Chapter 6

Data Types

*modified by
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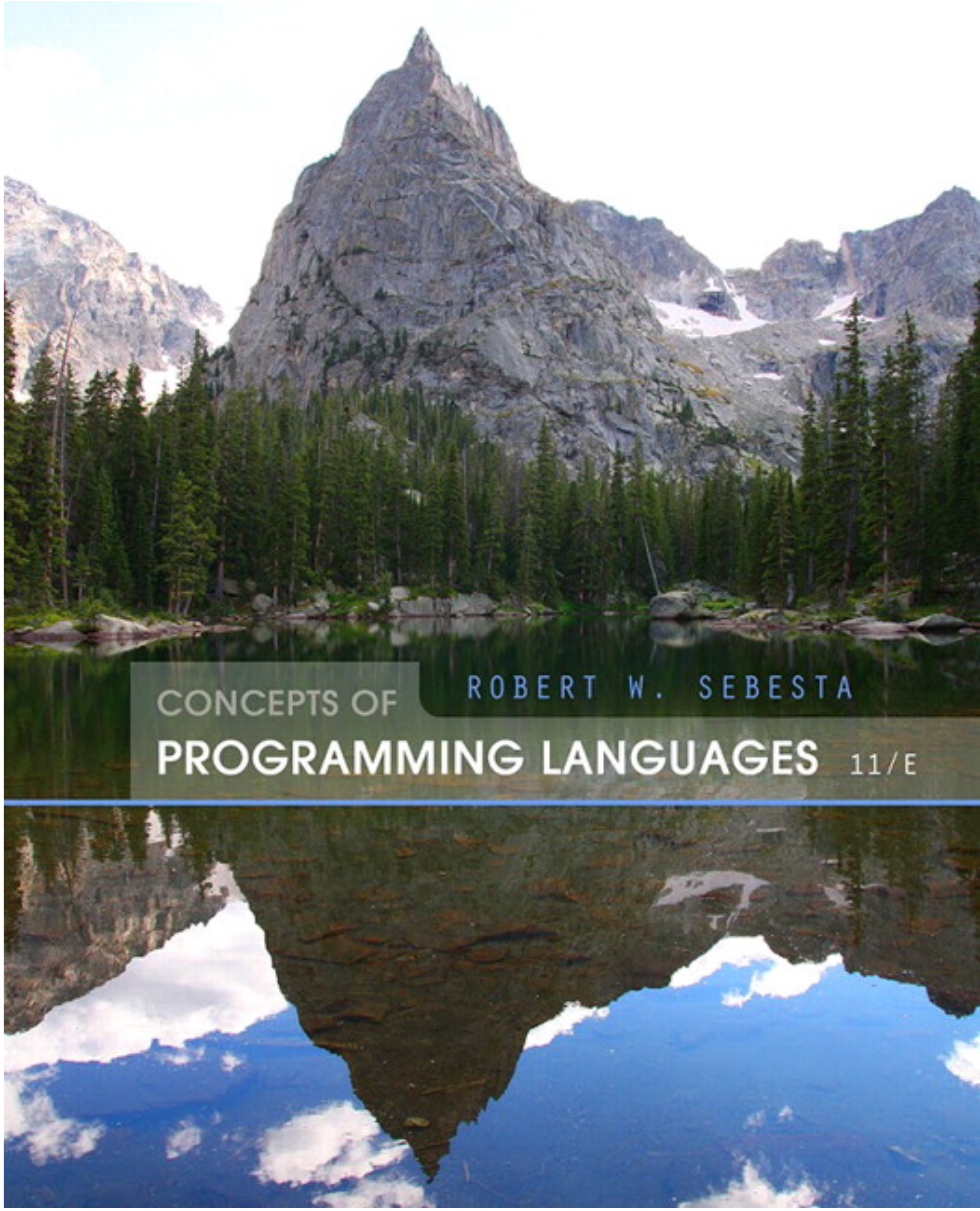

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Chapter 6 Topics

- Introduction
- Primitive Data Types
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- Array Types
- Associative Arrays
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- Tuple Types
- List Types
- Union Types
- Pointer and Reference Types
- Type Checking
- Strong Typing
- Type Equivalence
- Theory and Data Types

Introduction

- A *data type* defines a collection of data objects and a set of predefined operations on those objects
- An *abstract data type* (ADT) separates the interface/behavior of a type (visible to the user) from the representation (hidden from user)
  - In high-level programming languages, all data types are abstract data types

Type System

- The type system of a language:
  - Defines how a type is associated with each expression in a language
  - Includes rules for type equivalence and type compatibility
  - Allows for error detection

User-Defined Data Types

- User-Defined Data Types allow programmers to design new types
- First introduced in ALGOL 68
- Improved readability through meaningful names for types
- Improved reliability through type checking
- An *object* represents an instance of a user-defined or language-defined abstract data type (slightly different from the way we talk about objects in OOP)

Primitive Data Types

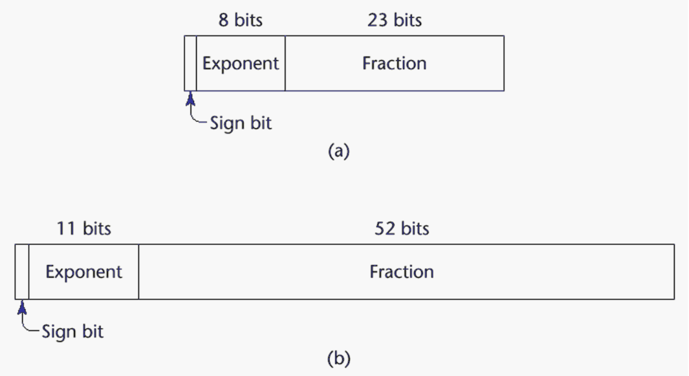
- Almost all programming languages provide a set of *primitive data types*
- Primitive data types: Those not defined in terms of other data types
- Some primitive data types are merely reflections of the hardware
- Others require only a little non-hardware support for their implementation

Primitive Data Types: Integer

- Almost always an exact reflection of the hardware so the mapping is trivial
- There may be as many as eight different integer types in a language
- Java's signed integer sizes: **`byte`**, **`short`**, **`int`**, **`long`**
- C++ and C# include unsigned integer types (often used for binary data)
- Python and F# include long integers (unlimited length)

Primitive Data Types: Floating Point

- Model real numbers, but only as approximations
- Languages for scientific use support at least two floating-point types (e.g., **`float`** and **`double`**; sometimes more
- Usually exactly like the hardware, but not always
- IEEE Floating-Point Standard 754



Primitive Data Types: Complex

- Some languages support a complex type, e.g., C99, Fortran, and Python
- Each value consists of two floats, the real part and the imaginary part
- Literal form (in Python):

  `(7 + 3j)`, where `7` is the real part and `3` is the imaginary part

Primitive Data Types: Decimal

- For business applications (money)
  - Essential to COBOL
  - C# offers a decimal data type
- Store a fixed number of decimal digits, in coded form (BCD)
- *Advantage*: accuracy
- *Disadvantages*: limited range, wastes memory

Primitive Data Types: Boolean

- Simplest of all
- Range of values: two elements, one for “true” and one for “false”
- Could be implemented as bits, but often as bytes
  - Advantage: readability

Primitive Data Types: Character

- Stored as numeric codings
- Most commonly used coding: ASCII
- An alternative, 16-bit coding: Unicode (UCS-2)
  - Includes characters from most natural languages
  - Originally used in Java
  - C# and JavaScript also support Unicode
- 32-bit Unicode (UCS-4)
  - Supported by Fortran, starting with 2003

Character String Types

- Values are sequences of characters
- Design issues:
  - Is it a primitive type or just a special kind of array?
  - Should the length of strings be static or dynamic?

Character String Types Operations

- Typical operations:
  - Assignment and copying
  - Comparison (=, >, etc.)
  - Concatenation
  - Substring reference
  - Pattern matching

Character String Type in Certain Languages

- C
  - Not primitive, use **`char`** arrays and a library of functions that provide operations
- C++
  - C-style strings available, but use string class in standard library
- SNOBOL4 (a string manipulation language)
  - Primitive
  - Many operations, including elaborate pattern matching
- Fortran and Python
  - Primitive type with assignment and several operations
- Java
  - via the `String` class (values are constant strings)
- Perl, JavaScript, Ruby, and PHP
  - Provide built-in pattern matching, using regular expressions

Character String Length Options

- Immutable: COBOL, Java's `String` class
- *Limited Dynamic Length*: C and C++
  - In these languages, a special character is used to indicate the end of a string's characters, rather than maintaining the length
- *Dynamic* (no maximum): SNOBOL4, Perl, JavaScript

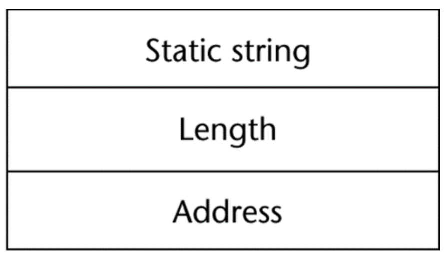
Character String Type Evaluation

- Aid to writability
- As a primitive type with static length, they are inexpensive to provide--why not have them?
- Dynamic length is nice, but is it worth the expense?

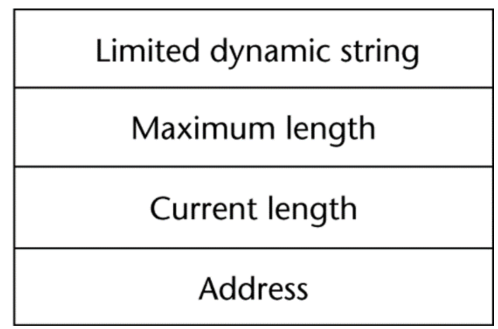
Character String Implementation

- Static length: compile-time descriptor
- Limited dynamic length: may need a run-time descriptor for length (but not in C and C++)
- Dynamic length: need run-time descriptor; allocation/deallocation is the biggest implementation problem

Compile- and Run-Time Descriptors



Compile-time descriptor for static strings



Run-time descriptor for limited dynamic strings

User-Defined Ordinal Types

- An ordinal type is one in which the range of possible values can be easily associated with the set of positive integers
- Examples of primitive ordinal types in Java
  - **`integer`**
  - **`char`**
  - **`boolean`**

Enumeration Types

- All possible values, which are named constants, are provided in the definition
- C# example

```
enum days {mon, tue, wed, thu, fri, sat, sun};
```

- Design issues
  - Is an enumeration constant allowed to appear in more than one type definition, and if so, how is the type of an occurrence of that constant checked?
  - Are enumeration values coerced to integer?
  - Any other type coerced to an enumeration type?

Evaluation of Enumerated Type

- Aid to readability, e.g., no need to code a color as a number
- Aid to reliability, e.g., compiler can check:
  - operations (don't allow colors to be added)
  - No enumeration variable can be assigned a value outside its defined range
  - C# and Java 5.0 provide better support for enumeration than C++ because enumeration type variables in these languages are not coerced into integer types

Array Types

- An array is a homogeneous aggregate of data elements in which an individual element is identified by its position in the aggregate, relative to the first element.
- If any of the subscript expressions include variables, then the reference will include a run-time calculation to determine the address of the memory location being referred to

Array Design Issues

- What types are legal for subscripts?
- Are subscripting expressions in element references range checked?
- When are subscript ranges bound?
- When does allocation take place?
- Are ragged or rectangular multidimensional arrays allowed, or both?
- What is the maximum number of subscripts?
- Can array objects be initialized?
- Are any kind of slices supported?

Array Indexing

- *Indexing* (or subscripting) is a mapping from indices to elements

  array_name (index_value_list) $\rightarrow$ an element
- Index Syntax
  - Fortran and Ada use parentheses
    - Ada explicitly uses parentheses to show uniformity between array references and function calls because both are *mappings*
  - Most other languages use brackets

Arrays Index (Subscript) Types

- FORTRAN, C: integer only
- Java: integer types only
- Index range checking
  - C, C++, Perl, and Fortran do not specify range checking
  - Java, ML, C# specify range checking

Subscript Binding and Array Categories

- *Static*: subscript ranges are statically bound and storage allocation is static (before run-time)
  - Advantage: efficiency (no dynamic allocation)
  - Disadvantage: storage is fixed for entire program execution
  - C and C++ arrays that include `static` modifier are static

Subscript Binding and Array Categories

- *Fixed stack-dynamic*: subscript ranges are statically bound, but the allocation is done at declaration time
  - Advantage: space efficiency
  - Disadvantage: Required allocation and deallocation time during execution
  - C and C++ arrays without `static` modifier are fixed stack-dynamic

Subscript Binding and Array Categories (continued)

- *Fixed heap-dynamic*: similar to fixed stack-dynamic: storage binding is dynamic but fixed after allocation (i.e., binding is done when requested and storage is allocated from heap, not stack)
  - Advantage: flexibility
  - Disadvantage: allocation time
  - C and C++ provide fixed heap-dynamic arrays
  - Java: all non-generic arrays are fixed heap-dynamic (and you can't allocate generic arrays)

Subscript Binding and Array Categories (continued)

- Heap-dynamic: binding of subscript ranges and storage allocation is dynamic and can change any number of times
  - Advantage: flexibility (arrays can grow or shrink during program execution)
  - Disadvantage: allocation time (and this may happen many times over program execution)
  - C# includes array class `List` and Java includes `ArrayList` that provide heap-dynamic
  - Perl, JavaScript, Python, and Ruby support heap-dynamic arrays (Perl, Ruby and Lua have negative subscripts, JS has sparse arrays

Array Initialization

- Some languages allow initialization at the time of storage allocation
  - C, C++, Java, C# example

```
int list [] = {4, 5, 7, 83}
```

  - Character strings in C and C++

```
char name [] = "freddie";
```

  - Arrays of strings in C and C++

```
char *names [] = {"Bob", "Jake", "Joe"};
```

  - Java initialization of String objects

```
String[] names = {"Bob", "Jake", "Joe"};
```

Array Initialization

- C-based languages
  - `int list [] = {1, 3, 5, 7}`
  - `char *names [] = {"Mike", "Fred", "Mary Lou"};`
- Python
  - List comprehensions

```
list = [x ** 2 for x in range(12) if x % 3 == 0]
    puts [0, 9, 36, 81] in list
```

Array Operations

- An *array operation* is one that operates on an array as a unit
- Most common: assignment, concatenation, comparison for equality/inequality and slices

Array Operations

- APL provides the most powerful array processing operations for vectors and matrixes as well as unary operators (for example, to reverse column elements)
- Python
  - Many array operations such as array assignments (only a reference change) and concatenation and element membership
  - Arrays are heterogeneous (list)
- C-based languages don’t provide built-in array operations – generally provided through methods of a class

Rectangular and Jagged Arrays

- A rectangular array is a multi-dimensioned array in which all of the rows have the same number of elements and all columns have the same number of elements
- A jagged matrix has rows with varying number of elements
  - Possible when multi-dimensioned arrays actually appear as arrays of arrays
- C, C++, and Java support jagged arrays
- F# and C# support rectangular arrays and jagged arrays

Slices

- A slice is some substructure of an array; nothing more than a referencing mechanism
- Slices are only useful in languages that have array operations

Slice Examples

- Python

```
vector = [2, 4, 6, 8, 10, 12, 14, 16]
mat = [[1, 2, 3], [4, 5, 6], [7, 8, 9]]
```

`vector (3:6)` is a three-element array

`mat[0][0:2]` is the first and second element of the first row of `mat`

`vector [0:7:2]` is `[2, 6, 10, 14]`

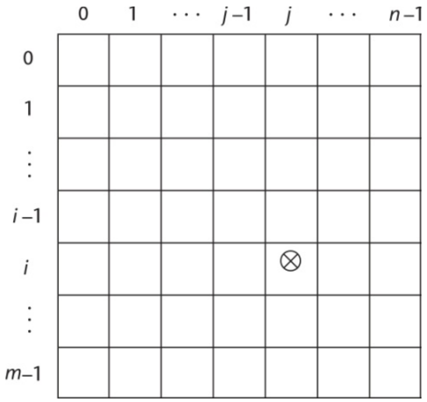
- Ruby supports slices with the `slice` method

`list.slice(2, 2)` returns the third and fourth elements of `list`

Implementation of Arrays

- Access function maps subscript expressions to an address in the array
- Access function for single-dimensioned arrays:

address(list[k]) = address (list[lower_bound])
+ ((k-lower_bound) * element_size)



Single-Dimensioned Arrays

- Compile-time descriptor includes information required to construct the access function
- What if subscript range checking isn't done?

Array
Element type
Index type
Index lower bound
Index upper bound
Address

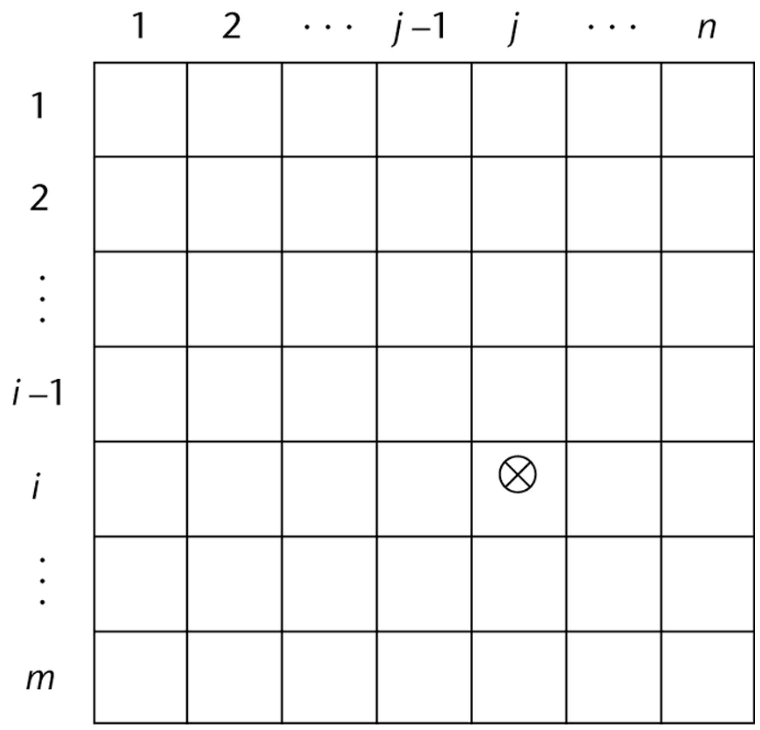
Accessing Multi-dimensioned Arrays

- Two common ways:
  - Row major order (by rows) – used in most languages
  - Column major order (by columns) – used in Fortran
  - A compile-time descriptor for a multidimensional array

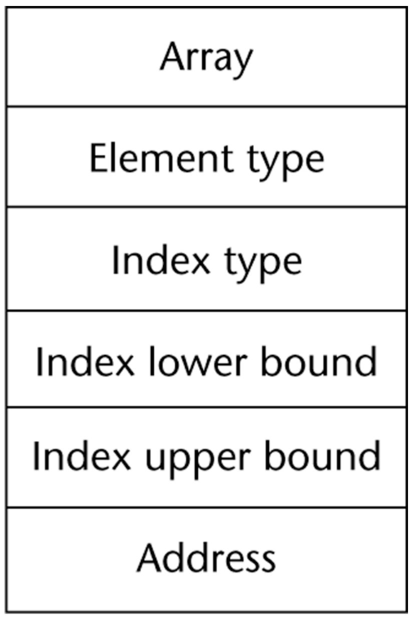
Multidimensioned array
Element type
Index type
Number of dimensions
Index range 0
⋮
Index range n – 1
Address

Locating an Element in a Multi-dimensioned Array

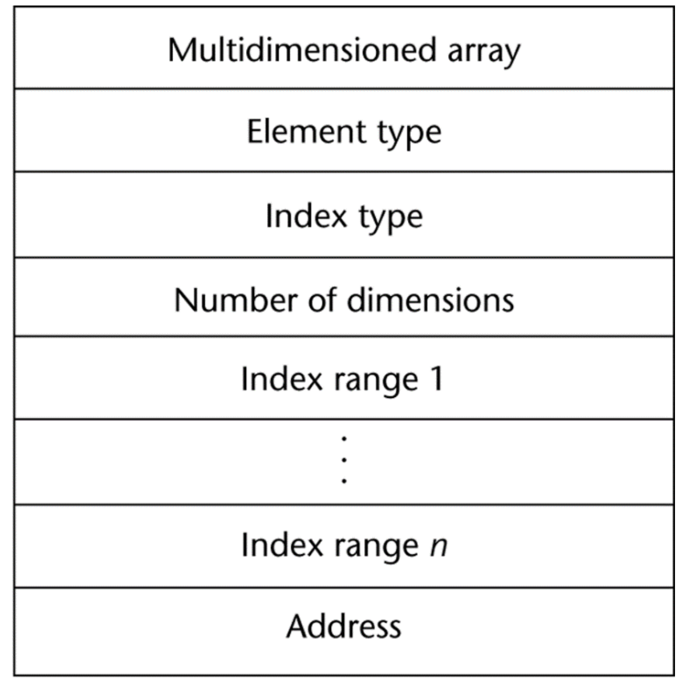
- General format
  Location (a[I,j]) = address of a [row_lb,col_lb] + (((I - row_lb) * n) + (j - col_lb)) * element_size



Compile-Time Descriptors



Single-dimensioned array



Multidimensional array

Associative Arrays

- An *associative array* is an unordered collection of data elements that are indexed by an equal number of values called *keys*
  - User-defined keys must be stored
- Design issues:
  - What is the form of references to elements?
  - Perl keys are strings, PHP keys can be ints or strings, Ruby keys can be any object
- Built-in type in Perl, Python, Ruby, and Lua
  - In Lua, they are supported by tables

Associative Arrays in Perl

- Names begin with `%`; literals are delimited by parentheses

```
%hi_temps = ("Mon" => 77, "Tue" => 79, "Wed" =>
  65, …);
```

- Subscripting is done using braces and keys

```
$hi_temps{"Wed"} = 83;
```

  - Elements can be removed with **`delete`**

```
delete $hi_temps{"Tue"};
```

Record Types

- A *record* is a possibly heterogeneous aggregate of data elements in which the individual elements are identified by names
- Design issues:
  - What is the syntactic form of references to the field?
  - Are elliptical references allowed

Definition of Records in COBOL

- COBOL uses level numbers to show nested records; others use recursive definition

```
01 EMP-REC.
   02 EMP-NAME.
      05 FIRST PIC X(20).
      05 MID   PIC X(10).
      05 LAST  PIC X(20).
   02 HOURLY-RATE PIC 99V99.
```

References to Records

- Record field references
  1. COBOL
  field_name OF record_name_1 OF ... OF record_name_n
  2. Others (dot notation)
  record_name_1.record_name_2. ... record_name_n.field_name

- Fully qualified references must include all record names

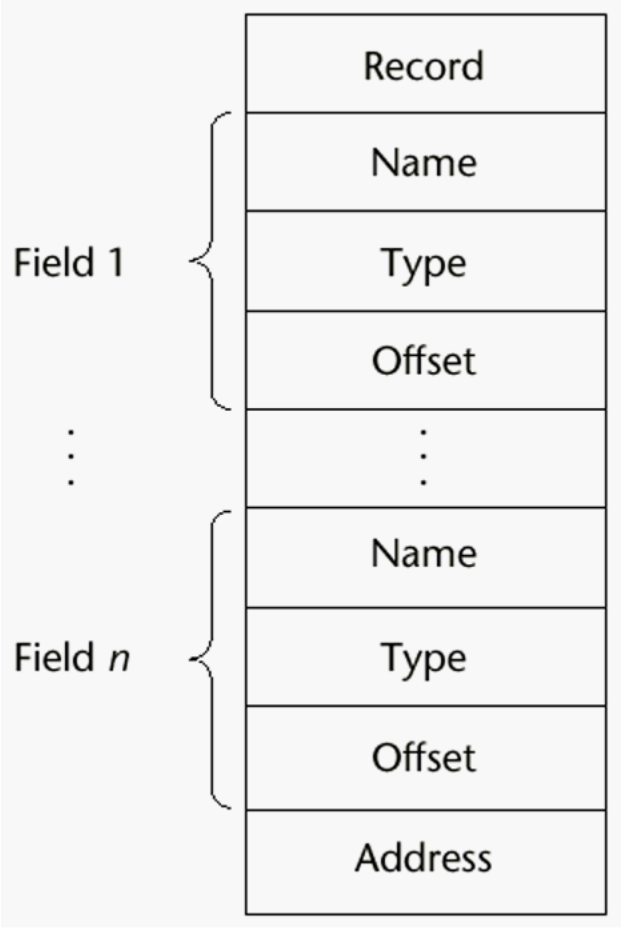
- Elliptical references allow leaving out record names as long as the reference is unambiguous, for example in COBOL `FIRST`, `FIRST OF EMP-NAME`, and `FIRST` of EMP-REC are elliptical references to the employee's first name

Evaluation and Comparison to Arrays

- Records are used when collection of data values is heterogeneous
- Access to array elements is much slower than access to record fields, because subscripts are dynamic (field names are static)
- Dynamic subscripts could be used with record field access, but it would disallow type checking and it would be much slower

Implementation of Record Type

Offset address relative to the beginning of the records is associated with each field



Tuple Types

- A tuple is a data type that is similar to a record, except that the elements are not named
- Used in Python, ML, and F# to allow functions to return multiple values
  - Python
    - Closely related to its lists, but immutable
    - Create with a tuple literal

      `myTuple = (3, 5.8, 'apple')`

      Referenced with subscripts (begin at `1`)

    Catenation with `+` and deleted with **`del`**

Tuple Types (continued)

- ML

```
val myTuple = (3, 5.8, 'apple');
```

  - Access as follows:

`#1(myTuple)` is the first element

  - A new tuple type can be defined

```
type intReal = int * real;
```

- F#

```
let tup = (3, 5, 7)
```

`let a, b, c = tup` This assigns a tuple to a tuple pattern `(a, b, c)`

List Types

- Lists in Lisp and Scheme are delimited by parentheses and use no commas

  `(A B C D)` and `(A (B C) D)`

- Data and code have the same form

  As data, `(A B C)` is literally what it is

  As code, `(A B C)` is the function `A` applied to the parameters `B` and `C`

- The interpreter needs to know which a list is, so if it is data, we quote it with an apostrophe

  `'(A B C)` is data

List Types (continued)

- List Operations in Scheme
  - `CAR` returns the first element of its list parameter

    `(CAR '(A B C))` returns `A`
  - `CDR` returns the remainder of its list parameter after the first element has been removed

    `(CDR '(A B C))` returns `(B C)`
  - `CONS` puts its first parameter into its second parameter, a list, to make a new list

    `(CONS 'A (B C))` returns `(A B C)`
  - `LIST` returns a new list of its parameters

    `(LIST 'A 'B '(C D))` returns `(A B (C D))`

List Types (continued)

- List Operations in ML
  - Lists are written in brackets and the elements are separated by commas
  - List elements must be of the same type
  - The Scheme `CONS` function is a binary operator in ML, `::`

    `3 :: [5, 7, 9]` evaluates to `[3, 5, 7, 9]`
  - The Scheme `CAR` and `CDR` functions are named `hd` and `tl`, respectively

List Types (continued)

- F# Lists
  - Like those of ML, except elements are separated by semicolons and `hd` and `tl` are methods of the `List` class
- Python Lists
  - The list data type also serves as Python's arrays
  - Unlike Scheme, Common Lisp, ML, and F#, Python's lists are mutable
  - Elements can be of any type
  - Create a list with an assignment

```
myList = [3, 5.8, "grape"]
```

List Types (continued)

- Python Lists (continued)
  - List elements are referenced with subscripting, with indices beginning at zero

    `x = myList[1]` Sets `x` to `5.8`
  - List elements can be deleted with `del`

    `del myList[1]`
  - List Comprehensions – derived from set notation

    `[x * x for x in range(6) if x % 3 == 0]`

    `range(6)` creates `[0, 1, 2, 3, 4, 5]`

    Constructed list: `[0, 9]`

List Types (continued)

- Haskell's List Comprehensions
  - The original

```
[n * n | n <- [1..10]]
```

- F#'s List Comprehensions

```
let myArray = [|for i in 1 .. 5 -> [i * i) |]
```

- Both C# and Java supports lists through their generic heap-dynamic collection classes, `List` and `ArrayList`, respectively

Unions Types

- A *union* is a type whose variables are allowed to store different type values at different times during execution
- Design issue
  - Should type checking be required?

Discriminated vs. Free Unions

- C and C++ provide union constructs in which there is no language support for type checking; the union in these languages is called *free union*
- Type checking of unions require that each union include a type indicator called a *discriminant*
  - Supported by ML, Haskell, and F#

Unions in F#

- Defined with a type statement using OR

```
type intReal =
     | IntValue of int
     | RealValue of float;;
```

`intReal` is the new type

`IntValue` and `RealValue` are constructors

To create a value of type `intReal`:

```
let ir1 = IntValue 17;;
let ir2 = RealValue 3.4;;
```

Unions in F# (continued)

- Accessing the value of a union is done with pattern matching

```
match pattern with
    | expression_list1 -> expression1
    | ...
    | expression_listn -> expressionn
```

  - Pattern can be any data type
  - The expression list can have wild cards (_)

Unions in F# (continued)

Example:

```
let a = 7;;
let b = "grape";;
let x = match (a, b) with
     | 4, "apple" -> apple
     | _, "grape" -> grape
     | _ -> fruit;;
```

Unions in F# (continued)

To display the type of the `intReal` union:

```
let printType value =
    match value with
        | IntValue value -> printfn "int"
        | RealValue value -> printfn "float";;
```

If `ir1` and `ir2` are defined as previously,

`printType ir1` returns `int`

`printType ir2` returns `float`

Evaluation of Unions

- Free unions are unsafe
  - Do not allow type checking

- Java and C# do not support unions
  - Reflective of growing concerns for safety in programming language

Pointer and Reference Types

- A *pointer* type variable has a range of values that consists of memory addresses and a special value, *nil*
- Provide the power of indirect addressing
- Provide a way to manage dynamic memory
- A pointer can be used to access a location in the area where storage is dynamically created (usually called a *heap*)

Design Issues of Pointers

- What are the scope of and lifetime of a pointer variable?
- What is the lifetime of a heap-dynamic variable?
- Are pointers restricted as to the type of value to which they can point?
- Are pointers used for dynamic storage management, indirect addressing, or both?
- Should the language support pointer types, reference types, or both?

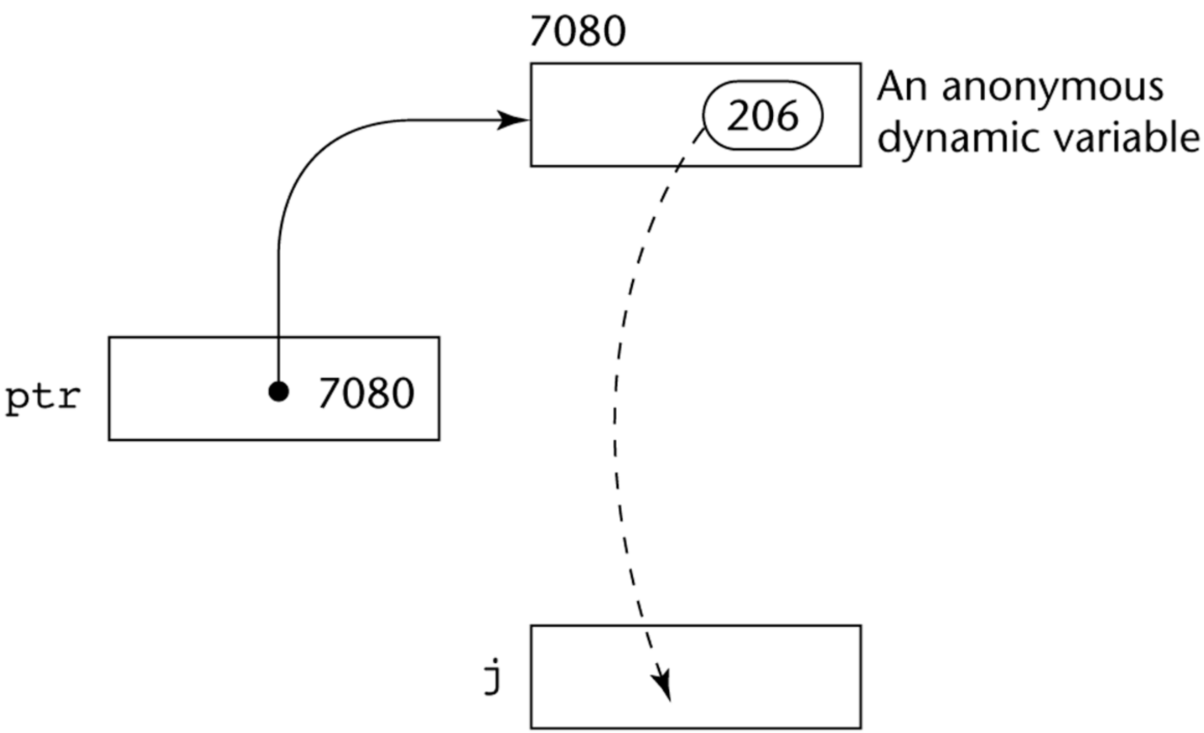
Pointer Operations

- Two fundamental operations: assignment and dereferencing
- Assignment is used to set a pointer variable's value to some useful address
- Dereferencing yields the value stored at the location represented by the pointer's value
  - Dereferencing can be explicit or implicit
  - C++ uses an explicit operation via *

    `j = *ptr`

    sets j to the value located at `ptr`

Pointer Assignment Illustrated



The assignment operation j = *ptr

Problems with Pointers

- Dangling pointers (dangerous)
  - A pointer points to a heap-dynamic variable that has been deallocated
- Lost heap-dynamic variable
  - An allocated heap-dynamic variable that is no longer accessible to the user program (often called *garbage*)
    - Pointer `p1` is set to point to a newly created heap-dynamic variable
    - Pointer `p1` is later set to point to another newly created heap-dynamic variable
    - The process of losing heap-dynamic variables is called *memory leakage*

Pointers in C and C++

- Extremely flexible but must be used with care
- Pointers can point at any variable regardless of when or where it was allocated
- Used for dynamic storage management and addressing
- Pointer arithmetic is possible
- Explicit dereferencing and address-of operators
- Domain type need not be fixed (`void *`)

  `void *` can point to any type and can be type checked (cannot be de-referenced)

Pointer Arithmetic in C and C++

```
float stuff[100];
float *p;
p = stuff;
```

`*(p+5)` is equivalent to `stuff[5]` and `p[5]`

`*(p+i)` is equivalent to `stuff[i]` and `p[i]`

Reference Types

- C++ includes a special kind of pointer type called a *reference type* that is used primarily for formal parameters
  - Advantages of both pass-by-reference and pass-by-value
- Java extends C++'s reference variables and allows them to replace pointers entirely
  - References are references to objects, rather than being addresses
- C# includes both the references of Java and the pointers of C++

Evaluation of Pointers

- Dangling pointers and dangling objects are problems as is heap management
- Pointers are like `goto`'s--they widen the range of cells that can be accessed by a variable
- Pointers or references are necessary for dynamic data structures--so we can't design a language without them

Representations of Pointers

- Large computers use single values
- Intel microprocessors use segment and offset

Dangling Pointer Problem

- *Tombstone*: extra heap cell that is a pointer to the heap-dynamic variable
  - The actual pointer variable points only at tombstones
  - When heap-dynamic variable de-allocated, tombstone remains but set to nil
  - Costly in time and space
- *Locks-and-keys*: Pointer values are represented as (key, address) pairs
  - Heap-dynamic variables are represented as variable plus cell for integer lock value
  - When heap-dynamic variable allocated, lock value is created and placed in lock cell and key cell of pointer

Heap Management

- A very complex run-time process
- Single-size cells vs. variable-size cells
- Two approaches to reclaim garbage
  - Reference counters (*eager approach*): reclamation is gradual
  - Mark-sweep (*lazy approach*): reclamation occurs when the list of variable space becomes empty

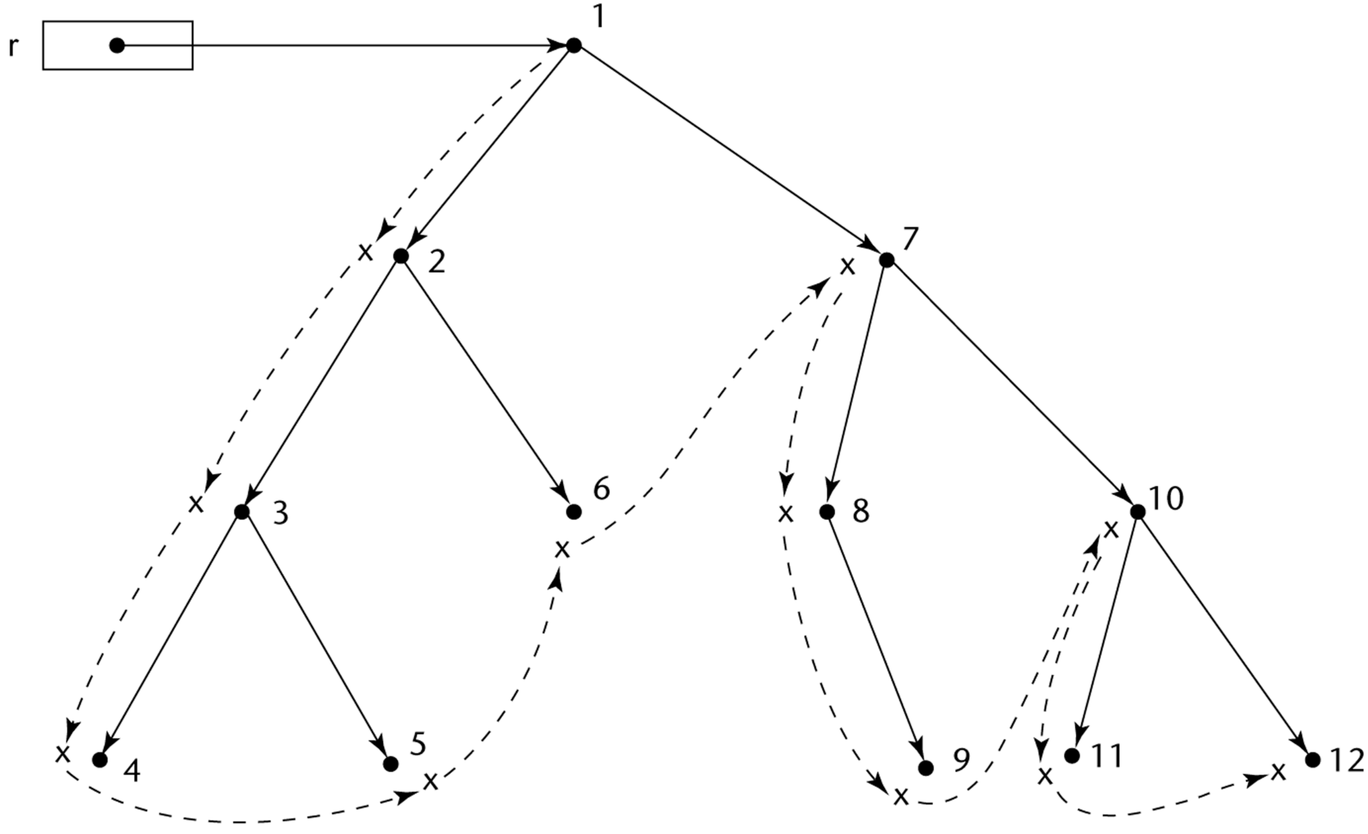
Reference Counter

- Reference counters: maintain a counter in every cell that store the number of pointers currently pointing at the cell
  - *Disadvantages*: space required, execution time required, complications for cells connected circularly
  - *Advantage*: it is intrinsically incremental, so significant delays in the application execution are avoided

Mark-Sweep

- The run-time system allocates storage cells as requested and disconnects pointers from cells as necessary; mark-sweep then begins
  - Every heap cell has an extra bit used by collection algorithm
  - All cells initially set to garbage
  - All pointers traced into heap, and reachable cells marked as not garbage
  - All garbage cells returned to list of available cells
  - Disadvantages: in its original form, it was done too infrequently. When done, it caused significant delays in application execution. Contemporary mark-sweep algorithms avoid this by doing it more often—called incremental mark-sweep

Marking Algorithm



Dashed lines show the order of node_marking

Variable-Size Cells

- All the difficulties of single-size cells plus more
- Required by most programming languages
- If mark-sweep is used, additional problems occur
  - The initial setting of the indicators of all cells in the heap is difficult
  - The marking process in nontrivial
  - Maintaining the list of available space is another source of overhead

Type Checking

- Generalize the concept of operands and operators to include subprograms and assignments
- *Type checking* is the activity of ensuring that the operands of an operator are of compatible types
- A *compatible type* is one that is either legal for the operator, or is allowed under language rules to be implicitly converted, by compiler- generated code, to a legal type
  - This automatic conversion is called a *coercion*.
- A *type error* is the application of an operator to an operand of an inappropriate type

Type Checking (continued)

- If all type bindings are static, nearly all type checking can be static
- If type bindings are dynamic, type checking must be dynamic
- A programming language is *strongly typed* if type errors are always detected
- Advantage of strong typing: allows the detection of the misuses of variables that result in type errors

Strong Typing

Language examples:

- C and C++ are not: parameter type checking can be avoided; unions are not type checked
- Java and C# are, almost (because of explicit type casting)
- ML and F# are

Strong Typing (continued)

- Coercion rules strongly affect strong typing--they can weaken it considerably (C++ versus ML and F#)

- Although Java has just half the assignment coercions of C++, its strong typing is still far less effective than that of ML and F#

Name Type Equivalence

- *Name type equivalence* means the two variables have equivalent types if they are in either the same declaration or in declarations that use the same type name
- Easy to implement but highly restrictive:
  - Subranges of integer types are not equivalent with integer types
  - Formal parameters must be the same type as their corresponding actual parameters

Structure Type Equivalence

- *Structure type equivalence* means that two variables have equivalent types if their types have identical structures
- More flexible, but harder to implement

Type Equivalence (continued)

- Consider the problem of two structured types:
  - Are two record types equivalent if they are structurally the same but use different field names?
  - Are two array types equivalent if they are the same except that the subscripts are different? (e.g. `[1..10]` and `[0..9]`)
  - Are two enumeration types equivalent if their components are spelled differently?
  - With structural type equivalence, you cannot differentiate between types of the same structure (e.g. different units of speed, both float)

Theory and Data Types

- Type theory is a broad area of study in mathematics, logic, computer science, and philosophy
- Two branches of type theory in computer science:
  - Practical – data types in commercial languages
  - Abstract – typed lambda calculus
- A type system is a set of types and the rules that govern their use in programs

Theory and Data Types (continued)

- Formal model of a type system is a set of types and a collection of functions that define the type rules
  - Either an attribute grammar or a type map could be used for the functions
  - Finite mappings – model arrays and functions
  - Cartesian products – model tuples and records
  - Set unions – model union types
  - Subsets – model subtypes

Summary

- The data types of a language are a large part of what determines that language's style and usefulness
- The primitive data types of most imperative languages include numeric, character, and Boolean types
- The user-defined enumeration and subrange types are convenient and add to the readability and reliability of programs
- Arrays and records are included in most languages
- Pointers are used for addressing flexibility and to control dynamic storage management