1. Rewrite the BNF grammar above to give + precedence over * and force + to be right associative.

2. Using the grammar provided above, show a parse tree and a leftmost derivation for each of the following statements:
   a. $A = (A + B) \times C$
   b. $A = B \times (C \times (A + B))$

3. Prove that the following grammar is ambiguous:

   \[
   \begin{align*}
   &<S> \rightarrow <A> \\
   &<A> \rightarrow <A> + <A> + <A> | <id> \\
   &<id> \rightarrow a | b | c
   \end{align*}
   \]

4. Consider the following grammar:

   \[
   \begin{align*}
   &<S> \rightarrow <A> a <B> b \\
   &<A> \rightarrow <A> b | b \\
   &<B> \rightarrow a <B> | a
   \end{align*}
   \]

   Which of the following sentences are in the language generated by this grammar?
   a. baab
   b. bbabab
   c. bbaaaaa
   d. bbaab

5. Write a grammar for the language consisting of strings that have $n$ copies of the letter a followed by the same number of copies of the letter b, where $n > 0$. 

\[
\begin{align*}
&<assign> \rightarrow <id> = <expr> \\
&id> \rightarrow A \mid B \mid C \\
<expr> \rightarrow <expr> + <term> \mid <term> \\
<term> \rightarrow <term> \times <factor> \mid <factor> \\
<factor> \rightarrow ( <expr> ) \mid <id>
\end{align*}
\]
6. Write an attribute grammar whose BNF basis is the grammar below but whose language rules are as follows: Data types cannot be mixed in expressions, but assignment statements need not have the same types on both sides of the assignment operator.

\[
\text{<assign> } \rightarrow \text{ <var> } = \text{ <expr>}
\]
\[
\text{<expr> } \rightarrow \text{ <var>[2] + <var>[3] | <var>}
\]
\[
\text{<var> } \rightarrow \text{ A | B | C}
\]

7. Consider the syntax rule and the semantic function below:

\[
<\text{bin} \_\text{num}> \rightarrow '0' | '1' | <\text{bin} \_\text{num}> '0' | <\text{bin} \_\text{num}> '1'
\]
\[
M_{\text{bin}}('0') = 0
\]
\[
M_{\text{bin}}('1') = 1
\]
\[
M_{\text{bin}}(<\text{bin} \_\text{num}> '0') = 2 * M_{\text{bin}}(<\text{bin} \_\text{num}> )
\]
\[
M_{\text{bin}}(<\text{bin} \_\text{num}> '1') = 2 * M_{\text{bin}}(<\text{bin} \_\text{num}> ) + 1
\]

What is the value of the sequence of characters ‘1001’? (Show your work)

8. Axiomatic Semantics

Compute the weakest precondition for each of the following:

a. \[a = 2 * (b - 1) - 1 \{a > 0\}\]

b. \[a = 2 * b + 1; \]
\[b = a - 3; \]
\[{b < 0}\]

Prove that the following program segment is correct:

\[c > 4 \]
\[c = b - 3; \]
\[a = c + 2; \]
\[{a >= 2}\]