Hierarchical Compartmental Reserving Models

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Motivation

Estimating future insurance claims

READY - PREDICT - AIM

vs.

AIM - READY - PREDICT

What are HCRM?

- Dynamical systems describing processes with differential equations
- A Bayesian framework to capture uncertainties in data and expert knowledge
- Best implemented in probabilistic programming language such as Stan (e.g. via ‘brms’ in R) or PyMC to model, fit and simulate

When might you consider HCRM for reserving?

- Data is poor, but expert knowledge is rich
- Paid and outstanding claims to be modelled simultaneously
- Insight into the underwriting cycle desired
- Full distribution around cash flows needed

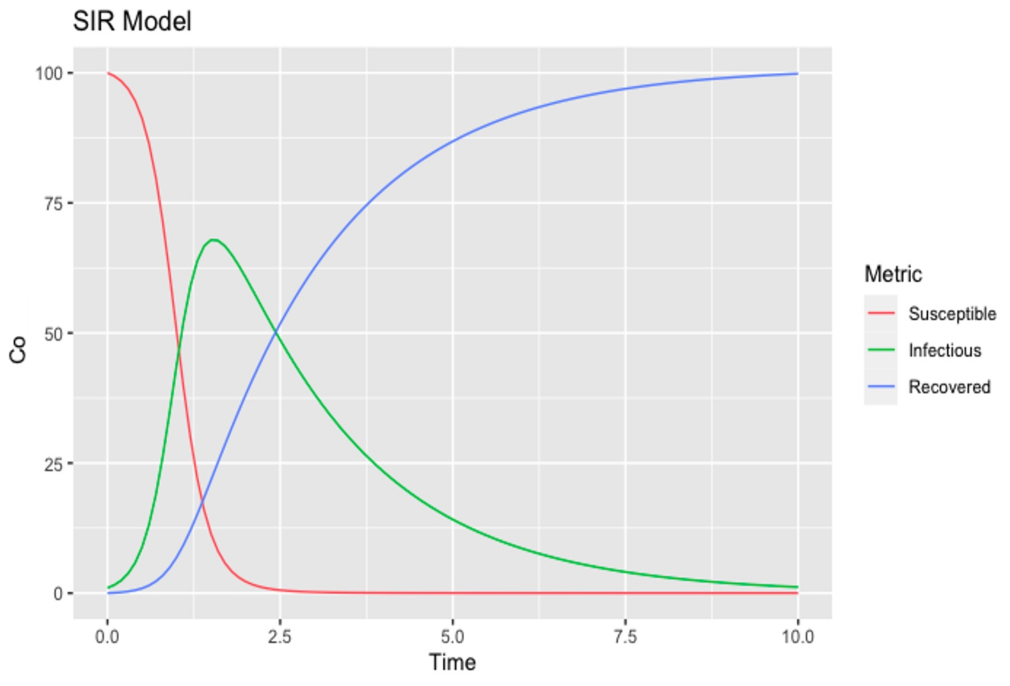
What are dynamical systems?

Dynamical systems are often used in physics, engineering and epidemiology to model a deterministic process

Very flexible, but requires expert knowledge to model a process with differential equations and to parameterise

Example: Modelling diseases

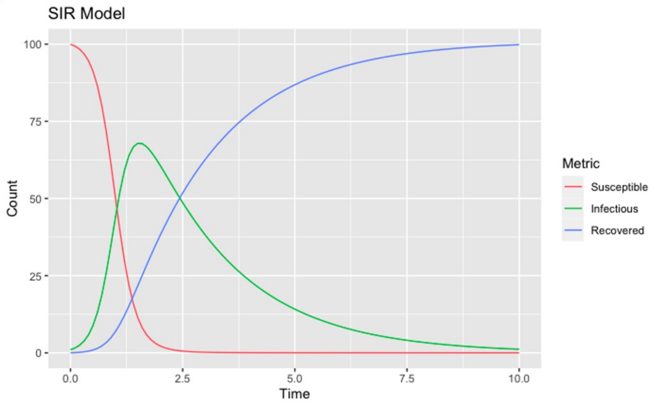
- Susceptible $$\frac{dS}{dt} = -\beta IS$$
- Infectious $$\frac{dI}{dt} = \beta IS - \gamma I$$
- Recovered $$\frac{dR}{dt} = \gamma I$$

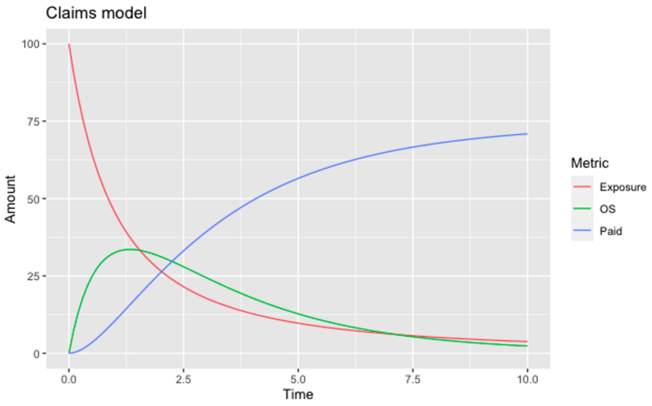


Modelling diseases and claims are alike

- Susceptible $\frac{dS}{dt} = -\beta IS$
- Infectious $\frac{dI}{dt} = \beta IS - \gamma I$
- Recovered $\frac{dR}{dt} = \gamma I$

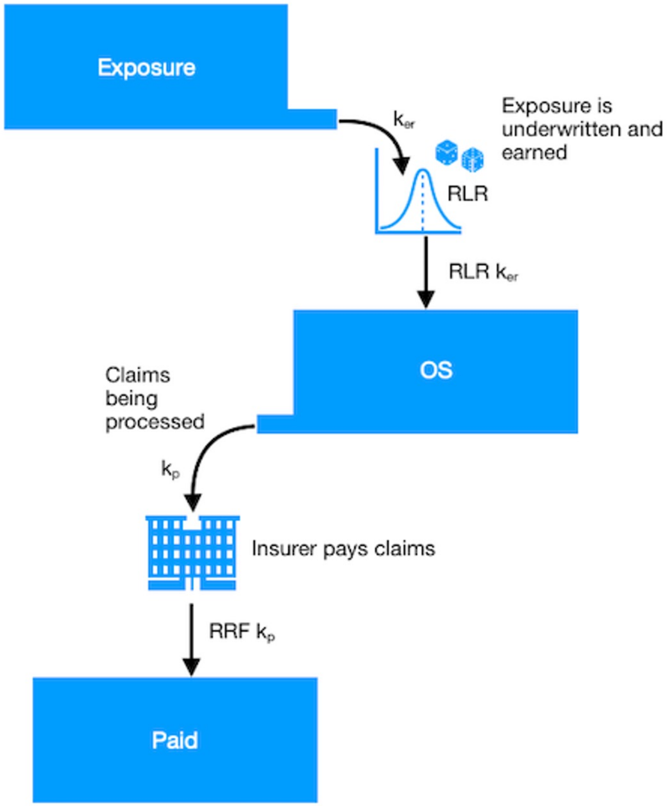
- Exposure $\frac{dEX}{dt} = -\beta \cdot EX$
- Outstanding claims $\frac{dOS}{dt} = \beta \cdot RLR \cdot EX - \gamma \cdot OS$
- Paid claims $\frac{dPD}{dt} = \gamma \cdot RRF \cdot OS$





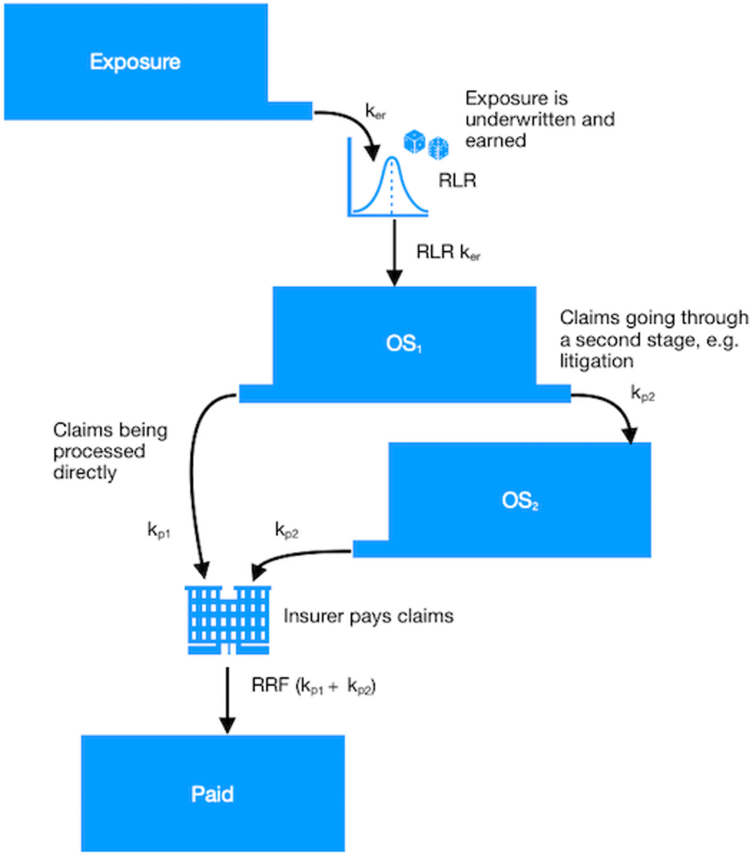
Insurance Model 1

$$dEX/dt = -k_{er} \cdot EX$$
$$dOS/dt = k_{er} \cdot RLR \cdot EX - k_p \cdot OS$$
$$dPD/dt = k_p \cdot RRF \cdot OS$$

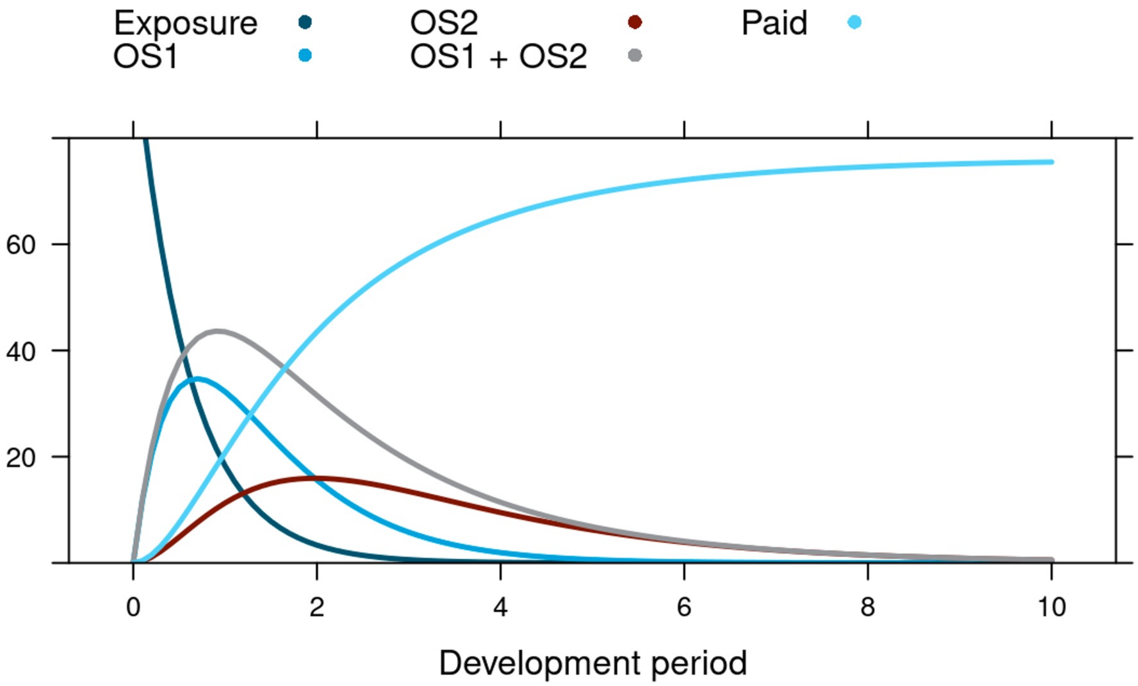


Insurance Model 2

$$dEX/dt = -k_{er} \cdot EX$$
$$dOS_1/dt = k_{er} \cdot RLR \cdot EX - (k_{p_1} + k_{p_2}) \cdot OS_1$$
$$dOS_2/dt = k_{p_2} \cdot (OS_1 - OS_2)$$
$$dPD/dt = RRF \cdot (k_{p_1} \cdot OS_1 + k_{p_2} \cdot OS_2)$$



Insurance Model 2



Bayesian framework

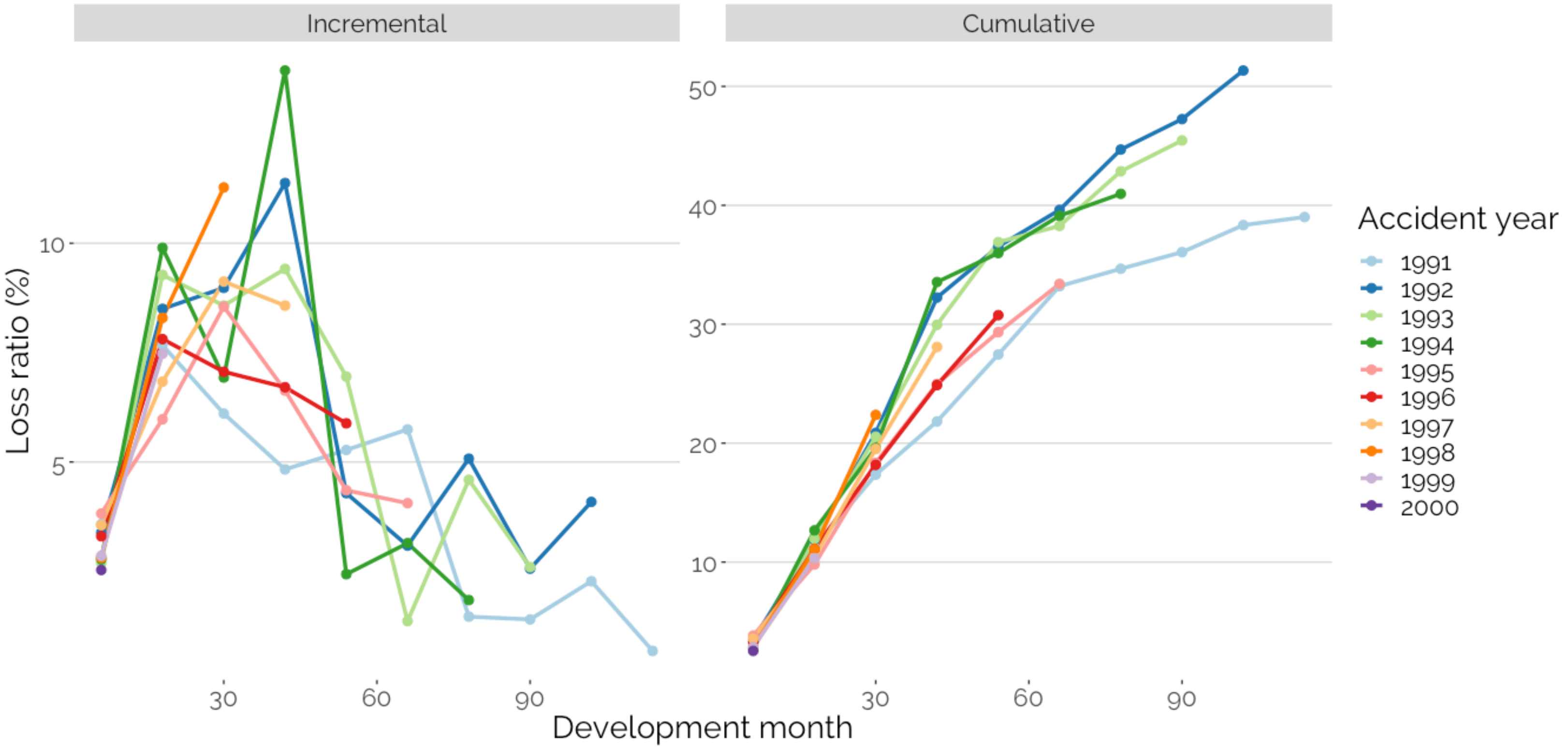
Create a data generating model first:

- Consider process and parameter distributions

$$y_j \sim D(f(t_j, \Theta), \Phi)$$

- Consider variance structure
- Consider hierarchical structure
  - Which parameters might have random effects, e.g. vary across accident years, development years, lines of business or entities?

Example: Paid loss ratio data only



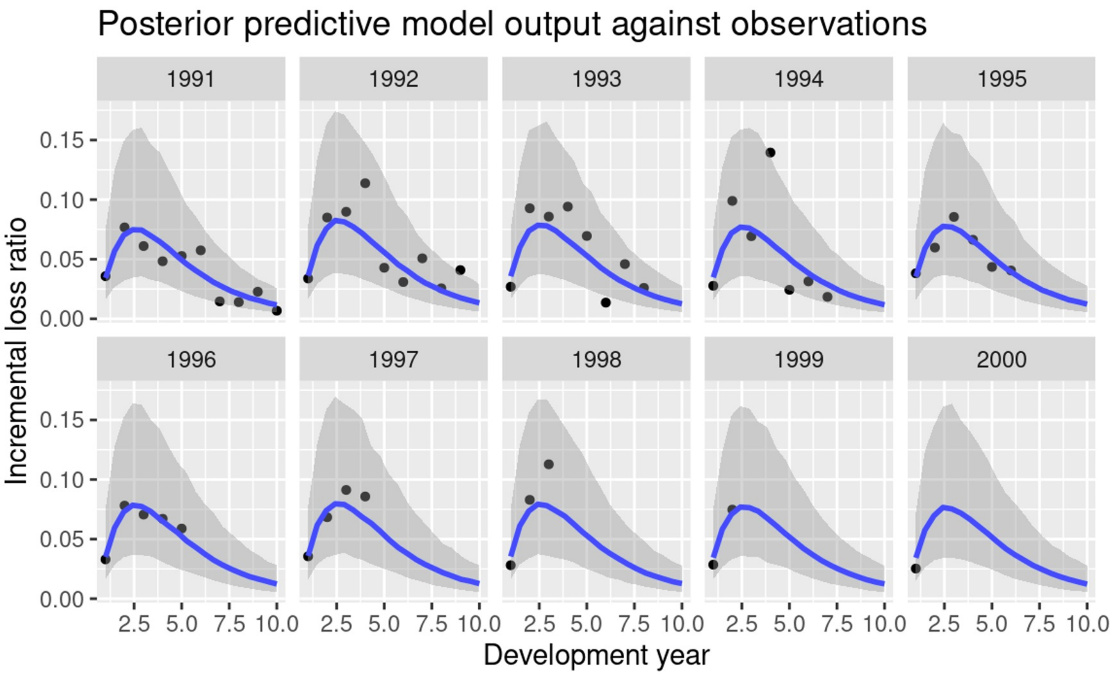
Hierarchical model candidate

Incremental paid loss ratio for accident year i, dev period j

Parametric growth curve G

$$
\begin{aligned}
\ell_{ij} &\sim \text{Lognormal}(\eta(t_j;\theta, ELR_{[i]}), \sigma) \\
\eta(t;\theta, ELR_{[i]}) &= \log(ELR_{[i]} \cdot (G(t_j;\theta) - G(t_{j-1};\theta))) \\
&= \log\big(ELR_{[i]}\big) + \log\big(G(t_j;\theta) - G(t_{j-1};\theta)\big) \\
\log(ELR)_{[i]} &= \mu_{ELR} + u_{[i]} \\
u_{[i]} &= \sigma_{[i]}\, z_{[i]} \\
\mu_{ELR} &\sim \text{Normal}(\log(0.6), 0.1) \\
\sigma_{[i]} &\sim \text{StudentT}(10, 0, 0.025)^{+} \\
z_{[i]} &\sim \text{Normal}(0, 1)
\end{aligned}
$$

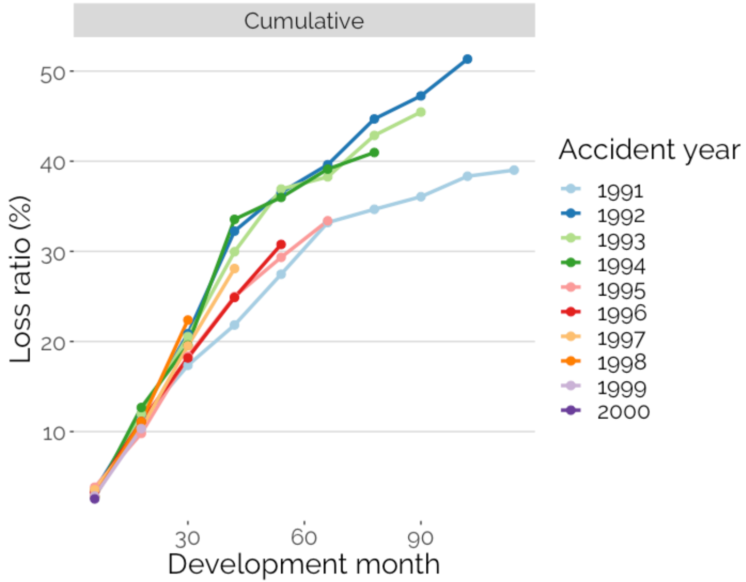
Simulated data vs observations



Reserves: Aggregate future payments from latest observation

Distinguish between ELR and ULR

AY	ELR (%)	Est. error	ULR (%)	Est. error
1991	46.6	4.4	43.2	1.4
1992	52.7	5.4	58.7	2.2
1993	49.7	4.6	53.5	2.4
1994	47.4	4.8	50.2	2.9
1995	49.0	4.9	47.0	3.8
1996	49.5	5.4	49.1	4.7
1997	50.5	6.1	52.2	5.7
1998	50.4	6.0	53.8	6.7
1999	48.7	6.2	49.1	7.7
2000	48.3	6.7	48.5	8.6



Estimated ELR for each AY, e.g. underlying pricing loss ratio

Similar across AY

Projection from latest observation, ie required for reserving

Increasing across AY

ELR: Expected Loss Ratio
ULR: Ultimate Loss Ratio, anchored to the latest actual data point

Summary

- HCRM provide transparent framework for reserving
- Expert knowledge is part of the model design, not an add-on or afterthought
- Model can be useful to extract historical pricing information from claims data
- Paper has more details and case studies implemented in R and Stan using the ‘brms’ package as an interface

Reference

Gesmann, M., and Morris, J. “Hierarchical Compartmental Reserving Models.” Casualty Actuarial Society, CAS Research Papers, 19 Aug. 2020, https://www.casact.org/sites/default/files/2021-02/compartmental-reserving-models-gesmannmorris0820.pdf

Bookdown: https://compartmentalmodels.gitlab.io/researchpaper/index.html