg, how many of you would make up a polar bear?



Solution: *For example, if your weight is 45.35 kg, then the solution is:*

450 / 45.35 = 9.92, so the polar bear is nearly 10 times as big as you. Imagine 10 of your friends all standing together, combined into one polar bear!

Nutrition Facts

Serv. size 1 bar (68g)

Calories per serving **240**

Amount/serving	% DV	Amount/serving	% DV	Amount/serving	% DV
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Polar bears really like to eat seals – they are full of yummy fat! One bearded seal provides approximately 3700 MJ (megajoules) of energy. It's easier to understand just how much energy that is if we change the units to something more familiar. In fact, 1 mJ = 239 calories (the unit that is used to measure energy on food packaging), roughly the number of calories in one Clif Bar.

3. Approximately how many Clif Bars are equivalent to one bearded seal?

Solution: *if one bearded seal is 3700 MJ, and a Clif bar is roughly 1 mJ, then the bearded seal is equivalent to approximately 3700 Clif bars. That's a lot of Clif bars.*

<table>
<tr>
<td>Ringed seals are smaller, and each ringed seal provides approximately 600 MJ.

4. How much energy will a polar bear gain if it hunts and eats 10 ringed seals? Do you have a guess for how long it might take a polar bear to catch 10 ringed seals?</td>
<td>

? =


Solution: energy from 10 ringed seals = (10)(600) = 6000 MJ. How quickly a polar bear can catch 10 ringed seals depends on where the bear is hunting and how many seals are in the area, so the number can vary wildly. In the spring, when hunting is good, if a bear caught a ringed seal every 3 days, it would take them roughly a month to catch 10 ringed seals.

Bearded Seal on ice, Svalbard, Hornsund, 2001. Photo by Michael Haferkamp</td>
</tr>
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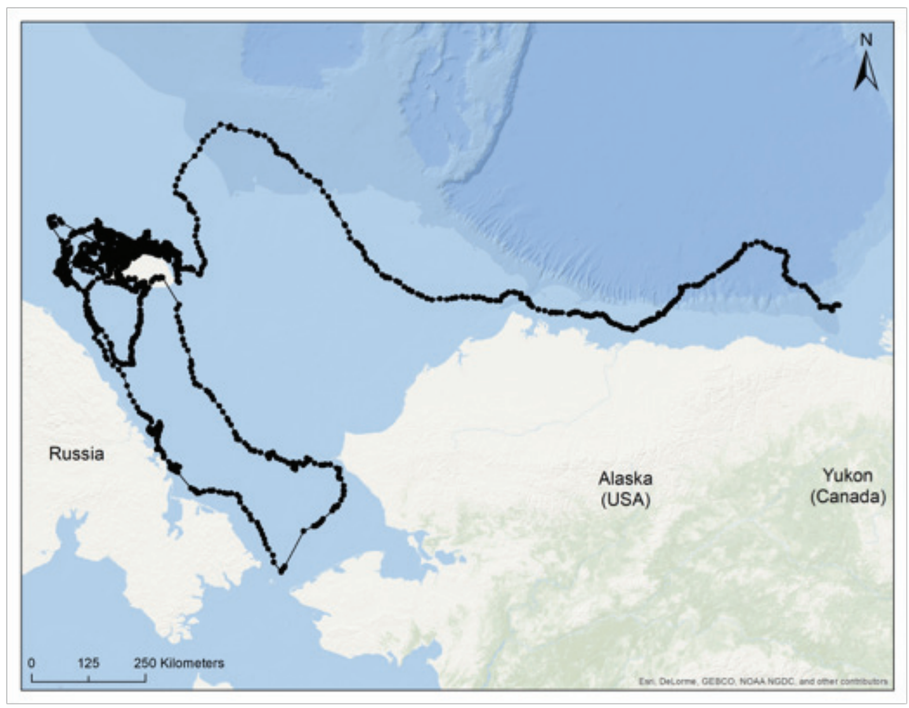
<td>Polar bears can travel really long distances. A female polar bear travelled a total of 11,686 km from Canada to Russia. The bear began traveling in the spring of 2009 and her recorded journey lasted 798 days, more than 2 years! Her path can be seen in the map on the right.</td>
<td>
</td>
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<tr>
<td>5. How much energy would she have used on this long journey if she uses 55 MJ per day?</td>
<td>Solution: If she was travelling for 798 days, and each day she used 55 MJ, then her total energy used was (798)(55) = 43,890 MJ.</td>
</tr>
<tr>
<td>6. How many bearded seals would she need to eat to fuel her trip?</td>
<td>Solution: Since a bearded seal provides 3,700 MJ of energy, she will need 43,890 MJ ÷ 3700 MJ per seal = 11.86 seals. Does that number seem reasonable to you, given how long she travelled? If she was eating smaller prey, such as ringed seals, do you think she would need to eat more or less?</td>
</tr>
</table>

POLAR BEARS INTERNATIONAL

Trends in Sea Ice Data Lesson Information

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Math Core Curriculum

Grades 9-12 **Interpreting Categorical & Quantitative Data**	**CCSS.MATH.CONTENT.HSS.ID.B.6:** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	**CCSS.MATH.CONTENT.HSS.ID.C.7:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Learning Objectives

To understand statistical principles using real world examples and to make inferences with the information that is presented.	Students will gain familiarity with the following ideas: • Estimating and interpreting trend lines • Extrapolation outside of observed values

Additional Materials

Conservation Concerns: Climate Warming https://polarbearsinternational.org/polar-bears-changing-arctic/conservation-concerns/	**Annual Arctic Sea Ice Minimum 1979-2020 with Area Graph** by NASA Climate Change https://youtu.be/Vphz0HbbQVo	**Sea ice Data Availability:** Sea ice data from the National Snow and Ice Data Center can be obtained on their website: https://nsidc.org/data/g02135#

Trends in Sea Ice Data

Looking for trends in data

We are often interested in how the world is changing over time. Statistics can help us understand what we observe and whether these observations are due to chance or if they represent something real about the world.

Is it just bad luck or is something suspicious happening?

Imagine that you have one box of candy to share with a friend. Your friend proposes a game with a coin she has in her pocket, to determine how much candy you both get. Every time the coin is "heads", your friend gets to eat a piece of candy. When it's "tails", you get a piece.

You flip the coin 5 times. The results are 4 heads, and 1 tails. It must just not be your day for luck! You flip the coin another 5 times; this time it's 5 heads and no tails. You are starting to get suspicious; is this game rigged?!

1. What kinds of numbers of heads and tails would make you suspicious about your friend's coin being weighted unfairly? How do you know it's not just chance?	
2. What kind of experiment could you do to convince yourself the coin is unfair and not that you are just unlucky?	

Statistics gives us the tools to figure out when things are likely just chance, or when something really is going on!

Changing sea ice

Climate change is causing us to observe lots of changes around the world. One change that affects all Arctic animals is a shrinking of the sea ice (see figure below).

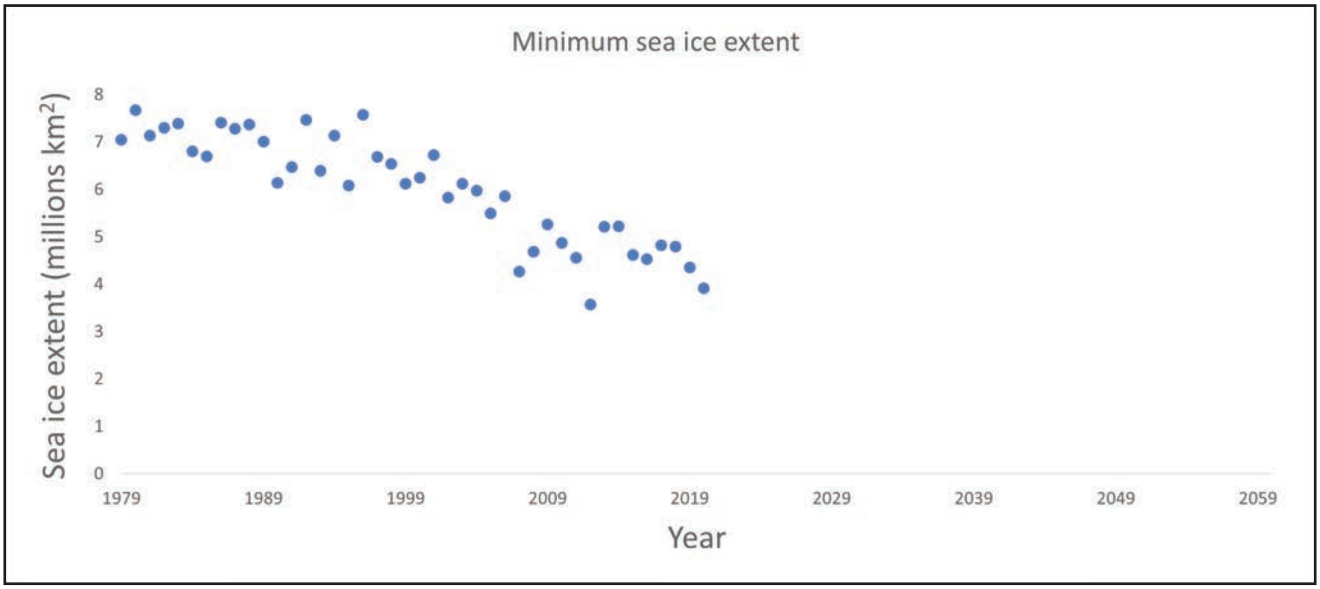

Figure 1: Sea ice extent, as calculated from satellite data since 1979 to 2020.

3. Do you think there is a "best" line we could draw through the data? Using your ruler, draw a straight line on the graph that seems to capture the main trend or pattern in the data.

4. Come up with at least one idea for how to determine which line fits the data "best."	
5. Do you think this pattern is happening by chance, or does the evidence seem strong enough to suggest real changes?	
6. From looking at the line you drew on the graph, when do you think that there will be no ice in September in the summer?	

7. Do you think this guess of when there will be an ice-free summer is realistic or like to happen? What other factors might influence the actual date? **(Bonus):** Using the internet to search, see if you can find the year in which scientists think that we will have an ice-free summer.	
8. Do you think a straight line did a good job capturing the patterns in the data?	
9. How do you think this trend in sea ice will affect polar bears?	

Trends in Sea Ice Data
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1. What kinds of numbers of heads and tails would make you suspicious about your friend's coin being weighted unfairly? How do you know it's not just chance?	***Solution:** There is no "right" answer, however it may be helpful to discuss concrete numbers (e.g., would 100 heads and no tails be suspicious? What about 1000 heads and 50 tails? Or 600 heads and 400 tails?)*
2. What kind of experiment could you do to convince yourself the coin is unfair and not that you are just unlucky?	***Solution:** Again, there is no right or wrong answer—the goal is to brainstorm and think about how we "know" things using data, and what counts as sufficient evidence.*

Statistics gives us the tools to figure out when things are likely just chance, or when something really is going on!

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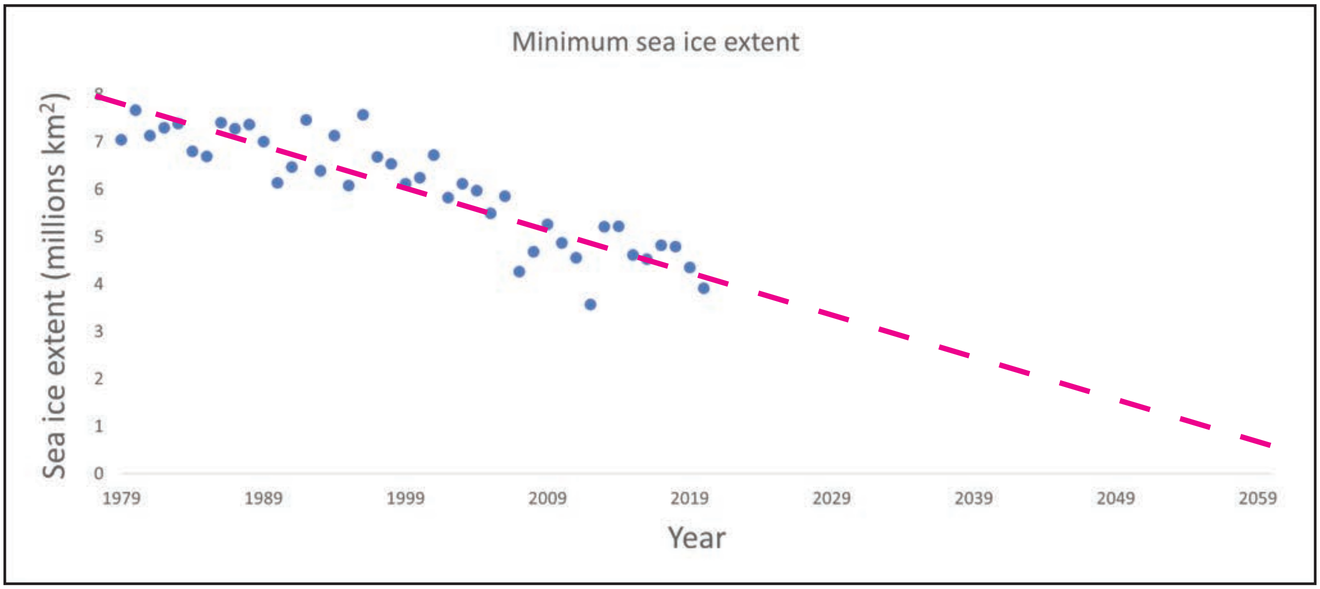

Figure 1: Sea ice extent, as calculated from satellite data since 1979 to 2020.

3. Do you think there is a "best" line we could draw through the data? Using your ruler, draw a straight line on the graph that seems to capture the main trend or pattern in the data.	***Solution:*** ***A possible "best fit" line is drawn using red dashes. The idea is to just provide something that looks somewhat reasonable.***
4. Come up with at least one idea for how to determine which line fits the data "best."	***Solution:*** ***This is another brainstorming question; there are many ways in which scientists decide which line fits "best" (e.g., maximum likelihood, minimizing the sum of squares). If students are stumped, you could suggest a line with really poor fit to the data, ask why they think it is a poor fit. One key idea to cover here is the idea of somehow minimizing how far away the line is from each of the points.***
5. Do you think this pattern is happening by chance, or does the evidence seem strong enough to suggest real changes?	***Solution:*** ***This is an opinion question. Most students will likely see this evidence as quite strong; you could propose other made-up data that is more or less compelling, to help develop student intuition.***

6. From looking at the line you drew on the graph, when do you think that there will be no ice in September in the summer?	***Solution:*** *Look at the x intercept; for the red line above, the line intersects the x axis at approximation year 2060.*
7. Do you think this guess of when there will be an ice-free summer is realistic or like to happen? What other factors might influence the actual date? **(Bonus):** Using the internet to search, see if you can find the year in which scientists think that we will have an ice-free summer.	***Solution:*** *Many factors might influence when there is actually an ice free summer. Scientists often talk about feedback loops and tipping points. One of these is related to sea ice albedo – water reflects less sunlight than ice, so as less and less ice is present, the air gets warmer faster and faster, causing an increase in the rate of sea ice decline. Other factors that may influence the rate of sea ice loss include the amount of emissions due to human activity, which could either slow down this rate or speed it up.* ***Bonus Solution:*** *There is no one scientific consensus on this; depending on assumptions made about the emissions scenario and the model used, possible estimates could include, for example the years 2035, 2048, or 2086. [See Guarino, M. V., Sime, L. C., Schröeder, D., Malmierca-Vallet, I., Rosenblum, E., Ringer, M., … & Sellar, A. (2020). Sea-ice-free Arctic during the Last Interglacial supports fast future loss. Nature Climate Change, 10(10), 928-932.] In general, model averages suggest sometime in the middle of our current century.*
8. Do you think a straight line did a good job capturing the patterns in the data?	***Solution:*** *students may answer yes or no; an alternate could be a line with a slight curve, or many curves. What about a line that goes through each of the points, like a connect-the-dots picture; do they think that is a useful line to look at? Why or why not? In general, we try to find patterns in data – if you draw a line connecting every dot, you cannot see the bigger pattern, which makes it difficult to understand why or how things are changing.*
9. How do you think this trend in sea ice will affect polar bears?	***Solution:*** *polar bears need sea ice as a platform for hunting, travel, for finding mates, and even for giving birth in some areas. If the sea ice is melting earlier and forming later in the fall, this shortens the amount of time polar bears can hunt on the ice. Additionally, as sea ice melts, the whole Arctic ecosystem will experience shifts, from the phytoplankton and algae at the bottom of the food web, through the zooplankton, fish, and marine mammals, which ultimately affects the available food for polar bears.* If students are interested in helping mitigate the loss of sea ice for polar bears (and for humans!), see the "Save Our Sea Ice" initiative here: https://polarbearsinternational.org/get-involved/save-our-sea-ice-campaign/

Counting Polar Bears Using Probability Lesson Information

Created by Dr. Jody Reimer and Linda Zhao, University of Utah, Department of Mathematics

Math Core Curriculum

Grades 9-12 **Making Inferences & Justifying Conclusions**	**CCSS.MATH.CONTENT.HSS.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population	**CCSS.MATH.CONTENT.HSS.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion

Learning Objectives

To explore how an understanding of probability can help us count the number of polar bears in a population.	Students gain familiarity with the following concepts: • Probability and random sampling • Experimental design

Additional Reading Material

Physical Mark-Recapture https://www.polarbearscanada.ca/en/research/population-inventories/physical-mark-recapture	**Capture-Mark-Recapture Science** https://www.usgs.gov/centers/eesc/science/capture-mark-recapture-science?qt-science_center_objects=0#	**Academic Sources:** Amstrup, S. C., McDonald, T. L., & Stirling, I. (2001). Polar bears in the Beaufort Sea: A 30-year mark-recapture case history. Journal of Agricultural, Biological, and Environmental Statistics, 6(2), 221-234.

Counting Polar Bears Using Probability

Pre-lesson reading: Pick one polar bear subpopulation to read about in this report (beginning on page 13): https://www.iucn-pbsg.org/wp-content/uploads/2021/11/July-2021-Status-Report-Web.pdf	Write down any questions or new vocabulary words here:

Guided Reading Questions

Using the pre-lesson reading, answer these questions:

1. Which subpopulation did you pick to read about?	
2. Has the subpopulation of polar bears increased or decreased?	
3. Why do you think the subpopulation has either increased or decreased?	
4. What is one way you can come up with for scientists to count how many polar bears there are in a big area? What do you think might be some challenges to counting polar bears?	

How to count polar bears – using probability!

To get a good estimate for the number of polar bears in an area, we use a method called "mark-recapture." There are two steps: a "mark" step and a "recapture" step, which we will explore through the activity on the following pages.

Hands on Activity

Materials you will need:

- Macaroni (or some other small pasta, beans, etc.)
- Marker in a different color than the macaroni (e.g. black)

STEP 1: Count out 50 pieces of macaroni and place them in a pile. Each macaroni represents one polar bear, and this pile represents the full, true population (i.e., 50 polar bears). After making the pile, we will pretend that we don't know how many there are, because in the real world, we don't know how many there are!

STEP 2: Randomly pick out 20 pieces of macaroni from the population pile and mark them each clearly with the marker. This is the "mark" part of "mark-recapture."

STEP 3: Return and mix the marked macaroni back into the pile.

5. If you now close your eyes and pick one macaroni from the pile, what is the probability that it has been marked?	
6. What information did you need to be able to answer that question? Could you have answered it without knowing how many macaroni you started with?	

STEP 4: *Randomly* select 20 new macaroni pieces from your pile; some will probably be marked, and some will not. This is the "recapture" part of "mark-recapture."

7. Without looking, how many of these 20 pieces do you expect to be marked?	

STEP 5: **Count how many of these 20 new macaroni are already marked (i.e., count how many you have "recaptured").**

STEP 6: **If x represents the number of macaroni already marked, then our estimate for the total number of bears in the pile can be calculated as:**

***Estimated number of bears* =**

Using this equation check how close your estimate is to the true number, which is 50 (remember: you wouldn't know that it is 50 for a real population!) Did your estimate do a good job?

8. What do you think we could do to improve how well our estimate did?	

STEP 7: **Take the average between your estimate and the estimates of a few friends or classmates.**

9. Is the average closer to the true number of bears or further? Why?	
10. Do you think this method will work well for polar bears in the wild? Why or why not?	

Counting Polar Bears Using Probability

Lesson Solutions

Created by Dr. Jody Reimer and Linda Zhao, University of Utah, Department of Mathematics

Math Core Curriculum

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Additional Reading Material

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STEP 2: Randomly pick out 20 pieces of macaroni from the population pile and mark them each clearly with the marker. This is the "mark" part of "mark-recapture."

STEP 3: Return and mix the marked macaroni back into the pile.

5. If you now close your eyes and pick one macaroni from the pile, what is the probability that it has been marked?	***Solution:*** *since 20 pieces out of 50 are marked, the probability of any given macaroni being marked is 20/50 = 0.4. That means there is a 40% chance the macaroni is marked.*
6. What information did you need to be able to answer that question? Could you have answered it without knowing how many macaroni you started with?	***Solution:*** *No, if you didn't know how many macaroni were in the pile, you could not answer the question since you wouldn't know the denominator of the fraction.*

STEP 4: *Randomly* select 20 new macaroni pieces from your pile; some will probably be marked, and some will not. This is the "recapture" part of "mark-recapture."

7. Without looking, how many of these 20 pieces do you expect to be marked?	***Solution:*** *Initially, students can answer with their gut feeling, maybe restricting the answer to a range such as "more than 5 but less than 15". The correct answer comes from multiplying each of the 20 pieces by the probability that piece is already marked, so (20)(0.4) = 8. So we should expect 8 pieces to be marked. (Note that this is the mean of the binomial distribution, where n=20 and p=0.4, the probability of "success" for each trial.)*

STEP 5: Count how many of these 20 new macaroni are already marked (i.e., count how many you have "recaptured").

STEP 6: If x represents the number of macaroni already marked, then our estimate for the total number of bears in the pile can be calculated as:

Estimated number of bears =

Using this equation check how close your estimate is to the true number, which is 50 (remember: you wouldn't know that it is 50 for a real population!) Did your estimate do a good job?

8. What do you think we could do to improve how well our estimate did?	***Solution:*** *the answer we are looking for here is that sampling more macaroni would help improve our estimate. In general, more data is usually better. The more data we have, the less likely it is that we see a pattern by chance (e.g., think about flipping a coin 3 times to see if it is fair, versus flipping it 3000 times! You would learn a lot more by flipping it 3000 times.)*

STEP 7: Take the average between your estimate and the estimates of a few friends or classmates.

9. Is the average closer to the true number of bears or further? Why?	***Solution:*** *it should typically be closer to the true number. This gets at the idea of "replicates"—if you can repeat an experiment many times, you will get a better estimate!*
10. Do you think this method will work well for polar bears in the wild? Why or why not?	***Solution:*** *there are lots of challenges to conducting mark-recapture studies in the wild. Bears are difficult to catch, since they are not all in a pile like the macaroni. Additionally, the true number of bears changes over time, as bears enter and leave the study area, as well as die and give birth. Students may come up with other ideas of why mark-recapture methods may be tricky to implement. Luckily, the statistical tools needed to deal with many of these challenges have been established so that we can still learn something about the size of polar bear populations!*