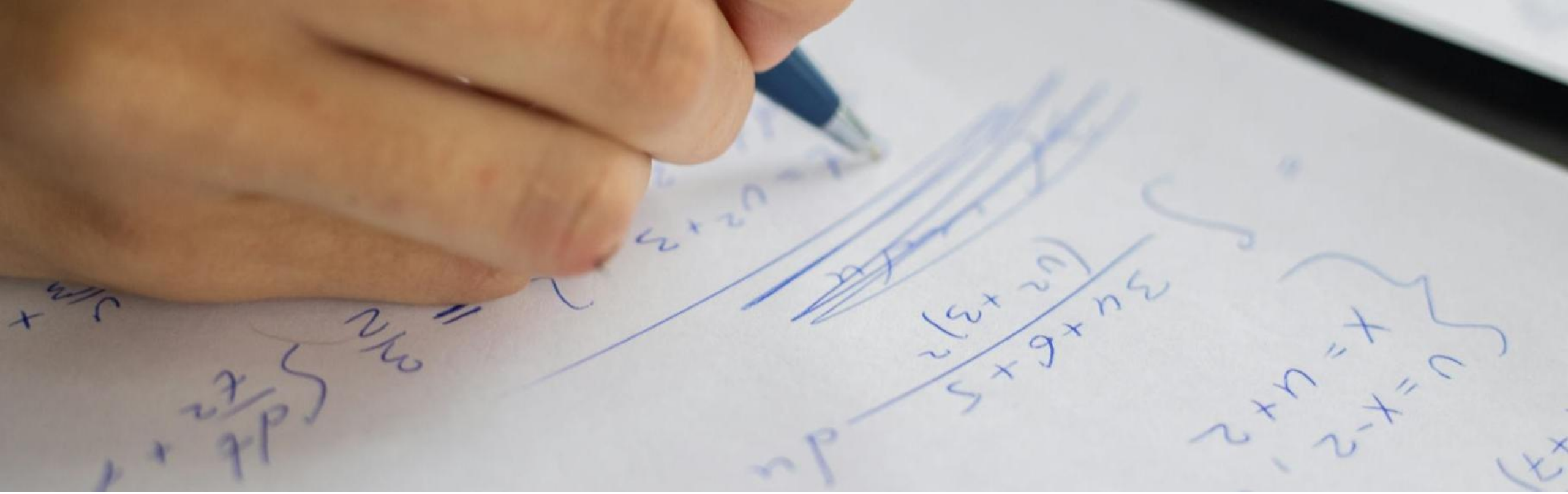


Demystifying AI in the math classroom:

Exploring machine learning concepts with high-school students

Stephan Kindler, Sarah Schönbrodt, Martin Frank

The era of AI

Students should be empowered to critically, reflectively, and creatively engage with AI technologies.

This requires early AI education on opportunities and design possibilities, limitations, and the functioning of AI.

UNESCO, 2024; European Commission, 2022; Long & Magerko, 2020, Touretzky et al., 2022

AI education in mathematics education

Reasonable? Possible? Even necessary?

- Data-driven ML methods are **based on mathematics and data!**
  mathematics education plays a crucial role in fostering a deeper understanding of AI technologies

- **Authentic** insight into ML methods is possible!
  connection to school mathematical contents (vectors, dot product, Euclidean distance, functions, lines, planes, ...)

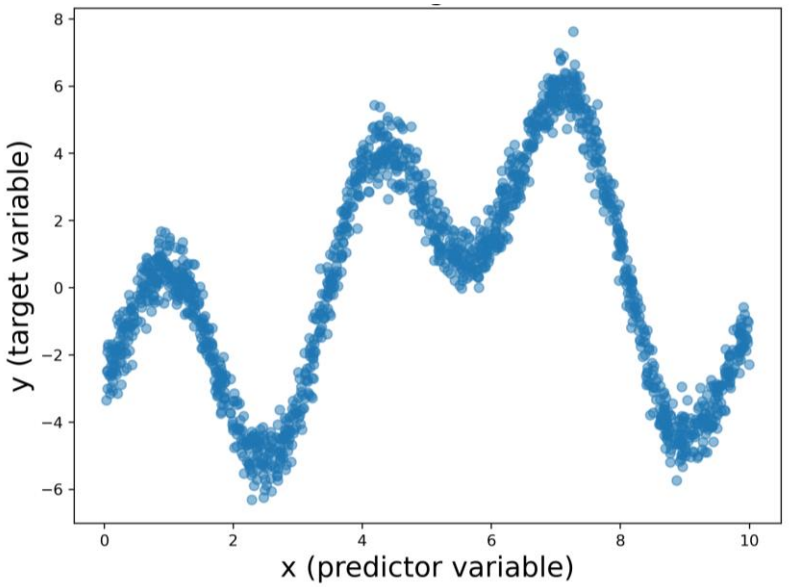
- Variety of suitable **real and relevant problems** exists!
  accessibility, availability of data

Math education is an essential component for developing AI skills.

Teaching machine learning in mathematics education

- Decision trees (Biehler & Fleischer, 2021)
- K-nearest neighbour (Hazzan & Mike, 2022; Bata, 2024; Schönbrodt et al., 2024)
- Support Vector Machine (Schönbrodt et al., 2025)
- Artificial Neural Networks (Kindler et al., 2024)
- Linear regression (Biehler et al., 2024)
- N-Gram model (Hofmann & Frank, 2022)
- …

Starting point: data
Supervised learning



Input data: (predictor variables):

$$x_1, \dots, x_N \in \mathbb{R}^n$$

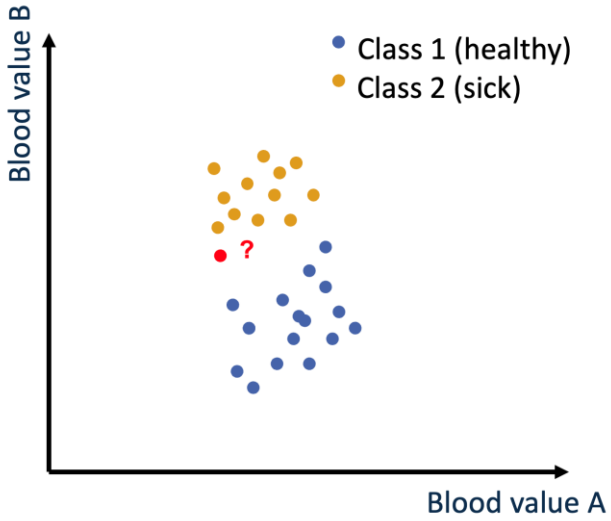
Output data: (values of the target variables):

$$y_1, \dots, y_N \in \mathbb{R}$$

Goal: mathematical model that best describes the unknown relationship between input and output data

→ a function $f: \mathbb{R}^n \longrightarrow \mathbb{R}$, such that predictions $\hat{y}_i := f(x_i)$ are close to the known outputs y_i.

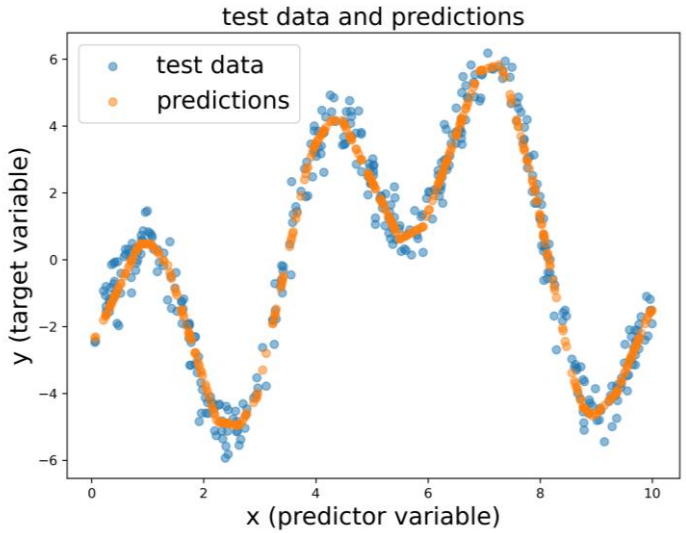
Two main problem classes



Output y_i

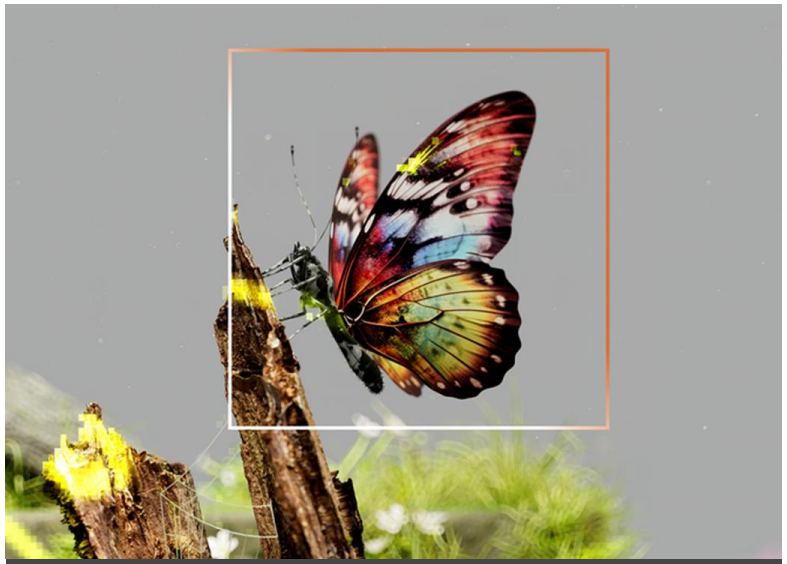
categorical → classification problem

numerical → regression problem



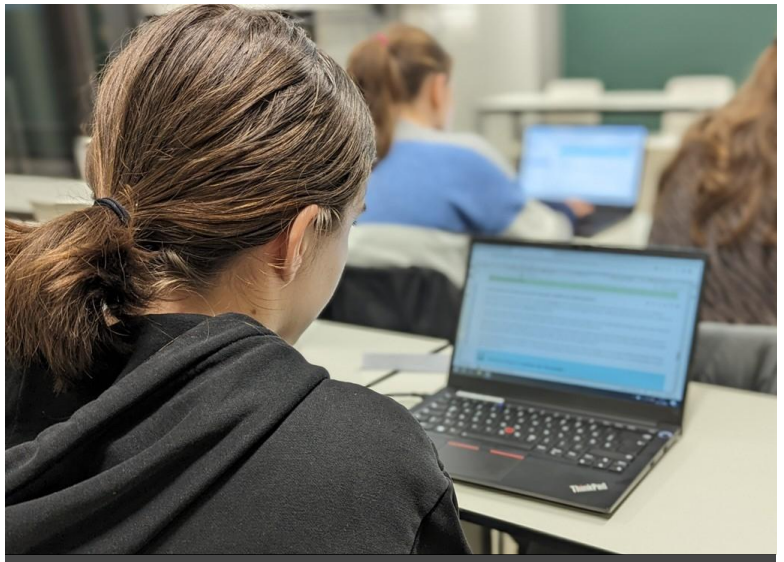
Biehler et al., 2024

Our focus today



A workshop on image classification and the Support Vector Machine

A learning path to better understand Artificial Neural Networks



Conclusions about AI education and ways to implement

CAMMP

Computational And Mathematical Modeling Program

Goal:
Allow students to solve
authentic, real and **relevant** problems
using mathematical modeling and computers

- Teaching and learning material on AI (and beyond!)
- Workshops for students and teachers
- Locations: Aachen, Karlsruhe, Stuttgart (Germany), Salzburg (Austria)

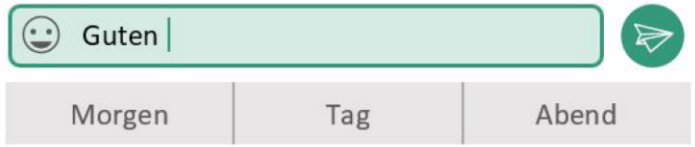
mathematical modeling = use of math to solve a real-world problem

AI workshops (blue = ML-method)

Privacy in social networks
Decision Trees

Sube (2019)

Word predictions
N-Gram model



Hofmann (2022)

Recommender systems
k-nearest-neighbour

Schönbrodt et al. (2021)

Topics

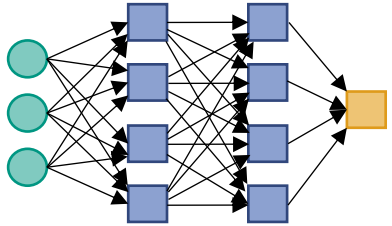
Human activity recognition
k-nearest-neighbour

phyphox
physical phone experiments

Running? Walking? Climbing stairs?

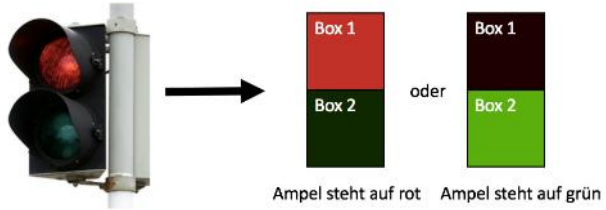
Schönbrodt et al. (2024)

Predicting life expectancy
lin. Regression & neural networks



Kindler et al. (2024)

(Image) Classification
Support Vector Machine

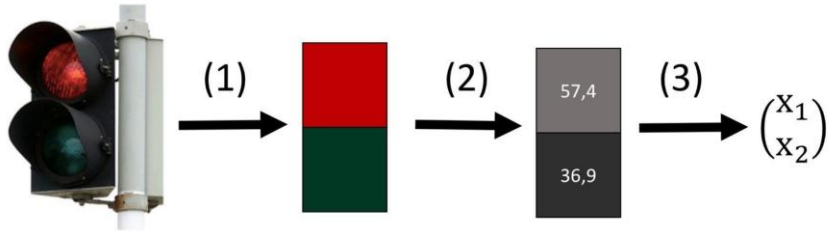


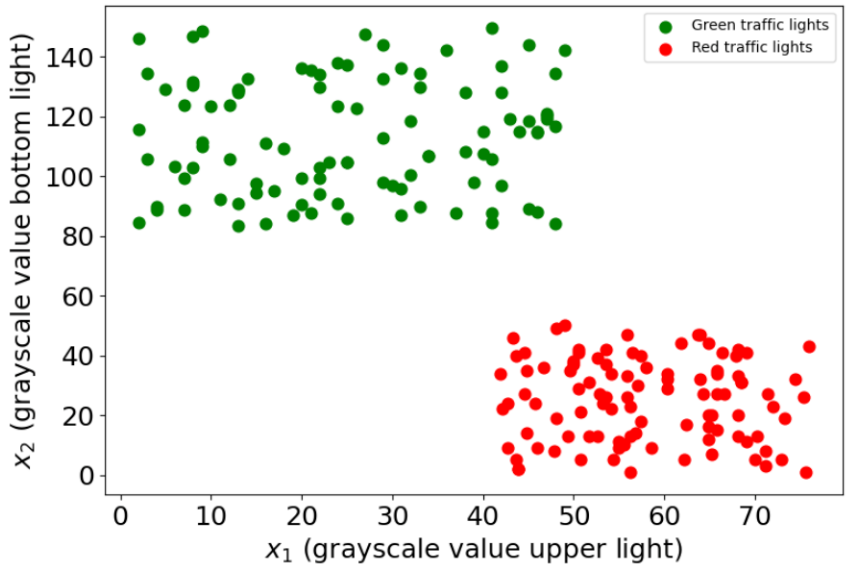
Schönbrodt et al. (2021)

Classification problems

The Support Vector Machine – upper secondary education

- Data points of two classes

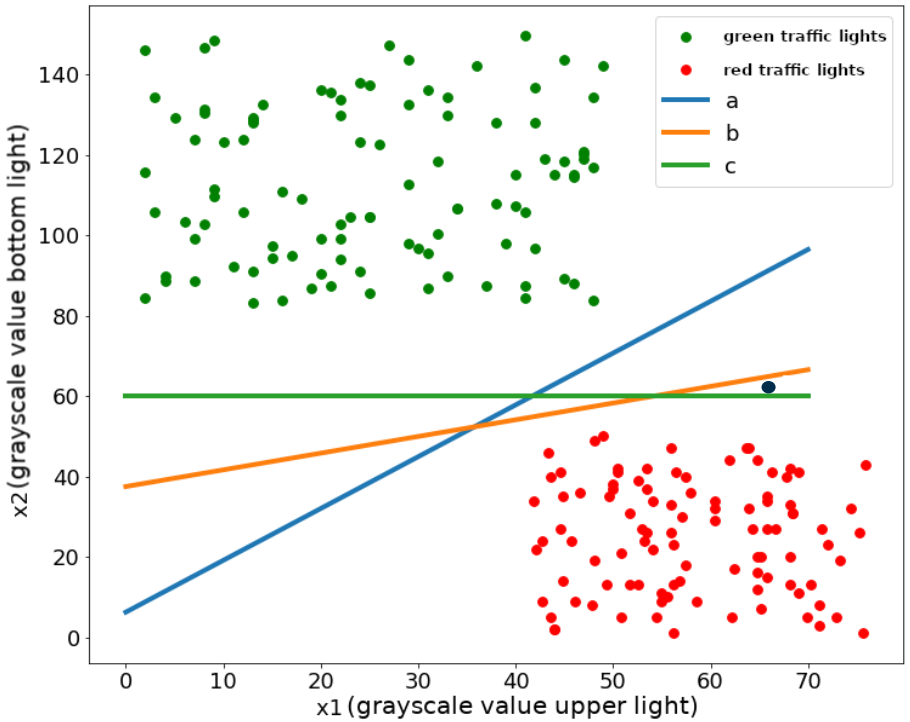




Classification problems

The Support Vector Machine – upper secondary education

- Data points of two classes
- Find a **line**, that best separates the data points of both classes.



Classification problems

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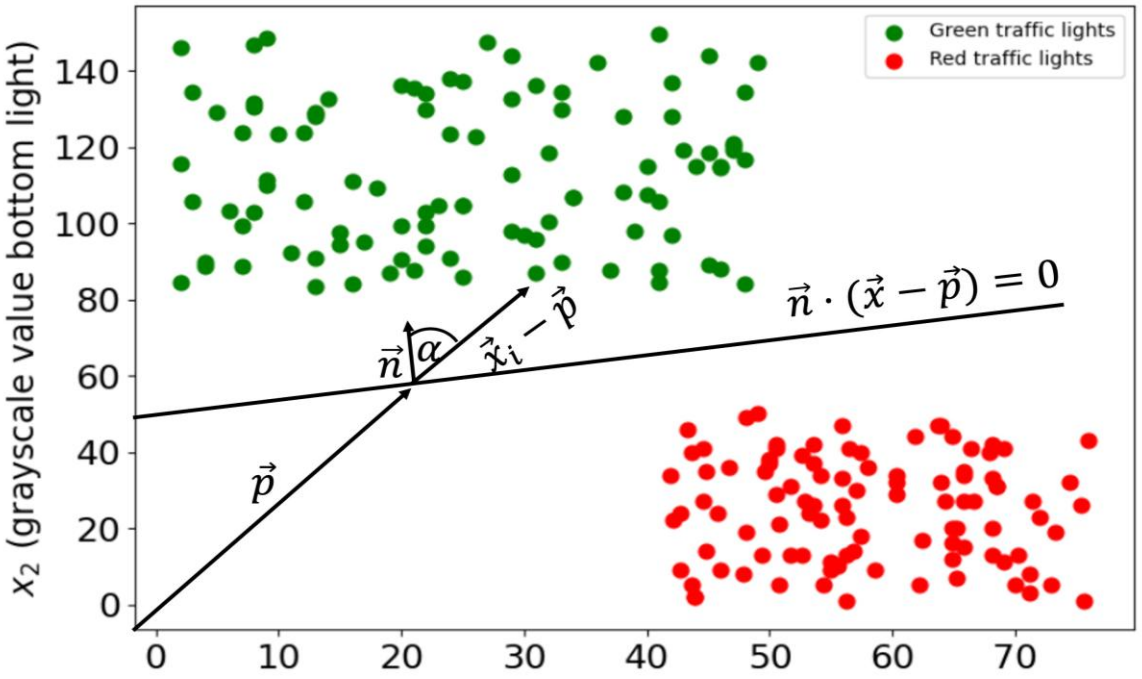
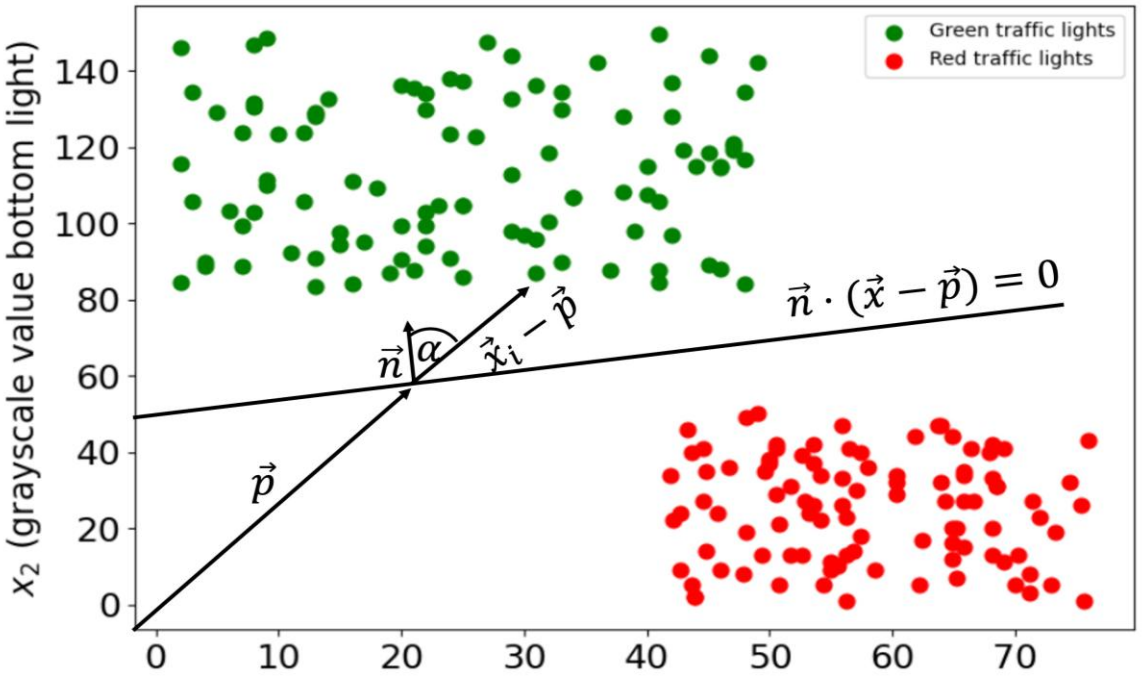


Classification problems

The Support Vector Machine – upper secondary education

- Data points of two classes
- Find a **line**, that best separates the data points of both classes.
- Define „best“.

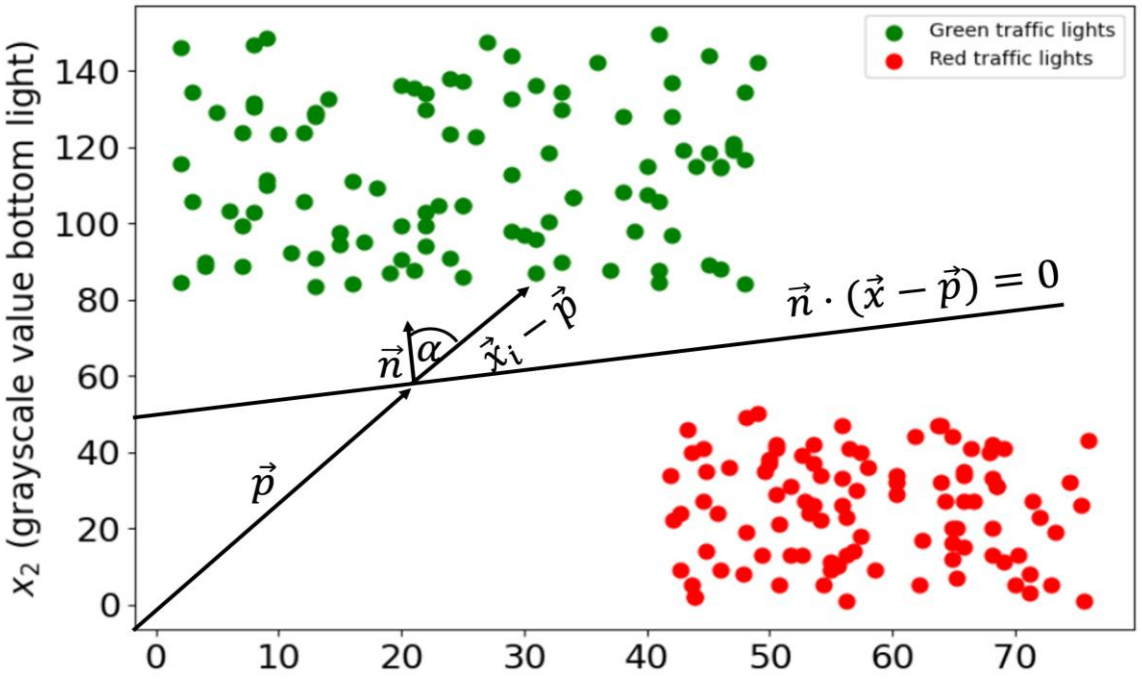


Select line with maximum distance to the nearest data points!

Classification problems

The Support Vector Machine – upper secondary education

- Data points of two classes
- Find a **line**, that best separates the data points of both classes.
- Define „best“.
- Classify data points according to their position in relation to the **separating line.**



Classification problems

The Support Vector Machine – upper secondary education

- Data points of two classes
- Find a **line**, that best separates the data points of both classes.
- Define „best“.
- Classify data points according to their position in relation to the **separating line.**
- Validate the model statistically.

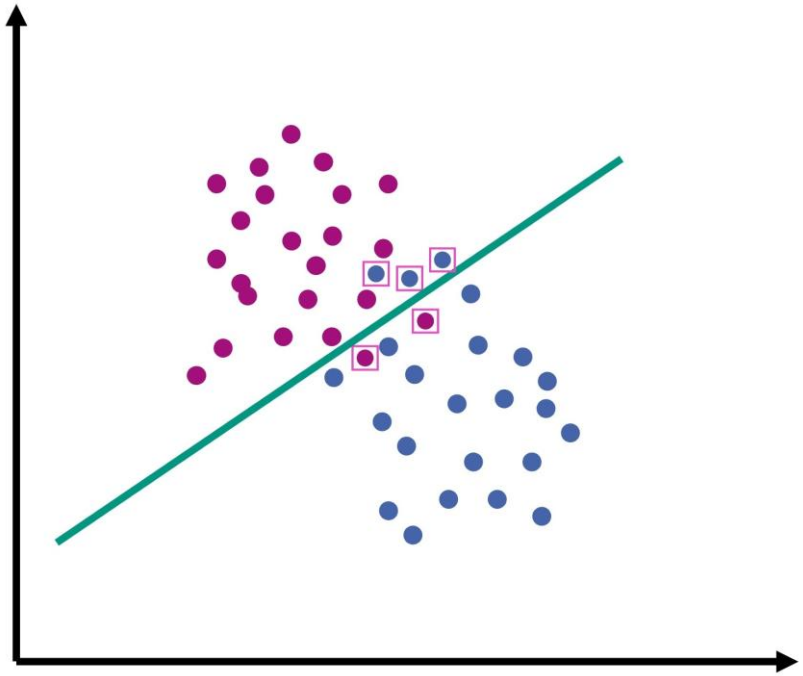
	Predicted class 1	**Predicted class 2**
Actual class 1	350	80
Actual class 2	20	550

Accuracy: 90%

Classification problems

The Support Vector Machine – upper secondary education

- Data points of two classes
- Find a **line**, that best separates the data points of both classes.
- Define „best“.
- Classify data points according to their position in relation to the **separating line.**
- Validate the model statistically.
- Discuss limitations.

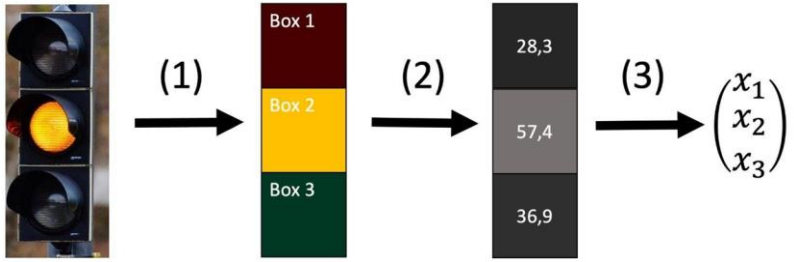


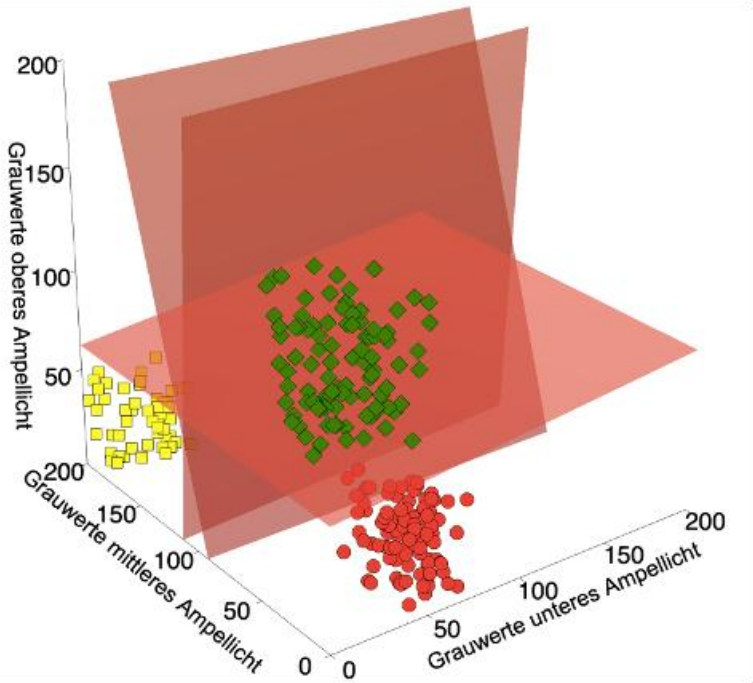
Classification problems

The Support Vector Machine – upper secondary education

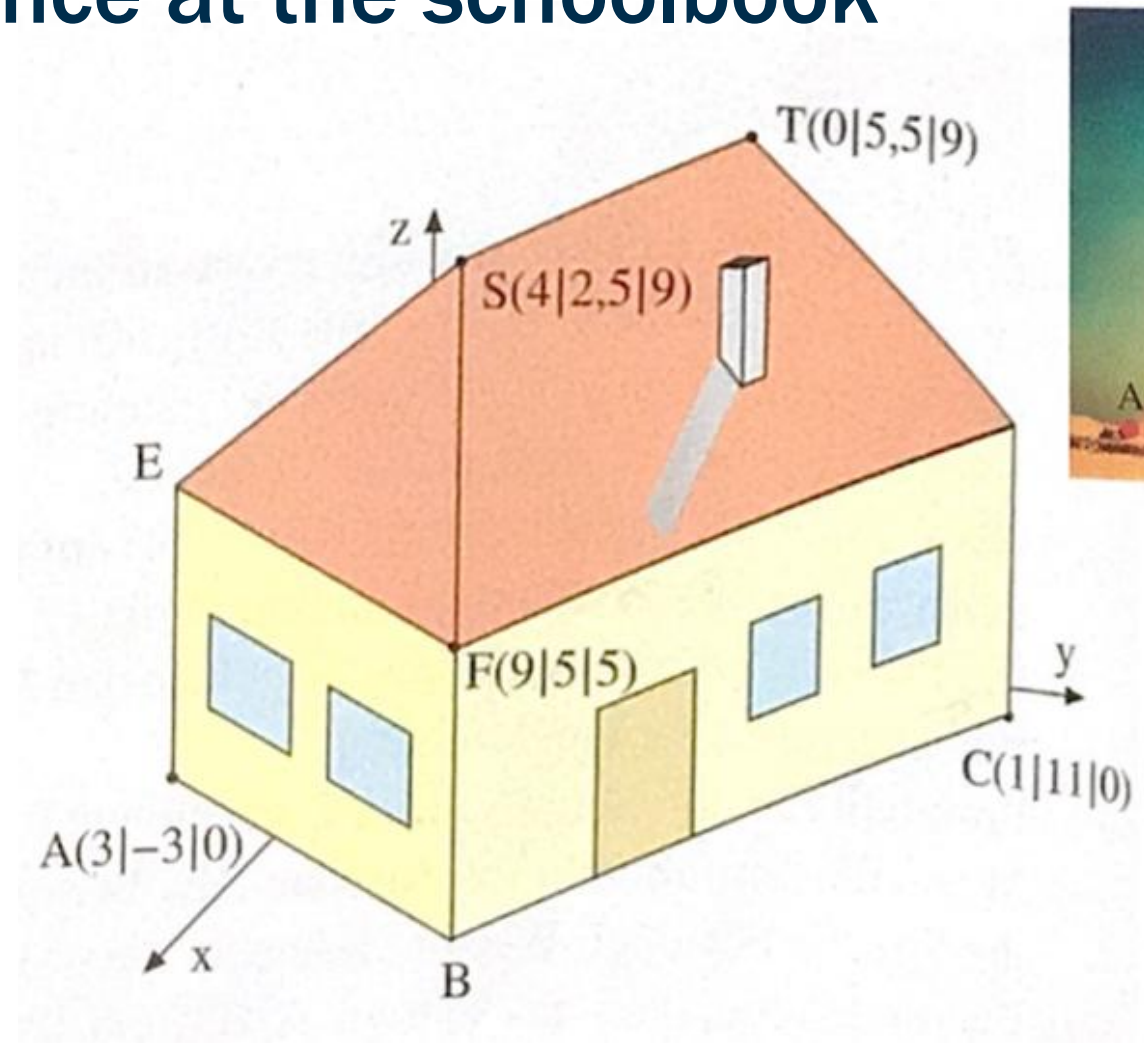
- Data points of three classes
- Find a **plane**, that best separates the data points of three classes.
- Define „best“.
- Classify data points according to their position in relation to the **separating plane.**

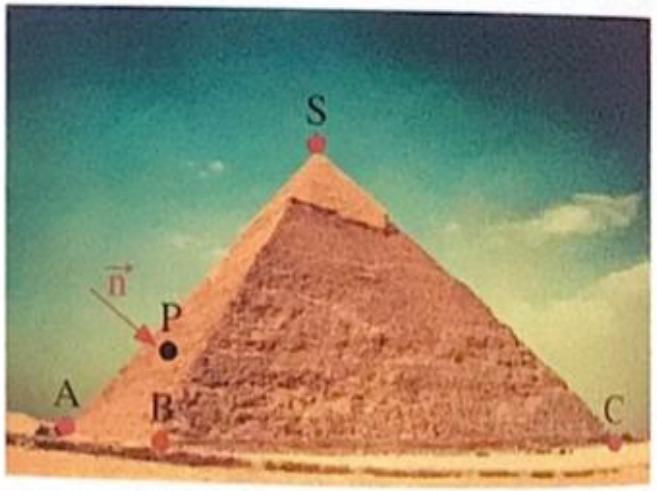
lines, planes, distances, dot product, statistical measures

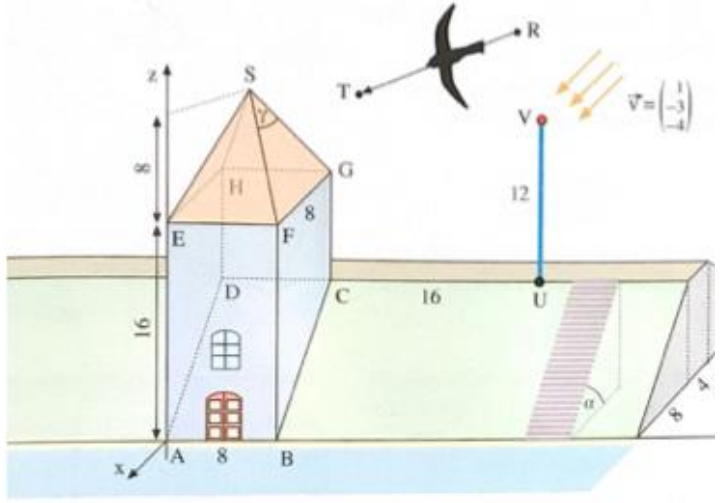




A glance at the schoolbook







Images from: Bigalke / Köhler. Mathematik Gymnasiale Oberstufe Nordrhein-Westfalen Qualifikationsphase Grundkurs Cornelsen, 2014.

Critical discussions

... in the mathematics classroom!

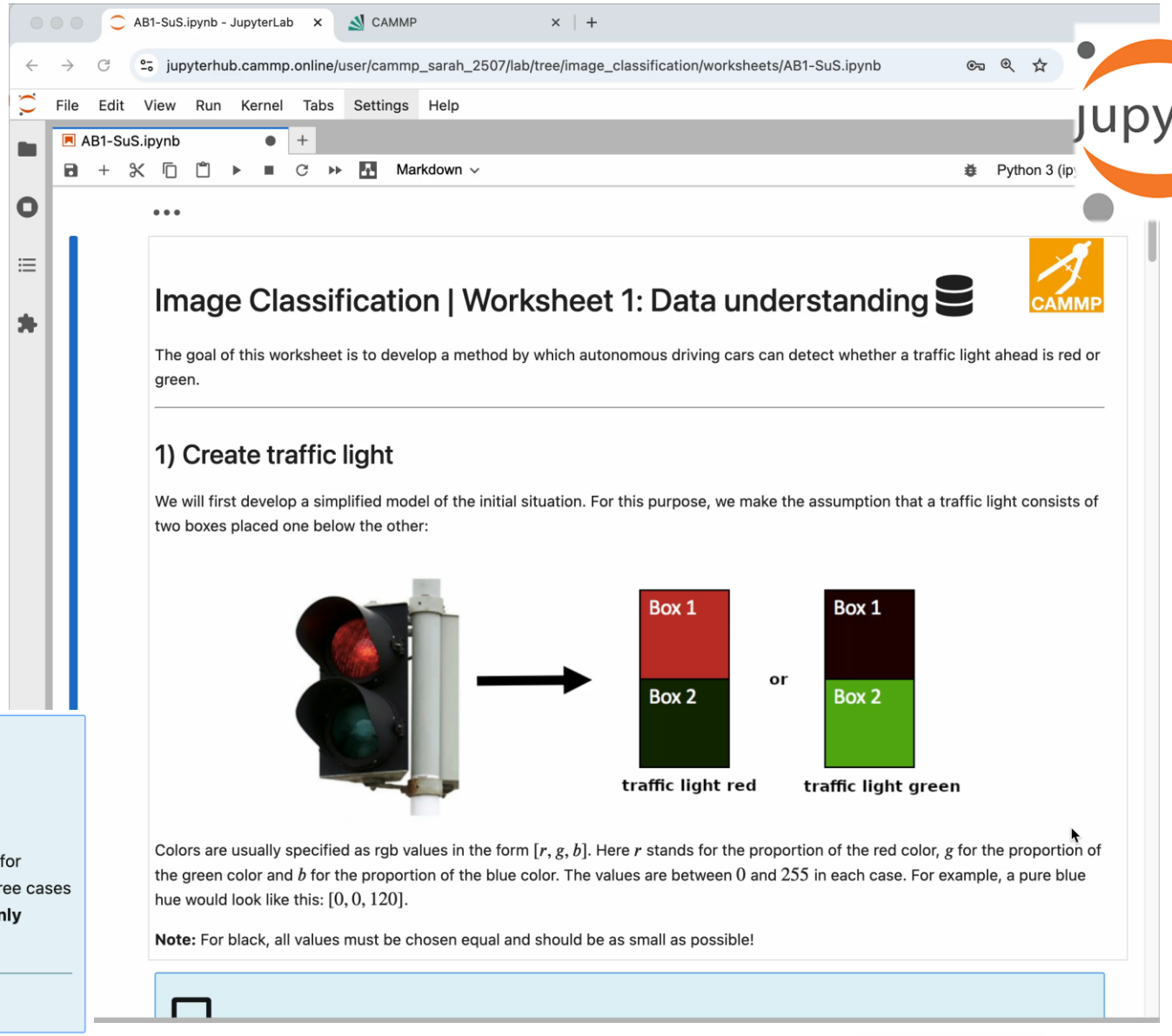
- Bias, discrimination, fairness
- Diversity of data
- Privacy and data protection
- Measurement errors; outliers

O'Neil, C. (2016)

Digital Learning material

... based on Jupyter Notebooks

- No programming skills needed (“fill in the gap” approach)
- No software installation
- Direct feedback
- In-depth tasks
- Staggered hints



a) Case distinction in the formula

Write on the answer sheet a general distinction of cases for formula $\vec{n} * (\vec{x} - \vec{p}) = cos(\alpha) \cdot |\vec{n}| \cdot |(\vec{x} - \vec{p})|$ for the three cases 1-3 shown above. Think why it is sufficient to consider **only angles between 0° and 180°**!

Hint

Access the teaching and learning material

Project Website: www.cammp.online/english

Learning material (based on Jupyter Notebooks): https://jupyterhub.cammp.online/

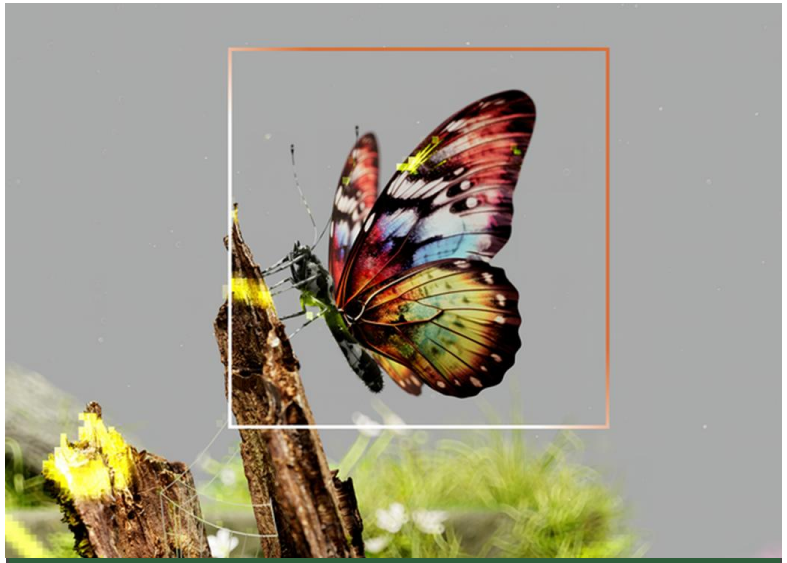
Useraccount: cammp_YOURPSEUDONYM
[prefix cammp_ is important]

Password: [can be determined freely]

Simple Simulation of an ANN (based on GeoGebra):

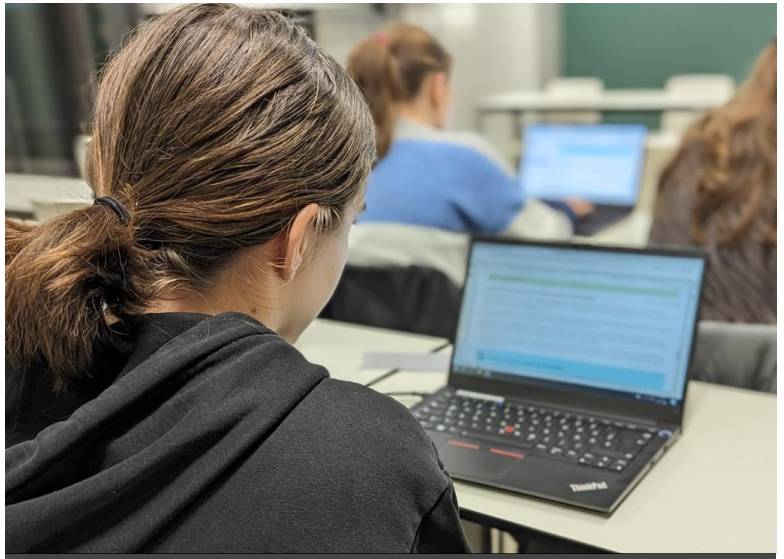
https://www.geogebra.org/calculator/kpjk8qb6

Our focus today



A workshop on image classification and the Support Vector Machine

A learning path to better understand Artificial Neural Networks

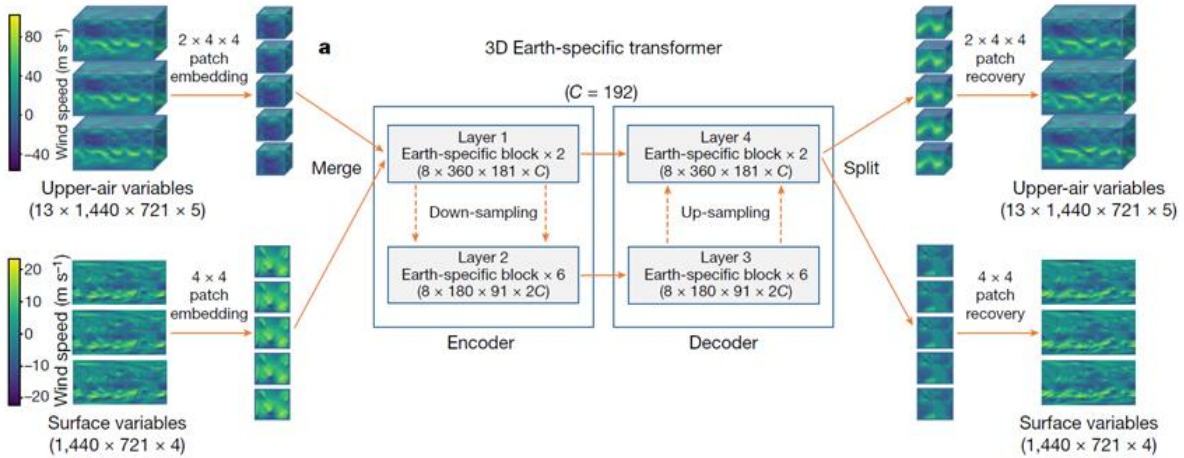


Conclusions about AI education and ways to implement

Artificial neural networks are big and powerful

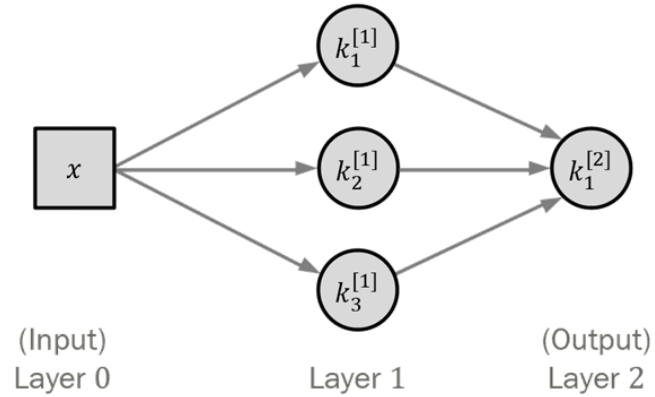
- ANN basis of many AI applications (GenAI like Chatbots, image generation, weather forecast, …)
- Current ANN models are complex and very large
- Focus on regression models rather than GenAI models: e.g. weather forecast

A current ANN structure



Architecture of the Pangu-Weather model (Bi et al., 2023)

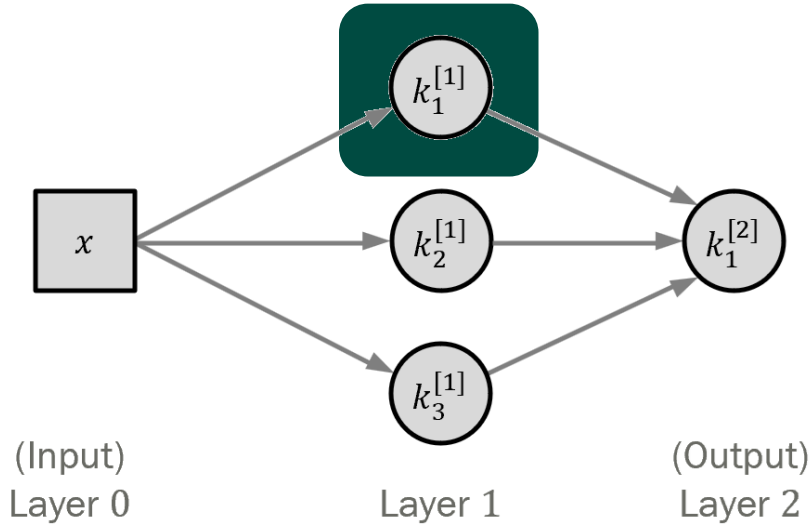
Simplified to



A toy ANN to tackle misconceptions

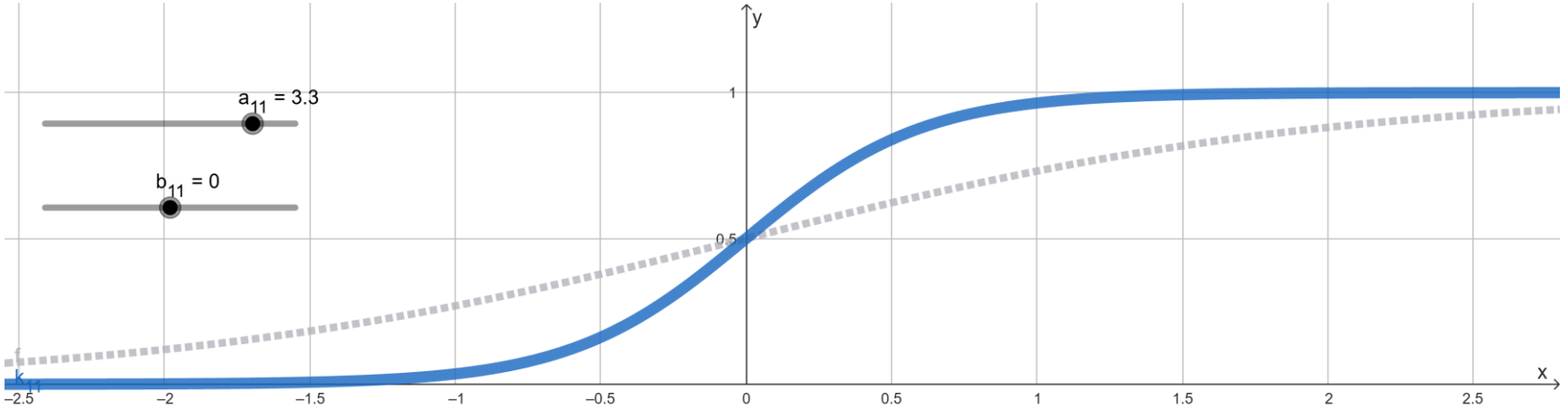
The ANN…

- *"… learns physics"*
- *"… recognizes patterns"*
- *"… also works on data it has not seen yet"*
- *"… is way to complex to understand"*
- *…*



⇒ Explore the functionality of this toy network to demystify misconceptions

One node of an ANN



$$k(x) = \sigma(w \cdot x + b)$$

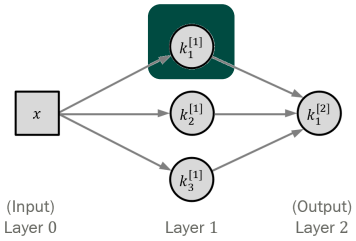
$$= \frac{1}{1 + e^{-(w \cdot x + b)}}$$

Linear function

Linear function with parameters $w, b \in \mathbb{R}$

Activation function

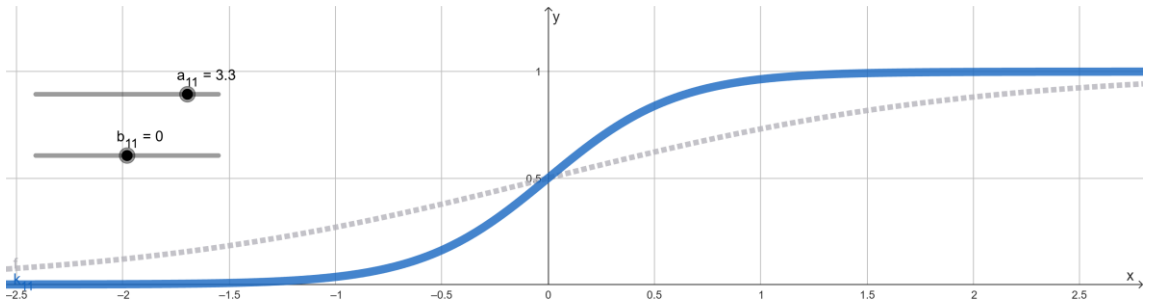
Non-linear function σ with $\sigma(x) = \frac{1}{1+e^{-x}}$



Kurvendiskussion of one node

Students can use mathematical tools to analyse this function

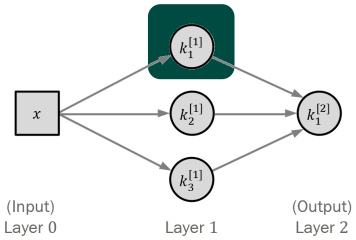
$$k(x) = \frac{1}{1 + e^{-(w \cdot x + b)}}$$



- Domain and codomain
- Limits of $k(x)$ for $x \to \pm\infty$

- Finding extremes

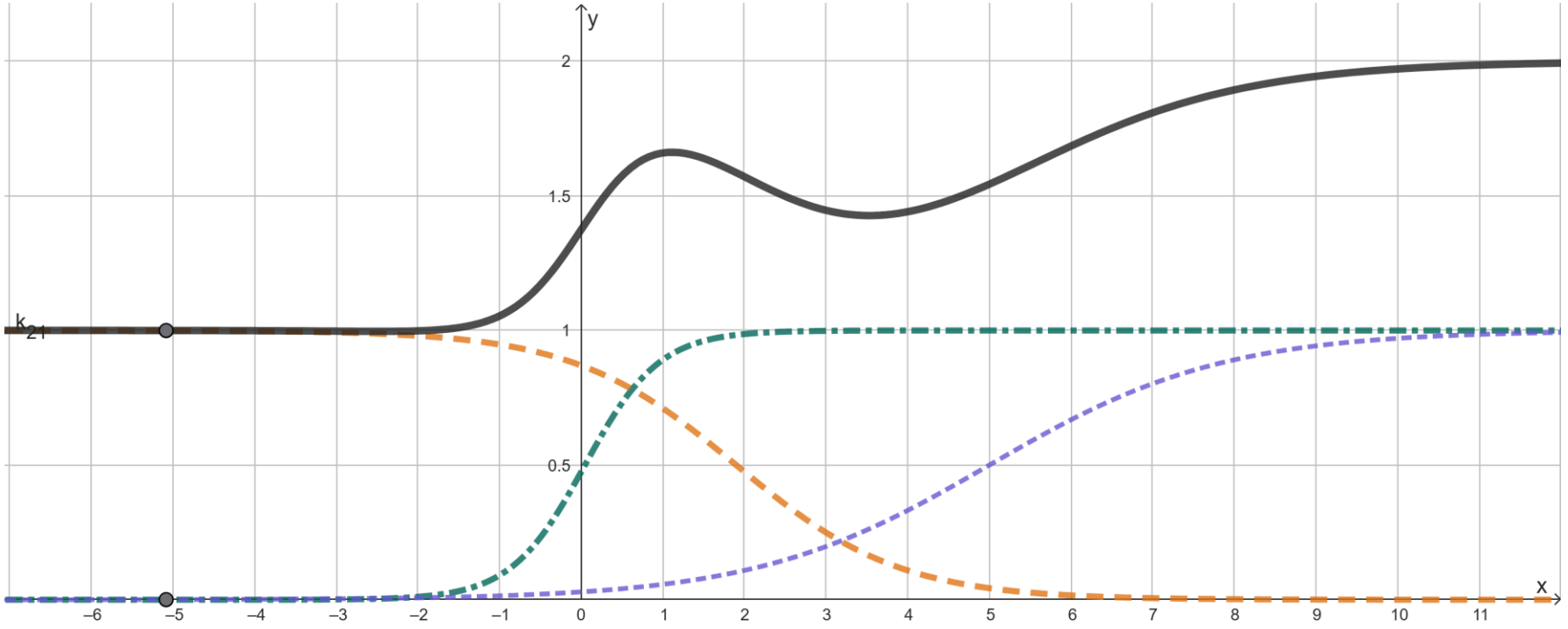
- Influence of the parameters w and b

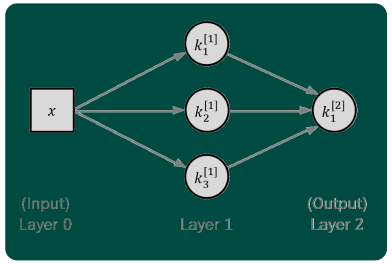


The toy network

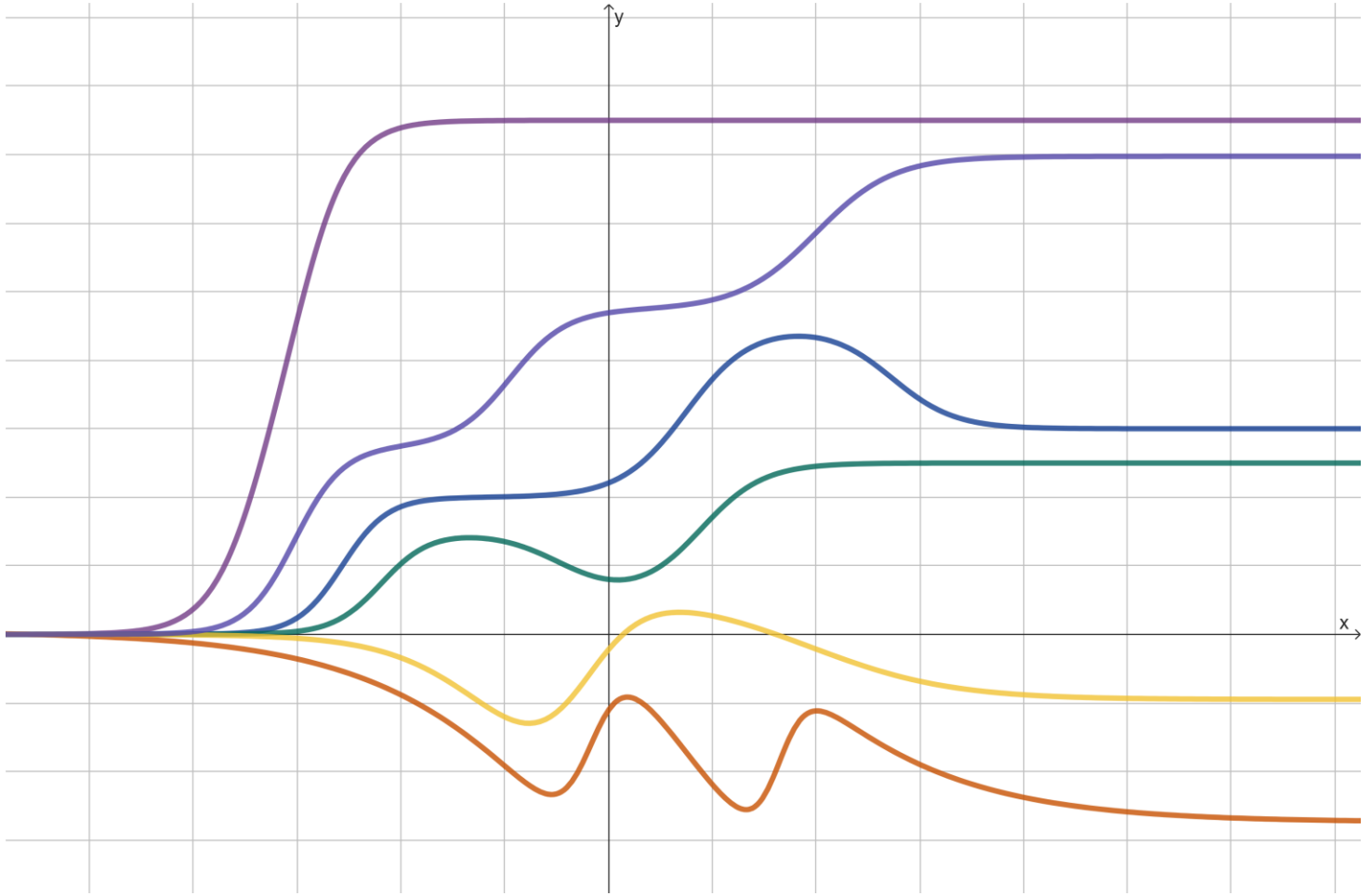
The network is the weighted sum of the previously considered functions

$$K(x) = w_1^{[2]} \cdot k_{1,1}(x) + w_2^{[2]} \cdot k_{1,2}(x) + w_3^{[2]} \cdot k_{1,3}(x) + b_{2,1}$$



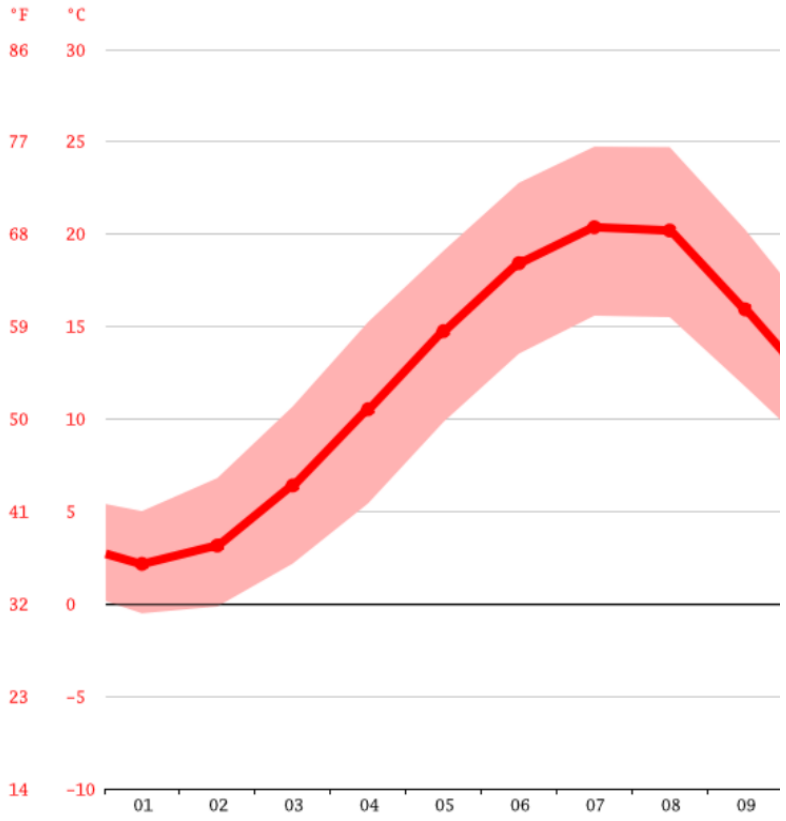


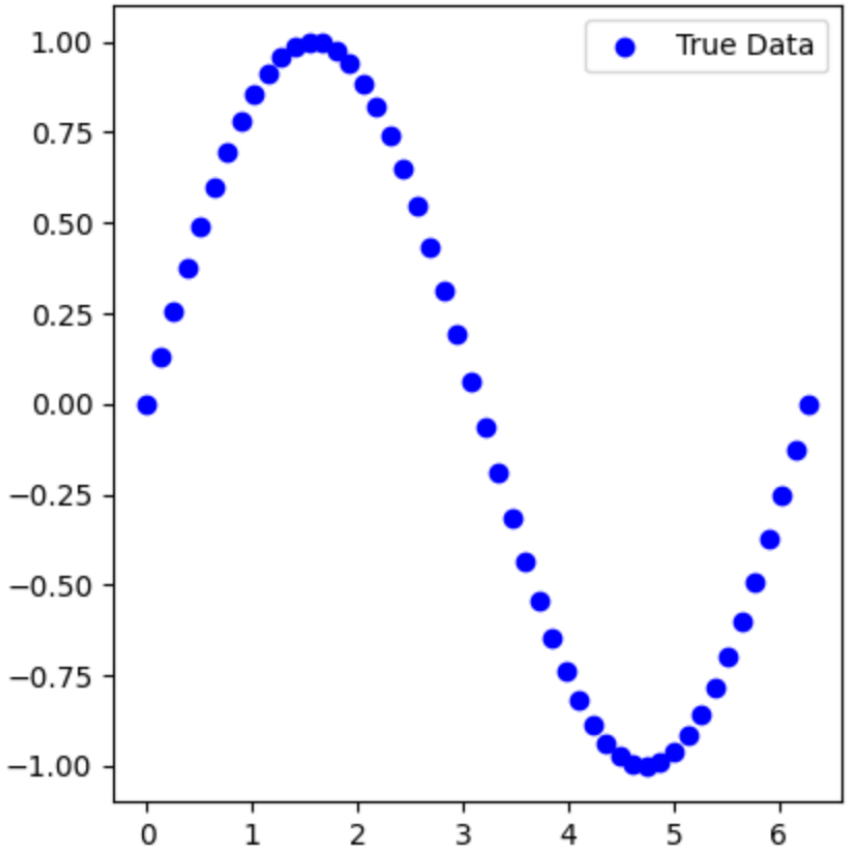
Flexibility of the toy network



Approximating periodic data

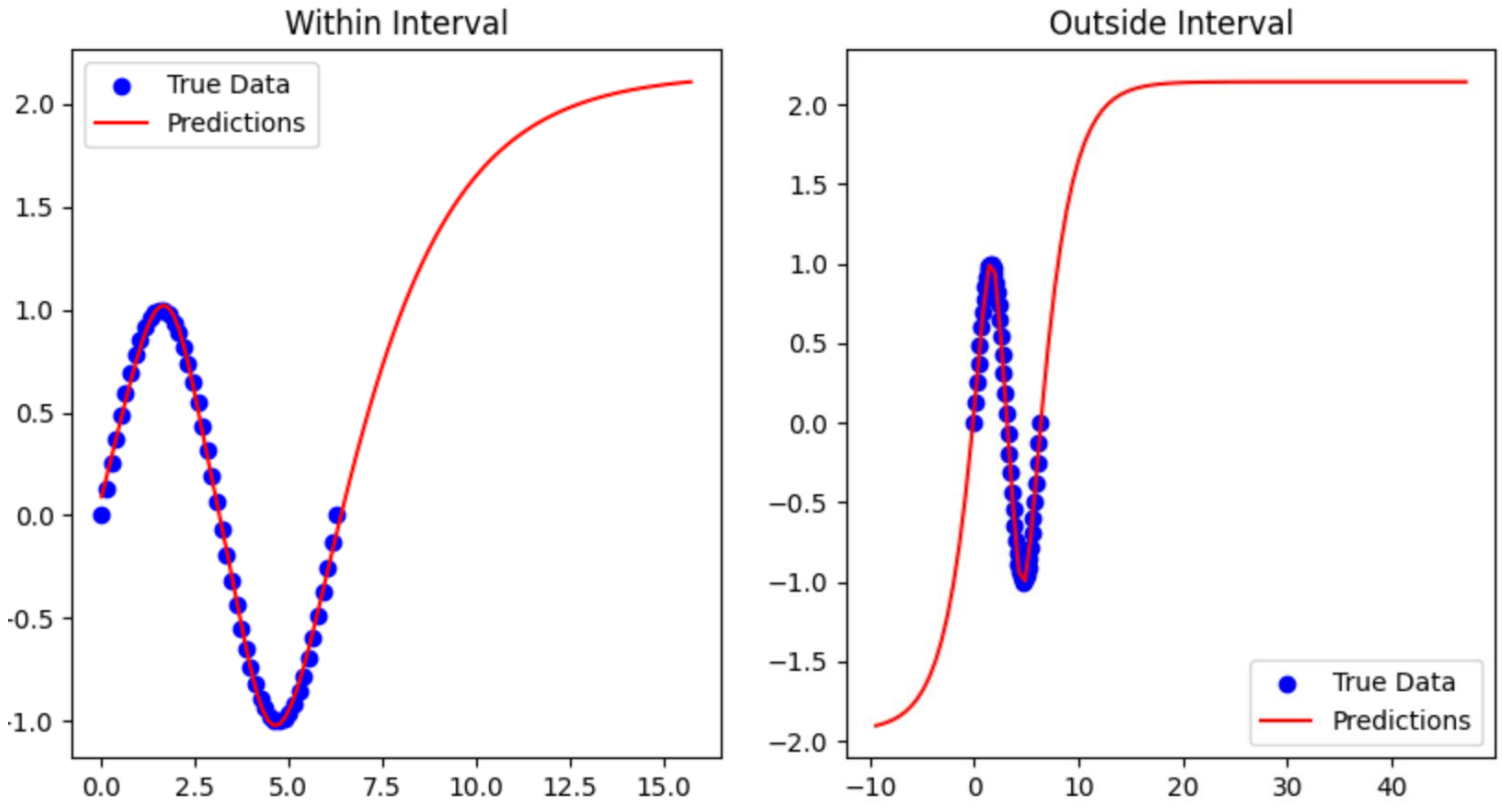
A regression problem





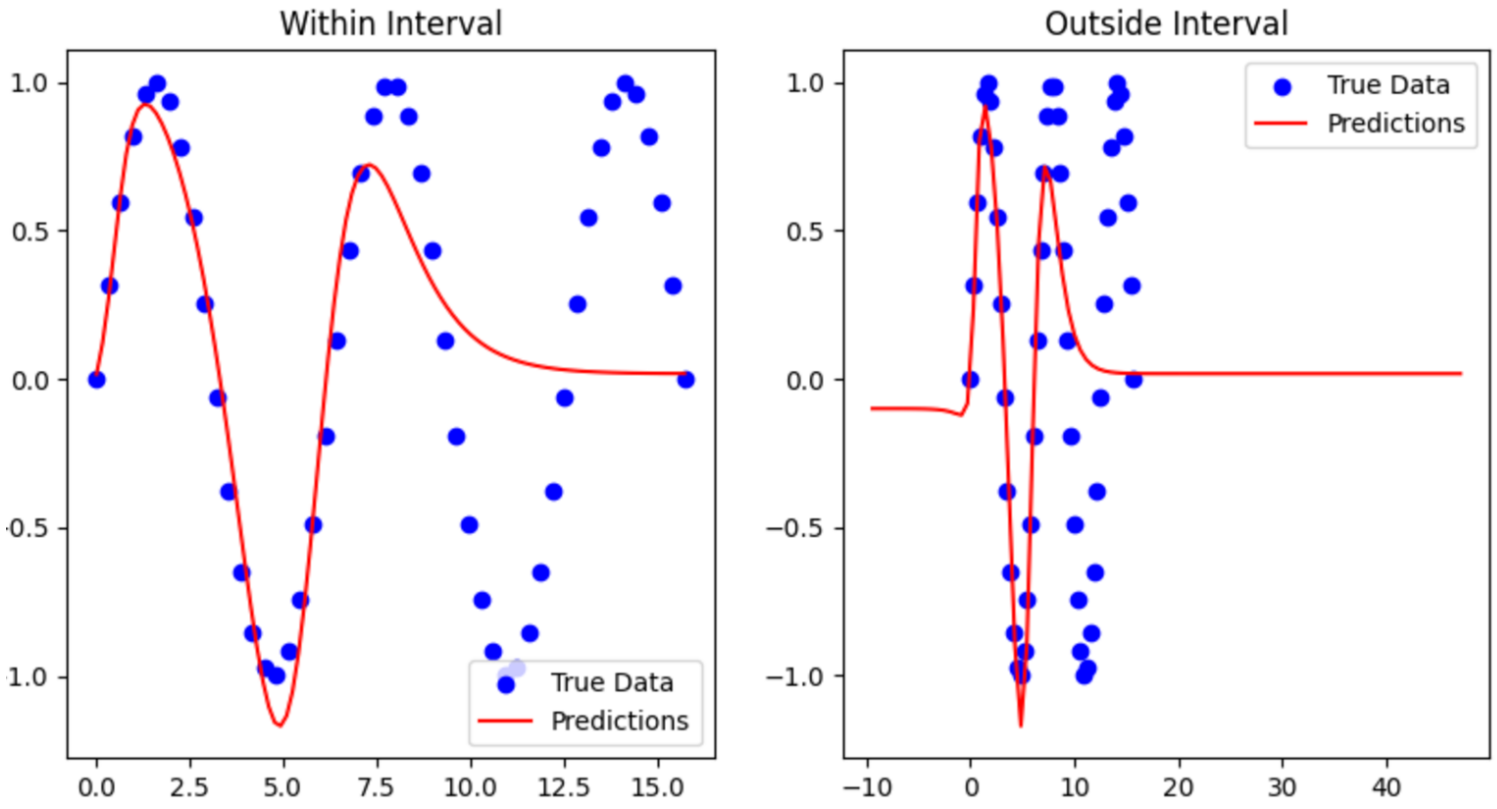
Approximating periodic data

A regression problem



Approximating periodic data

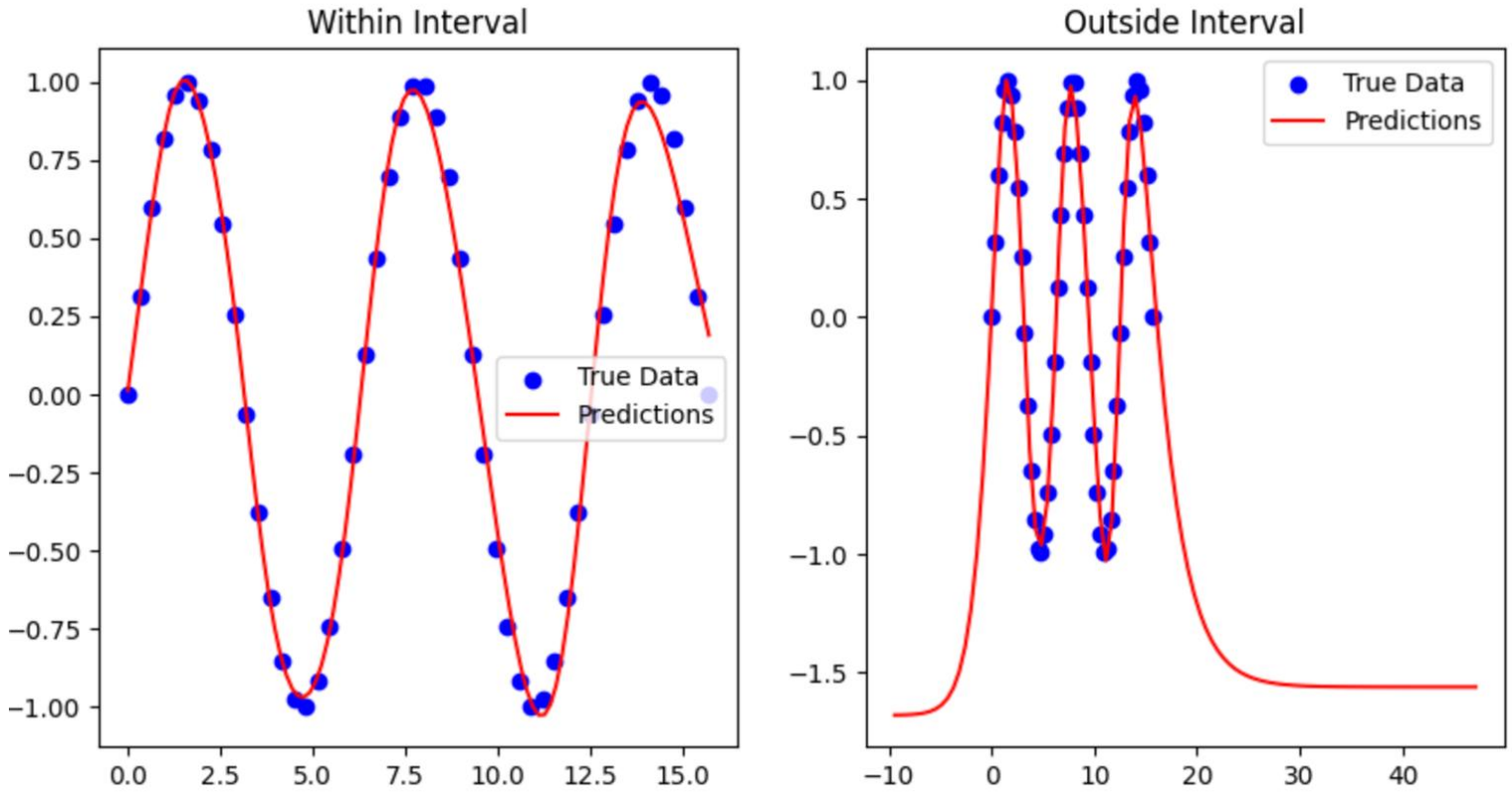
A regression problem



How can we modify the network for this task?

Approximating periodic data

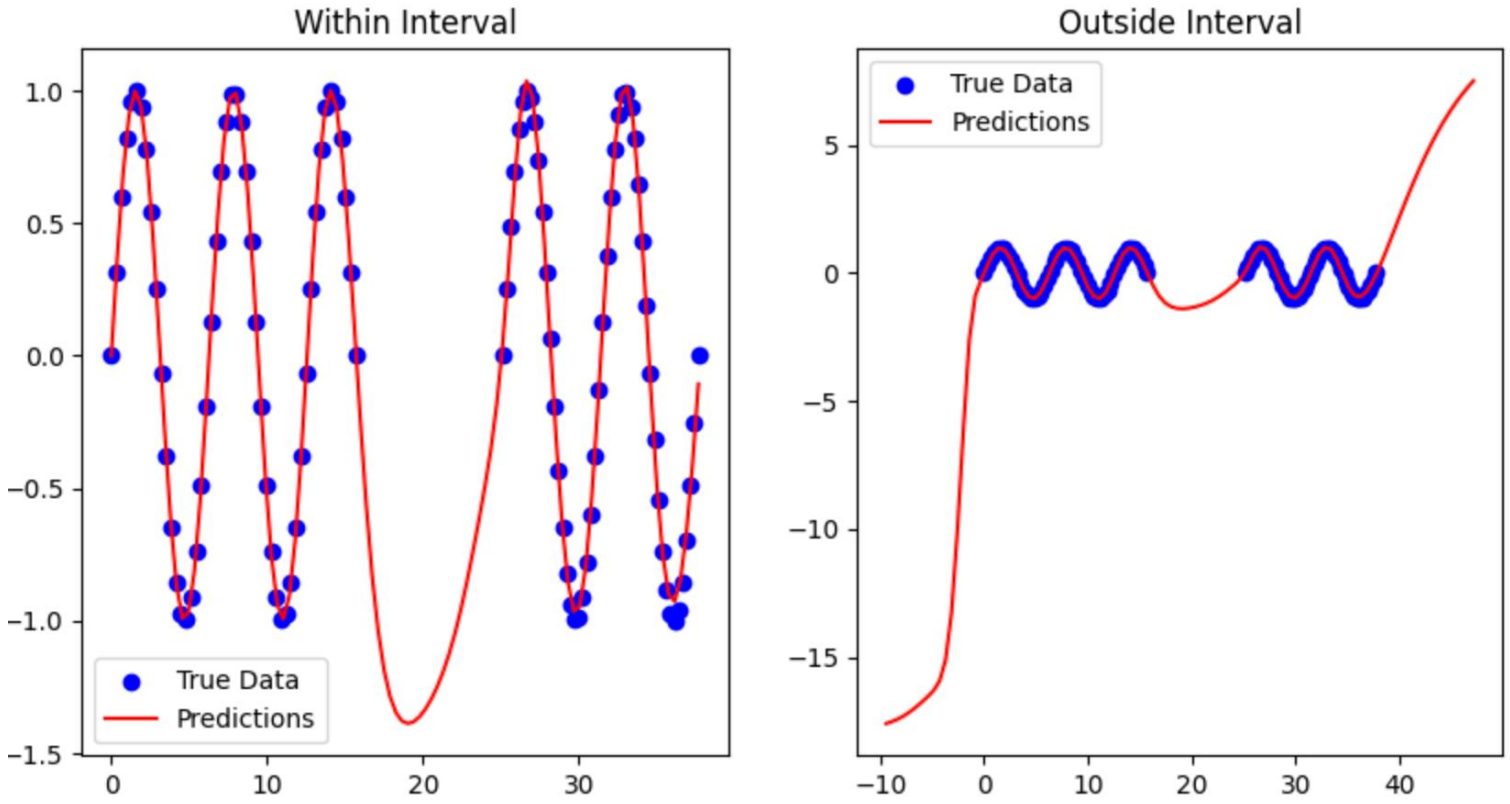
A regression problem



Network structure: 1 - 6 - 1

Approximating periodic data

A regression problem



Network structure: 1 - 500 - 1

What can students learn from this?

ABOUT ANN

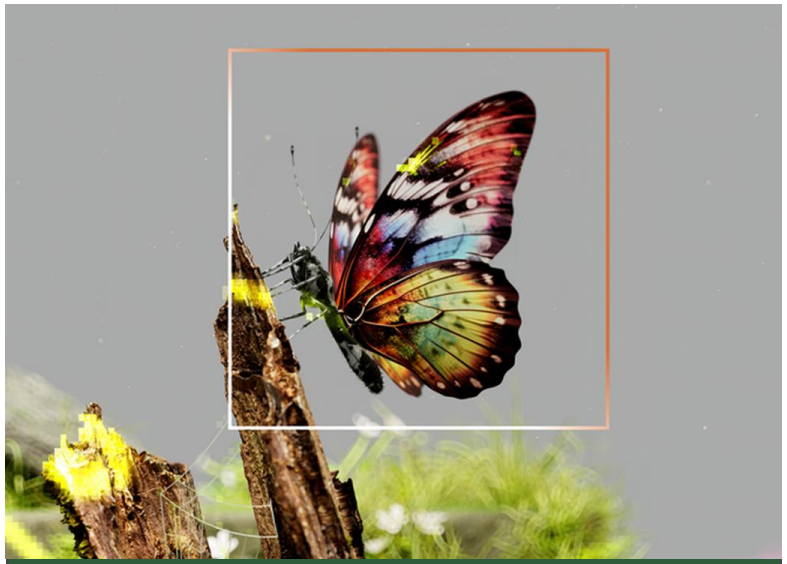
- Recognize ANN as highly adjustable mathematical functions:
  - By adding nodes, any function can be approximated as precisely as desired, but only on a bounded set
  - The functions can approximate relationships in given data BUT the approximation only works in the intervals in which data points are given

- ANN models don’t “understand”
- ANN models don’t “know” when they don’t know

ABOUT MATHEMATICS

- Develop and train mathematical skills
  - e.g. functional thinking, finding extremes, …

Our focus today



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Conclusions about AI education and ways to implement

Black box vs. White box

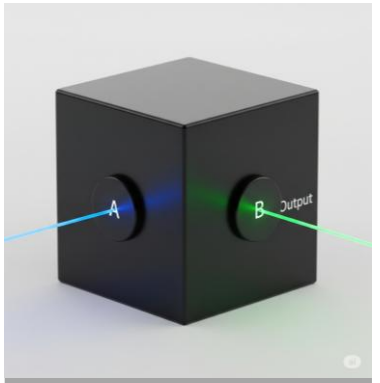
Balancing black and white box approaches of AI models

SVM

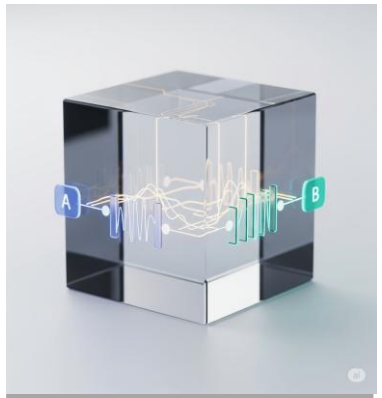
- White Box: Mathematical description of separating lines / planes; idea of maximizing the “margin” (data-point free area)
- Black Box: Optimization algorithm

ANN

- White Box: Functionality of the ANN as a mathematical function
- Black Box: Optimization (“Training”) of the ANN



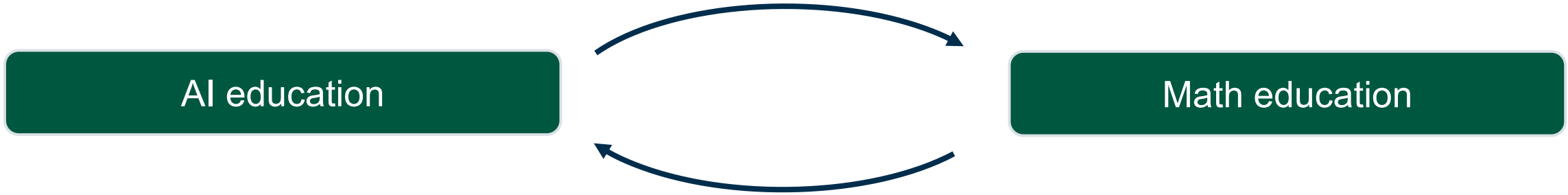
Black Box



White Box

AI education in the math classroom: mutual benefits

In accordance with the AI competencies by Long & Magerko (2020)



Understanding the mathematical concepts of AI models enables…

- a deeper understanding of their results.
- a realistic assessment of opportunities and risks.
- an idea of the human role in their development.
- …

AI education in mathematics education enables …

- the classification of mathematics as a key qualification for modern applications.
- the application and development of school mathematical skills.
- interdisciplinary links with other school subjects.
- …

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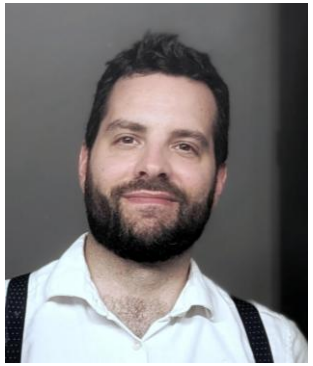
Thank you !

Access to learning material?
Suggestions and exchange?

www.cammp.online/english



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Discussion

REFLECTING THE APPROACH

- Are AI models a good learning content for the math classroom?
- What advantages and challenges do you see?

- What can be transferred into your context?
- What ethical or societal conversations does this type of hands-on understanding open up for students? (e.g. regarding last month's talk)

INTERDISCIPLINARY APPROACH

- How much math when teaching about AI in the CS classroom?
- How much Programming when teaching about AI in the math classroom?