An Open Quantum System for Discrete Optimization

L. Nguyen,* W. Dyk, M.A. Miri, M. Begliarbekov, R.J. Rupert, S. Wu, N. Vrahoretis, I. Huang, P. Mahamuni, C.M. Delgado, D. Haycraft, Y.M. Sua, and Y. Huang

Quantum Computing Inc (QCi), 5 Marine View Plaza, Hoboken, NJ 07030

**lnguyen@quantumcomputinginc.com*

Abstract: We propose a novel framework for photonic computing specialized in solving discrete optimization problems by leveraging the quantum Zeno effect. We demonstrate the efficiency of this computing paradigm within a hybrid quantum optimization machine.

Discrete optimization problems involve finding an optimal object from a finite and discrete set of objects. Many of the well-known discrete optimization problems such as the traveling salesman, knapsack, graph coloring, spanning tree, set covering, set packing, etc [1]. are classified as NP-hard, meaning no known algorithm can solve all instances in polynomial time with respect to the problem size. Conventional von Neumann computers are hitting the limit due to the saturation of semiconductor miniaturization. Thus, a logical progression is shifting away from universal Turing Machines and exploring alternative computing methodologies for specific tasks. Significant progress has been made in unconventional computing approaches where the mathematical formulation can be mapped into the evolution of physical systems. A notable development portion of alternative computing research, particularly in the area of quantum annealing, focuses on solving Ising problems. However, there are many NP problems do not naturally map to binary spin states and two-body interaction, but rather higher discrete space and multi-body interaction. Developing an effective mapping that accurately represents the original problem within the Ising framework can be highly non-trivial. Modern quantum technologies using matter are designed as closed quantum systems to isolate from environmental interactions, which constrains scalability for practical implementation and energy efficiency. Here, we propose and demonstrate a novel room temperature hybrid quantum optimization machine that builds on interaction with a reservoir in conjunction with the quantum Zeno blockade to efficiently search for ground states of complex Hamiltonians.

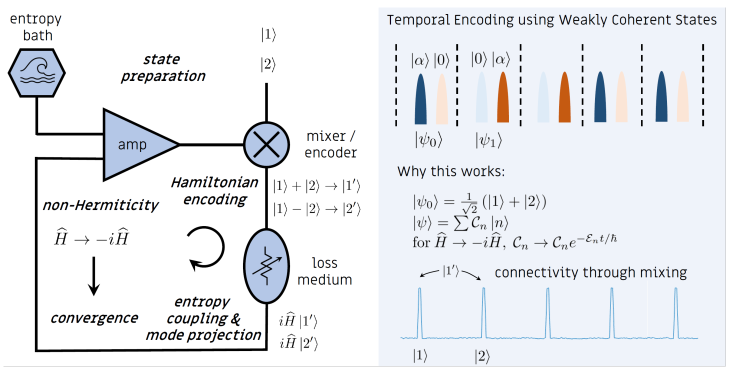

Fig. 1. A schematic of the proposed hybrid quantum optimization machine.

The working principle of the proposed technology is schematically depicted in Fig. 1. It consists of a hybrid optical-electronic feedback loop, which includes an optical amplifier, a photon-mode mixer/encoder, and a loss medium with both linear and nonlinear loss mechanisms. The quantum states are encoded into a train of time-bin states of light in the photon-number Hilbert space [2]. During each loop, optical signals are amplified by a fixed-power optical amplifier. The output is sent into the mixer and encoder that performs linear transformation of the time modes according to the desirable Hamiltonian, by using a series of beamsplitters, optical delay lines, and optical switches, all controlled opto-electronically. The transformed signals are sent to the loss medium, which causes differential loss to each time mode. The loss rate for each mode is the sum of a constant term, corresponding to linear "chemical potential energy" of that mode, and a photon-number dependent term, corresponding to the nonlinear interaction energy. The nonlinear loss can be realized by two-photon absorption, second-harmonic generation, and quantum Zeno blockade. In the current machine offering, it is emulated by time correlated single photon counting and feedback through an electro-optical modulation. Together, the mixer/encoder and loss medium realize an open quantum system governed by a non-Hermitian Hamiltonian. As a result, after many loops, the system will relax to and stabilize on a quantum state with the least loss, which corresponds to the lowest energy state: the ground state. This approach induces loss or decoherence into the system to suppress the evolution of unwanted states while promotes least loss states evolution. We thus called this ”entropy computing”. This optimization machine is governed by the minimization of the following cost function E over variables V_i:

$$E = \sum_i C_i V_i + \sum_{i,j} J_{ij} V_i V_j + \sum_{i,j,k} T_{ijk} V_i V_j V_k + \sum_{i,j,k,l} Q_{ijkl} V_i V_j V_k V_l + \sum_{i,j,k,l,m} P_{ijklm} V_i V_j V_k V_l V_m \ . \quad (1)$$

Here, V_i $(i = 1, 2, 3, \cdots, N)$ are real numbers over a discrete space, C_i is the linear return of each variable which must be real numbers, J_{ij}, T_{ijk}, Q_{ijkl}, P_{ijklm} represent interaction coefficients that are real numbers subject to the tensors J, T, Q, and P being symmetric under all permutations of the indices.

In stark contrast with the Ising Hamiltonian, which is the basis model for the majority of quantum annealers, the above objective function involves polynomial terms (up to fifth order) over discrete variables. In this regard, the proposed hybrid quantum optimization machine offers two immediate advantages over an Ising solver; (i) it can naturally represent non-binary optimization problems, and (ii) it involves k-body interaction terms ($k = 2, 3, 4, 5$). Accordingly, it offers great potential in efficiently solving continuous and integer variables as well as problems that naturally involve higher-order interaction terms such as the satisfiability boolean, without requiring additional complex encoding or incorporating auxiliary variables that adds to the size of the problem in case of an Ising solver. Furthermore, the proposed machine naturally allows for dense long-range interactions in all orders of the interactions which alleviates the requirement for complex embedding algorithms. Here, we report results from our first commercially available machine, which we call Dirac-3. Dirac-3 is a discrete optimization solver which implements the entropy computing paradigm discussed above. As a first example, we consider QPLIB_0018, a non-convex quadratic optimization problem with 50 continuous variables over a fully connected weighted graph, selected from QPLIB, a library of quadratic programming instances. Linear terms are added to the problem which produce an offset of 1 from the original formulation's objective value for all solutions. Figures 2(a-c) shows the energy distribution over 50 runs and its evolution with iteration on Dirac-3 (red) compared with a gradient descent algorithm (blue). Dirac-3 successfully lands in the ground state in 84% of instances. As another example, we consider the MAX-CUT problem on a 100-node graph, g05_100.6, from the Biq Mac Library. For 100 runs on Dirac-3, the results distribute around the ground state as shown in Fig. 2(d).

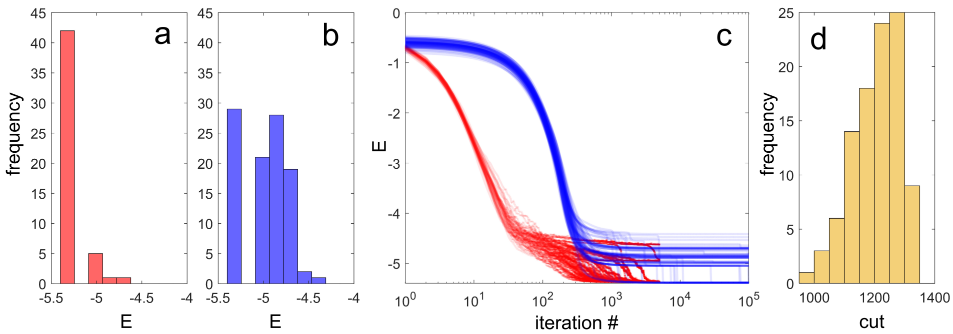

Fig. 2. (a,b) Energy distribution over 50 runs of Dirac-3 (red) and gradient descent algorithm (blue). (c) Energy evolution versus the number of iterations on Dirac-3 (red) and gradient descent (blue). (d) Energy distribution (cut value) for the MAX-CUT problem over 100 runs of Dirac-3.

In summary, we demonstrated discrete optimization with a hybrid quantum optimization machine Dirac-3. Our results highlight the performance of Dirac-3 for efficiently solving non-convex polynomial problems as well as combinatorial and integer programming problems. Our next-generation of entropy computing technology will focus on implementing optical interactions to realize an all-optical quantum optimization machine [3].

References

1. R Gary Parker and Ronald L Rardin. *Discrete optimization*. Elsevier, 2014.
2. JH Kim, JW Chae, YC Jeong, and YH Kim. *APL Photonics*, 7(1), 2022.
3. S. Kumar, H. Zhang, Y. Huang, and Bu T., 2023. US Patent App. 17/924,638.