Optimizing Risk-Adjusted Return in Constructing Portfolios of Alphas

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Summary

- Modern Portfolio Theory (MPT), developed by Harry Markowitz in the 1950s, is essential in investment management, providing a framework for optimizing the balance between risk and return through diversification.
- Alphas in statistical arbitrage refer to systematic trading signals that predict relative asset price movements.
- Constructing a portfolio of alphas involves combining multiple alphas to create a diversified and optimized strategy, reducing the risk associated with any single alpha and benefiting from internal trade crossing and position netting.
- Risk-adjusted return maximization measures investment efficiency by considering both return and risk.
- The empirical analysis, conducted using over 800 alphas in the Binance USDⓈ-M market, compares different weighting methodologies. Results show that strategies optimized for risk-adjusted returns outperform equal-weighted strategies, highlighting the importance of optimization in portfolio construction.

Modern Portfolio Theory [1], developed by Harry Markowitz in the 1950s, is a foundational concept in investment management and financial theory. It provides a framework for constructing a portfolio of assets to maximize the expected return for a given level of risk or to minimize risk for a given level of expected return. On the other hand, the maximization of risk-adjusted return focuses on maximizing returns after adjusting for risk, using specific performance metrics like the Sharpe or Sortino ratio. This article will assess various models that extend MPT by substituting the risk term in constructing a portfolio of alphas to maximize its risk-adjusted return, where each alpha is treated as an investment asset.

Key Concepts

- **Risk and Return:** At the heart of MPT is the idea that the return on investment is directly related to the risk we are willing to take. Typically, higher risk is associated with the potential for higher returns, and lower risk correlates with lower returns. Risk is often measured by the volatility of returns, indicating how much an asset's return fluctuates.
- **Diversification:** This is a central principle of MPT. Diversification involves spreading investments across various assets to reduce risk. The theory suggests that holding multiple assets is less risky than concentrating on one, as different assets often do not move in the same direction simultaneously. If one investment performs poorly, another might perform well, offsetting the loss.
- **Efficient Frontier:** Imagine a graph with risk (volatility) on the X-axis and expected return on the Y-axis. The efficient frontier is a curved line on this graph representing the set of portfolios that offer the highest expected return for a given level of risk. These portfolios are considered "efficient" because it is impossible to achieve a higher return without taking on more risk.

Alphas in Statistical Arbitrage

In statistical arbitrage, an alpha represents a systematic trading signal designed to generate returns by predicting relative price movements of assets. Various alphas are implemented by a combination of mathematical expressions based on different concepts, such as mean-reversion, momentum, and more. For more details of alphas, refer to [2].

Portfolio Construction on Alphas

Each alpha can be considered as an investment vehicle. Constructing a portfolio of alphas involves combining multiple alphas to create a unified strong signal (a.k.a. combo alpha) that benefits from diversification and optimized performance. By combining various alphas, we can diversify the alpha portfolio to hedge against any subset of alphas performing poorly during any given period. Additionally, the portfolio benefits from internal crossing of trades and netting of positions, which boosts expected return, reduces trading costs, and improves profitability. This approach also reduces the risk associated with any single alpha by spreading it across multiple alphas.

Modern Portfolio Theory

MPT focuses on creating an optimal portfolio that maximizes expected return for a given level of risk, or equivalently, minimizes risk for a given level of expected return. There are three types of optimization problems.

1. Maximize Return Given Portfolio Risk Target

$$\max_{\mathbf{w}} \boldsymbol{\mu}^\top \mathbf{w}$$
$$s.t.\ \mathbf{w}^\top \mathbf{C} \mathbf{w} \leq \sigma_p$$
$$\sum_i w_i = 1$$

, where $\boldsymbol{\mu} \in \mathbb{R}^n$ is the expected return vector of alphas, n is the number of alphas, $\mathbf{w} \in \mathbb{R}^n$ is the weight vector, $\mathbf{C} \in \mathbb{R}^{\boldsymbol{n} \times \boldsymbol{m}}$ is the covariance matrix of alphas, and σ_p is the target of the portfolio risk.

2. Minimize Risk Given Portfolio Return Target

$$\min_{\mathbf{w}} \mathbf{w}^\top \mathbf{C} \mathbf{w}$$
$$s.t.\ \boldsymbol{\mu}^\top \mathbf{w} \leq \mu_p$$
$$\sum_i w_i = 1$$

, where μ_p is the target return of the portfolio.

3. Maximize Return while Minimizing Risk

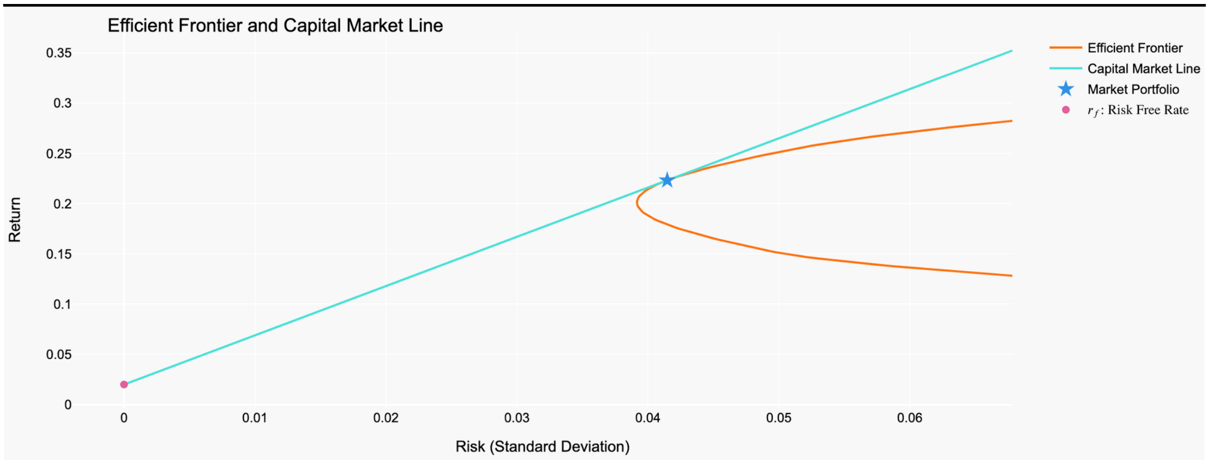
$$\max_{\mathbf{w}} \boldsymbol{\mu}^\top \mathbf{w} \ - \gamma \mathbf{w}^\top \mathbf{C} \mathbf{w}$$
$$s.t. \sum_i w_i = 1$$

, where γ is the risk-aversion coefficient.

Using any of the problems described above, an efficient frontier curve can be constructed using an iterative method. This curve represents the set of optimal portfolios offering the highest expected return for a given level of risk. The straight line starting from the risk-free rate and tangent to the efficient frontier is the Capital Market Line (CML). The tangency point of the CML and the efficient frontier represents the portfolio with the highest Sharpe ratio, indicating the best risk-adjusted return. This is illustrated in Figure 1.

Figure 1. Efficient Frontier and Capital Market Line to Find the Portfolio of Maximum Sharpe Ratio

Source: Presto Research



Maximization of Risk-Adjusted Return

Risk-adjusted return is a concept used in finance to measure the return on a strategy relative to the amount of risk taken to achieve that return. It helps comparing the performance of different strategies by considering not just the returns they generate, but also the risks involved in generating those returns. Here's a detailed explanation of risk-adjusted return and its importance:

1. **Comparative Analysis**: It allows comparing strategies with different levels of risk. For example, a high-return investment might not be preferable if it comes with extremely high risk.
2. **Efficiency Assessment**: It helps in assessing the efficiency of a strategy. A higher risk-adjusted return indicates a more efficient investment, as it generates higher returns per unit of risk taken.
3. **Portfolio Management**: It aids in constructing and managing a diversified portfolio by helping to balance the trade-off between risk and return.

Frequently used metrics for risk-adjusted return include Sharpe ratio, Sortino ratio, and average return over MDD (Maximum Drawdown).

Sharpe Ratio Maximization

Instead of constructing the efficient frontier using an iterative method to find the tangency point that maximizes the Sharpe ratio, we can directly aim to solve the problem of maximizing the ratio. In constructing the portfolio of alphas for statistical arbitrage, which is dollar neutral and, thus, self-financing, subtracting the risk-free rate from the portfolio return to calculate the Sharpe ratio is not usually considered. Therefore, the problem can be formulated as follows:

$$\max_{\mathbf{w}} \frac{\boldsymbol{\mu}^\top \mathbf{w}}{\sqrt{\mathbf{w}^\top \mathbf{C} \mathbf{w}}}$$

$$s.t. \sum_i w_i = 1$$

Usually, the problem is solved with lower and upper bound on the weight of each alpha:

$$\mathbf{w}_L \leq \mathbf{w} \leq \mathbf{w}_U$$

, where $\mathbf{w}_L$ is the lower bound and $\mathbf{w}_U$ is the upper bound vector of the weights.

To solve this optimization problem using either quadratic programming or conic programming solvers, we need to transform the problem. Since the objective function is scale invariant with respect to $\mathbf{w}$, we may define a scaled vector, $\mathbf{y}$, proportional to the weight vector to simplify the function:

$$\mathbf{y} \coloneqq \kappa \mathbf{w}$$

which satisfies $\boldsymbol{\mu}^\top \mathbf{y} = 1$ for $\kappa \geq 0$.

With this change, the problem becomes

$$\min_{\mathbf{y},\kappa} \mathbf{y}^\top \mathbf{C} \mathbf{y}$$

$$s.t. \boldsymbol{\mu}^\top \mathbf{y} = 1$$

$$\sum_i y_i = \kappa$$

$$\kappa \mathbf{w}_L \leq \mathbf{y} \leq \kappa \mathbf{w}_U$$

$$\kappa \geq 0$$

Sortino Ratio Maximization

The Sortino Ratio is a risk-adjusted performance metric used to evaluate the return of an investment relative to its downside risk. Similar to the Sharpe Ratio, the Sortino Ratio focuses on downside deviation rather than total volatility, penalizing only the negative returns that fall below a specific target or threshold. This makes the Sortino Ratio particularly useful for investors who are more concerned with downside risk rather than total volatility. The problem of maximizing the Sortino Ratio can be formulated as:

$$\max_{\mathbf{w}} \frac{\boldsymbol{\mu}^\top \mathbf{w}}{\mathrm{DR}(\mathbf{Rw})}$$

, where $R \in \mathbb{R}^{m \times n}$ is the matrix of alpha returns and m is the number of time intervals. $\mathrm{DR}(\cdot)$ is the function that outputs the downside deviation:

$$\mathrm{DR}(\mathbf{r}) \coloneqq \frac{1}{m-1} \sqrt{\sum_{t=0}^{m-1} \max(-r_t, 0)^2}$$

, where $\mathbf{r} \in \mathbb{R}^m$ is a vector of returns.

Note that the objective function is scale invariant. Thus, we can use the same technique we exploited in Sharpe ratio maximization:

$$\begin{aligned}
&\min_{\mathbf{y},\kappa} \mathrm{DR}(\mathbf{Ry})^2 \\
&s.t.\ \boldsymbol{\mu}^\top \mathbf{y} = 1 \\
&\sum_i y_i = \kappa \\
&\kappa \mathbf{w}_L \leq \mathbf{y} \leq \kappa \mathbf{w}_U \\
&\kappa \geq 0
\end{aligned}$$

We define the downside return vector as:

$$\mathbf{d} \coloneqq \mathrm{maximum}(-\mathbf{Ry}, \mathbf{0})$$

, where $\mathrm{maximum}(\cdot,\cdot)$ is an element-wise max function that works on vectors.

Then the constraints for the downside risk will be:

$$\frac{1}{\sqrt{m-1}} \|\mathbf{d}\|_2 \leq q$$

Now the problem can be formulated as:

$$\min_{\mathbf{y},\kappa,\mathbf{d},q} q^2$$
$$s.t.\ \boldsymbol{\mu}^\top \mathbf{y} = 1$$
$$\sum_i y_i = \kappa$$
$$\kappa \mathbf{w}_L \le \mathbf{y} \le \kappa \mathbf{w}_U$$
$$\kappa \ge 0$$
$$\text{maximum}(-\mathbf{R}\mathbf{y}, \mathbf{0}) \le \mathbf{d}$$
$$\frac{\mathbf{1}}{\sqrt{\boldsymbol{m} - \mathbf{1}}} \|\mathbf{d}\|_2 \le q$$

Maximization of Average Return over MDD

Another frequently used measure of risk is MDD. The problem of maximizing the risk-adjusted return using MDD can be formulated as:

$$\max_{\mathbf{w}} \frac{\boldsymbol{\mu}^\top \mathbf{w}}{\text{MDD}(\mathbf{R}\mathbf{w})}$$

This problem is also scale invariant with respect to $\mathbf{w}$. Using the same transformation we have used before, we can reformulate it as:

$$\min_{\mathbf{y},\kappa} \text{MDD}(\mathbf{R}\mathbf{y})$$
$$s.t.\ \boldsymbol{\mu}^\top \mathbf{y} = 1$$
$$\sum_i y_i = \kappa$$
$$\kappa \mathbf{w}_L \le \mathbf{y} \le \kappa \mathbf{w}_U$$
$$\kappa \ge 0$$

Now we need to transform the objective function into a combination of a convex objective and constraints. First, we need to form a time series of the cumulative portfolio return:

$$\mathbf{c} \coloneqq \text{cumsum}(\mathbf{R}\mathbf{y})$$

, which is an affine function.

With the cumulative return vector, we define the record high vector, $\mathbf{h}$, as:

$$\max(c_0, 0) \le h_0$$
$$\max(c_t, h_{t-1}) \le h_t, \qquad \forall t \in \{1, 2, \dots, m-1\}$$

Don’t worry about the inequality because we will minimize it in the end. Therefore, it doesn't matter whether it is an equality or inequality constraint. However, if we use equality constraints, the constraints cannot be translated into convex constraints.

With the record high vector, we define the MDD, d, as:

$$h_t - c_t \leq d, \qquad \forall t \in \{0, 1, \dots, m-1\}$$

Now we can minimize the MDD of $\mathbf{y}$:

$$\min_{\mathbf{y},\kappa,\mathbf{h},\mathbf{c},d} d$$
$$s.t.\ \boldsymbol{\mu}^\top \mathbf{y} = 1$$
$$\sum_i y_i = \kappa$$
$$\kappa \mathbf{w}_L \leq \mathbf{y} \leq \kappa \mathbf{w}_U$$
$$\kappa \geq 0$$
$$\mathbf{c} = \mathrm{cumsum}(\mathbf{R}\mathbf{y})$$
$$\max(c_0, 0) \leq h_0$$
$$\max(c_t, h_{t-1}) \leq h_t, \qquad \forall t \in \{1, 2, \dots, m-1\}$$
$$h_t - c_t \leq d, \qquad \forall t \in \{0, 1, \dots, m-1\}$$

This leads to the maximization of the average return over MDD of the portfolio.

Comparison

More than 800 alphas in production are used for constructing strategies with a target GMV of 10 million USDT in the Binance USDⓈ-M market, each targeting 5-minute intervals and the 50 most liquid symbols for signal generation. The in-sample part of alphas is not used for the strategy construction. The simulation summary considers transaction costs, including slippage, under-fill, and funding fees. Metrics such as return, Sharpe ratio, Sortino ratio, and return/MDD are annualized.

Figure 2. PNL Plots of Strategies Using Different Weighting

Source: Presto Research

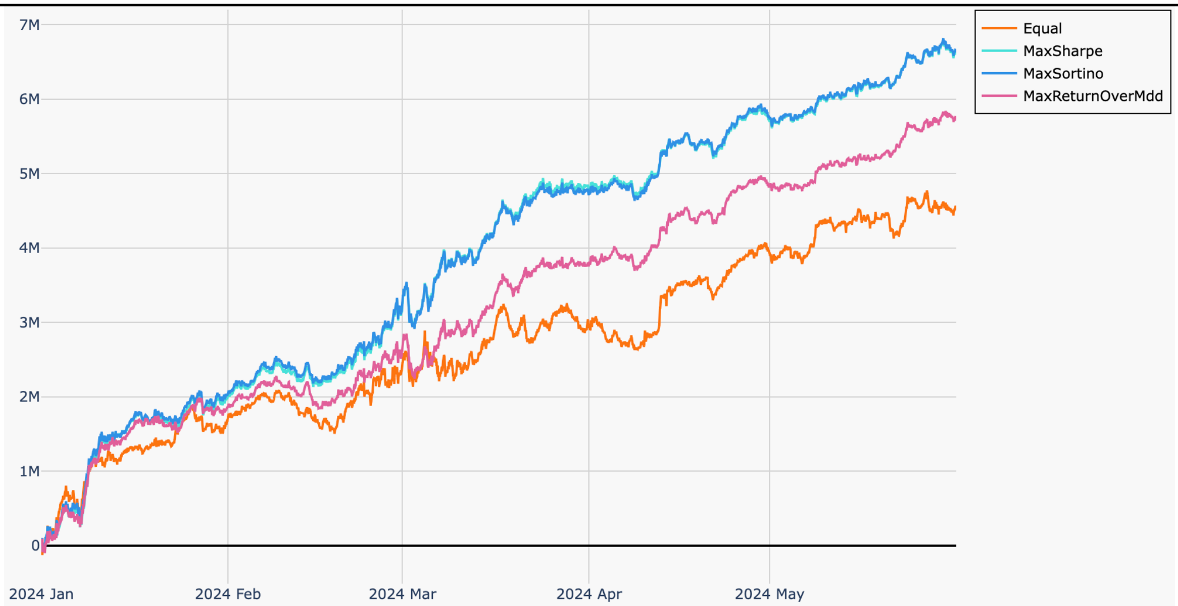


Table 1. Stats of the Strategy Using Equal Weight

Equal

	Booksize	Return (%)	Sharpe	Sortino	Return/MDD	MDD (%)	Daily Win Ratio (%)	Daily Turnover (%)	Return/Trade (bp)
20240101-20240201	9.94E+06	202.14	9.39	16.05	46.53	4.34	67.74	408.42	13.56
20240201-20240301	1.02E+07	86.23	5.08	8.58	14.98	5.76	65.52	394.40	5.99
20240301-20240401	1.01E+07	64.21	3.17	5.23	9.48	6.77	58.06	392.47	4.48
20240401-20240501	9.04E+06	131.47	6.37	11.02	26.09	5.04	66.67	398.92	9.03
20240501-20240531	9.38E+06	78.65	4.74	7.87	18.28	4.30	58.06	391.19	5.51
Total	9.72E+06	112.38	5.65	9.50	16.04	7.01	63.16	397.11	7.75

Table 2. Stats of the Strategy Using Sharpe Ratio Maximization

MaxSharpe

	Booksize	Return (%)	Sharpe	Sortino	Return/MDD	MDD (%)	Daily Win Ratio (%)	Daily Turnover (%)	Return/Trade (bp)
20240101-20240201	1.02E+07	230.05	10.30	16.82	71.05	3.24	77.42	370.18	17.03
20240201-20240301	9.95E+06	152.81	8.46	14.36	41.96	3.64	58.62	351.44	11.91
20240301-20240401	9.76E+06	193.48	8.87	15.37	30.77	6.29	61.29	338.87	15.64
20240401-20240501	8.80E+06	124.32	7.03	12.30	32.56	3.82	66.67	352.16	9.67
20240501-20240531	8.66E+06	122.86	7.17	12.19	43.05	2.85	64.52	360.84	9.33
Total	9.48E+06	167.42	8.42	14.26	25.87	6.47	65.79	354.81	12.93

Table 3. Stats of the Strategy Using Sortino Ratio Maximization

MaxSortino

	Booksize	Return (%)	Sharpe	Sortino	Return/MDD	MDD (%)	Daily Win Ratio (%)	Daily Turnover (%)	Return/Trade (bp)
20240101-20240201	1.02E+07	235.72	10.32	16.82	84.22	2.80	70.97	374.17	17.26
20240201-20240301	9.87E+06	150.78	8.40	14.27	40.47	3.73	62.07	351.82	11.74
20240301-20240401	9.70E+06	185.68	8.59	14.87	28.44	6.53	64.52	333.26	15.26
20240401-20240501	8.75E+06	137.72	7.77	13.59	38.24	3.60	66.67	352.64	10.70
20240501-20240531	8.62E+06	123.01	7.17	12.21	45.31	2.71	64.52	360.95	9.34
Total	9.43E+06	169.14	8.48	14.34	25.16	6.72	65.79	354.70	13.06

Table 4. Stats of the Strategy Using Return/Mdd Maximization

MaxReturnOverMdd

	Booksize	Return (%)	Sharpe	Sortino	Return/MDD	MDD (%)	Daily Win Ratio (%)	Daily Turnover (%)	Return/Trade (bp)
20240101-20240201	9.51E+06	230.59	9.91	16.23	73.74	3.13	64.52	373.01	16.94
20240201-20240301	9.52E+06	101.39	5.84	9.84	21.65	4.68	58.62	357.84	7.76
20240301-20240401	9.26E+06	148.58	6.92	11.88	22.73	6.54	61.29	327.90	12.41
20240401-20240501	8.29E+06	150.14	8.15	14.29	38.01	3.95	63.33	350.38	11.74
20240501-20240531	8.51E+06	126.56	7.66	13.02	71.83	1.76	67.74	347.92	9.97
Total	9.02E+06	152.69	7.70	13.00	22.74	6.71	63.82	351.56	11.90

Table 5. Correlation between Strategies with Different Weighting Methodologies

	0	1	2	3
0: Equal	1.000000	0.663371	0.659124	0.650733
1: MaxSharpe	0.663371	1.000000	0.996157	0.932304
2: MaxSortino	0.659124	0.996157	1.000000	0.935317
3: MaxReturnOverMdd	0.650733	0.932304	0.935317	1.000000

Compared to the simple methodology of allocating equal weight to each alpha, optimizing the weight using risk-adjusted metrics yields significantly better results in various aspects, including average return, risk-adjusted metrics, MDD, return per trade. While the correlations between the equal weighting strategy and the other strategies are quite low, the correlations among the risk-adjusted weighting strategies are remarkably high. Among the risk-adjusted metric optimization strategies, maximizing Sharpe ratio and Sortino ratio clearly outperforms equal weighting or return/MDD maximization.

Conclusion

The integration of Modern Portfolio Theory and risk-adjusted return maximization offers a robust framework for constructing efficient portfolios in quantitative finance. MPT, with its emphasis on diversification and the efficient frontier, provides a foundational approach to balancing risk and return. By optimizing portfolios to either maximize expected return for a given level of risk or minimize risk for a given level of return, investors can systematically achieve more efficient investment outcomes.

The maximization of risk-adjusted return, using metrics such as the Sharpe ratio, Sortino ratio, and average return over MDD, further refines this process by directly targeting the efficiency of returns relative to risk. This approach is particularly valuable in constructing portfolios composed of multiple alphas, where each alpha represents a distinct trading signal. By combining these alphas into a unified portfolio, overall performance can be enhanced through diversification and optimization, thereby reducing the impact of any single underperforming alpha.

The empirical analysis conducted on the Binance USDⓈ-M market demonstrates the practical application of these approaches. The results indicate that strategies optimized for risk-adjusted returns, such as those maximizing the Sharpe ratio, Sortino ratio, or return over MDD, outperform those using equal weighting. These optimized strategies exhibit higher returns and improved risk-adjusted return and risk metrics, validating the effectiveness of risk-adjusted return maximization.

Overall, risk-adjusted return maximization provides a powerful toolkit for portfolio management, enabling investors to achieve superior performance relative to risk in diverse and dynamic market environments. Future research and advancements in computational methods are likely to further enhance these approaches, providing even greater adaptability in portfolio construction and management.

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