CSCI 340: Computational Models

Post Machines

Chapter 20

Department of Computer Science

An Aside on Algorithms

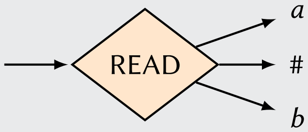
- "An algorithm is a procedure with instructions so detailed that no further information is necessary"
- **Goal:** Create a "universal algorithm machine"
- In 1936, Emil Post created a Post machine – which he hoped would be a "universal algorithm machine"
- *Requires:* Universal algorithm machines must accept *any language* which can be defined by humans

Post Machines

Definition

A **Post Machine**, denoted PM, is a collection of five things:

1. An alphabet Σ of input letters and the special symbol #
2. A linear storage location called the **STORE**. We can *read* the leftmost character in the store and *add* a new character to the "end" (rightmost location) of the STORE. We allow for characters **not** in Σ to be used in the STORE — usually denoted as Γ.
3. READ states which remove the leftmost character from the STORE and branch accordingly

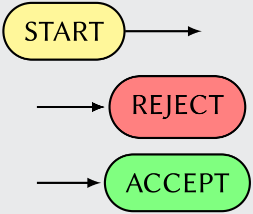


Post Machines

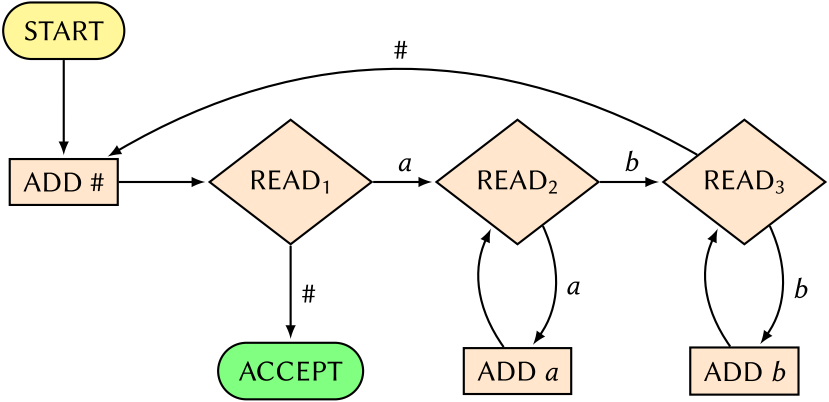
Definition (continued)

4. ADD states which concatenate a character onto the *right* end of the string in the STORE. (This is the "opposite" of a PDA PUSH state). No branching can take place. Letters from Σ and Γ can be ADDed to the STORE.

5. A START state (unenterable) and some halt states called ACCEPT and REJECT. REJECT states are optional.

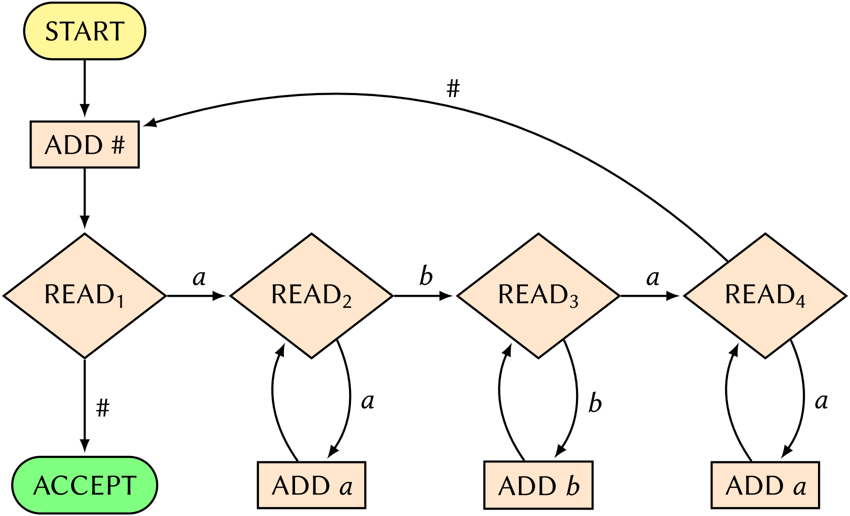


Example



Trace: *aaabbb*

Example #2


START
ADD #
#
READ$_1$
a
READ$_2$
b
READ$_3$
a
READ$_4$
a
b
a
#
ACCEPT
ADD a
ADD b
ADD a

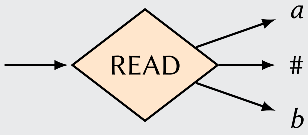
Simulating a PM on a TM

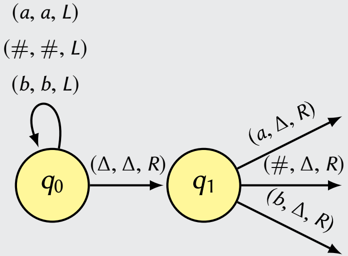
Theorem

Any language that can be accepted by a PM can be accepted by some TM

Proof.

- START states remain unchanged
- ACCEPT states can be renamed to HALT
- REJECT states can be removed
- READ states should move the TAPE-HEAD to the first non-Δ character on the TAPE.

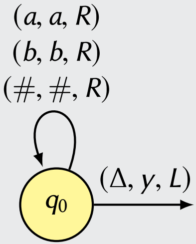




Simulating a PM on a TM

Proof.

- ADD states should move the TAPE-HEAD to the “end” of the tape and insert the character to the END



□

Simulating a TM on a PM

Theorem

Any language that can be accepted by a TM can be accepted by some PM

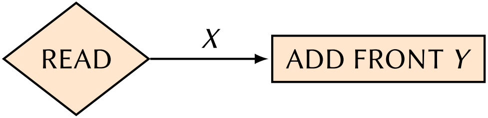
Proof.

- Key: use # to indicate the "tape" boundary separator
- TAPE may store any of $\Sigma,\ \Gamma,\ \#,\ \Delta$
- TAPE-HEAD will always be the *front* of the STORE
- When we *read* from the TM, we READ from the PM
- When we *write* to the TM, we ADD to the PM
- When we move to the *left*, we have to rotate all of the elements in our STORE right (cyclically)
- When we move to the *right*, we don't have to do anything
- START needs a secondary *ADD #* state immediately after. Any cycles will go to this new ADD state

□

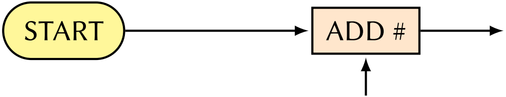
Simulating a TM on a PM

Converting transition of (X, Y, R)



Converting transition of (X, Y, L)

Changing START



TM = PM

Proof.

- $PM \subseteq TM$ because we can show how to convert a PM to a TM
- $TM \subseteq PM$ because we can show how to convert a TM to a PM
- $PM = TM$ because of the above two claims

□