CSCI 340: Computational Models

CFG = PDA

Chapter 15

Department of Computer Science

Building a PDA for Every CFG

Theorem

Given a CFG that generates the language L, there is a PDA that accepts exactly L

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Given a PDA that accepts the language L, there exists a CFG that accepts exactly L

Both of these theorems were discovered independently by Schützenberger, Chomsky, and Evey

CFG to PDA Algorithm

Note: We assume the CFG grammar is defined in CNF

$X_1 \to X_2X_3$

$X_1 \to X_3X_4$

$X_2 \to X_2X_2$

$\dots$

$X_3 \to a$

$X_4 \to a$

$X_5 \to b$

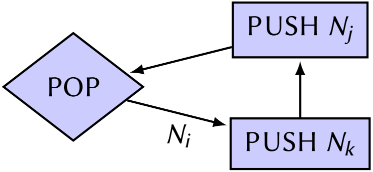
$\dots$

Two forms:

$N_i \to N_jN_k$

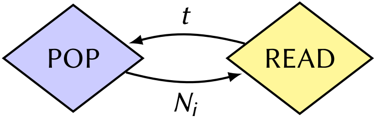
$N_i \to t$

Handling form $N_i \to N_jN_k$:



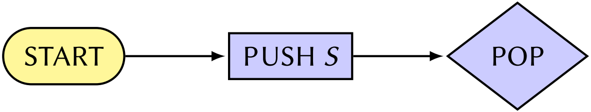
Note: non-terminals are pushed in reverse order

Handling form $N_i \to t$:

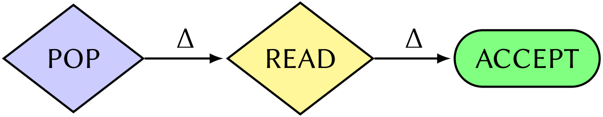


CFG to PDA Algorithm

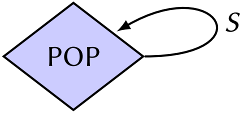
Start of machine:



End of machine:



If a language should accept λ, include:

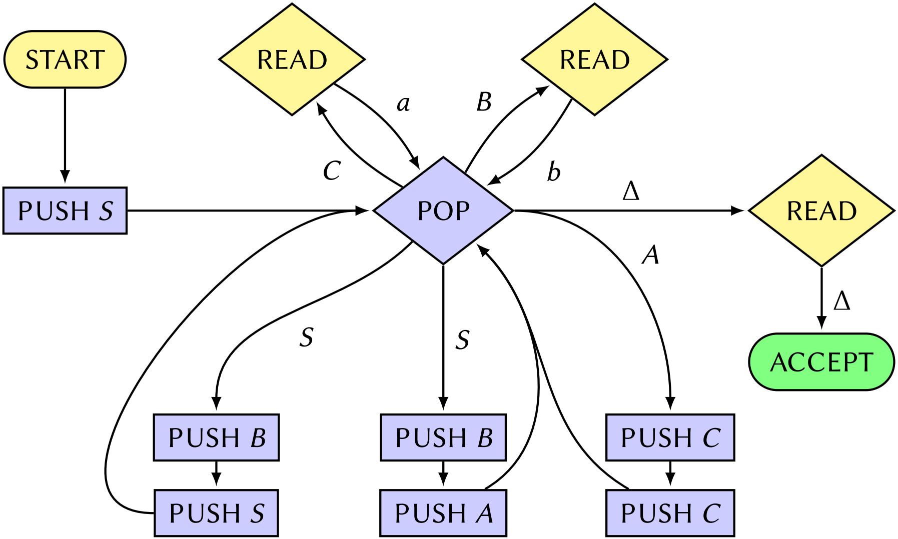


Example

Consider the following grammar (in CNF):

$$
\begin{aligned}
S &\longrightarrow SB \\
S &\longrightarrow AB \\
A &\longrightarrow CC \\
B &\longrightarrow b \\
C &\longrightarrow a
\end{aligned}
$$

Example


START
PUSH S
READ
READ
POP
READ
ACCEPT
a
B
C
b
Δ
A
Δ
S
S
PUSH B
PUSH S
PUSH B
PUSH A
PUSH C
PUSH C

PDA to CFG

"This is a long proof by constructive algorithm. In fact, it is unquestionably the most torturous proof in the book; parental consent is required"

Pages 327 – 347

PDA to CFG

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