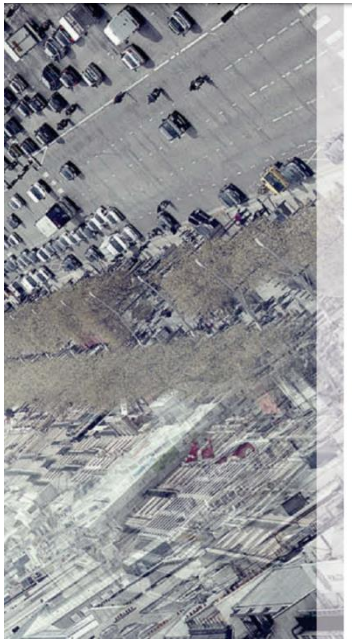
Оптимизация: От Джорджа Данцига до продакшена



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The world maker

Is time running out for the clever piece of maths that runs modern life, asks Richard Elwes

YOU MIGHT not have heard of the algorithm that runs the world. Few people have, though it can determine much that goes on in our day-to-day lives: the food we have to eat, our schedule at work, when the train will come to take us there. Somewhere, in some server basement right now, it is probably working on some aspect of

The simplex algorithm directs wares to their destinations the world over

cosmos. The tetrahedron, cube, octahedron and 20-sided icosahedron embodied the "elements" of fire, earth, air and water, and the 12-faced dodecahedron the shape of the universe itself.

Things have moved on a little since then. Theories of physics today regularly invoke strangely warped geometries unknown to

One of the first insights he arrived at was that the optimum value of the "target function" – the thing we want to maximise or minimise, be that profit, travelling time or whatever – is guaranteed to lie at one of the corners of the polytope. This instantly makes things much more tractable: there are infinitely many points within any polytope, but only ever a finite number of corners.

If we have just a few dimensions and constraints to play with, this fact is all we need. We can feel our way along the edges of the polytope, testing the value of the target function at every corner until we find its sweet spot. But things rapidly escalate. Even just a 10-dimensional problem with 50 constraints – perhaps trying to assign a schedule of work to 10 people with different expertise and time

"Probably tens or hundreds of thousands of calls are made of the simplex algorithm every minute"

solver indeed, typically reaching an optimum solution after a number of pivots comparable to the number of dimensions in the problem. That means a likely maximum of a few hundred steps to solve a 50-dimensional problem, rather than billions with a suck-it-and-see approach. Such a running time is said to be "polynomial" or simply "P", the benchmark for practical algorithms that have

Задача линейного программирования

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$$f(x) = \sum_{i=1}^{n} c_i x_i = c_1 x_1 + c_2 x_2 + c_3 x_3 + .. \rightarrow \min$$

$$\sum_{i=1}^{n} a_i x_i \leq b_i$$

$$x_i \geq 0, i = 1..n$$

Движки ЛП

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IBM CPLEX

FICO® Xpress Optimization

Целочисленные переменные	**~ 100 000 переменных**
Бинарные переменные	**~ 1000 000 переменных**
Вещественные переменные	**~ 100 000 000 переменных**

Проблема перевозки на длинных маршрутах

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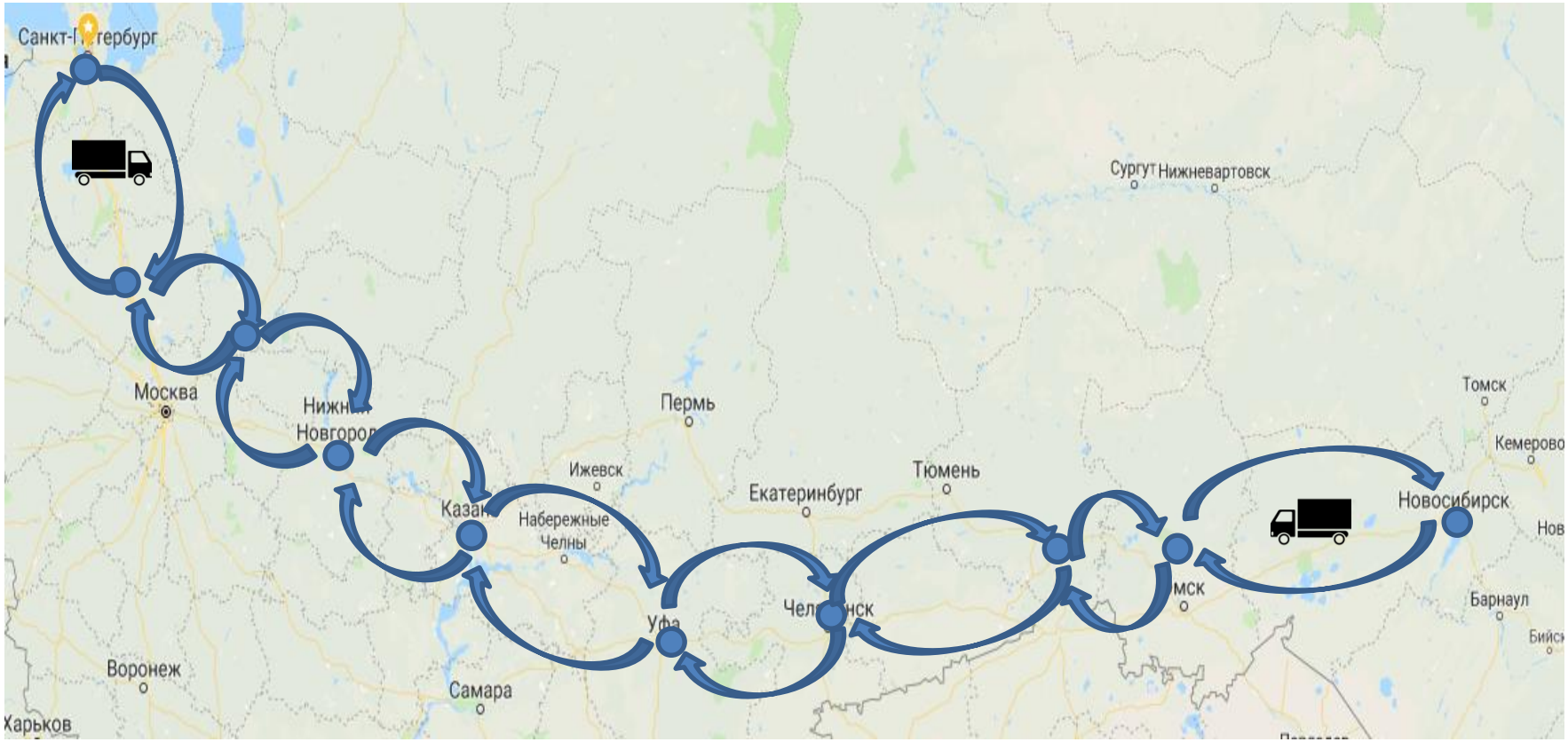

Расписание водителей

Минимизация затрат на перевозку с пересменкой водителей

```
from docplex.mp.model import Model
m = Model(name = '…')
```

$$\sum_{state=100}^{101} \sum_{i=firstPoint}^{lastPoint} \sum_{t=0}^{totalTime} truckInactivityPenalty \quad m(t,i,state)$$
$$(+ trailerInactivityPenalty \quad p(t,i,state)) +$$
$$driverCost \quad dN_1 + truckCost \quad cN_1 + trailerCost \quad pN_1 \rightarrow min$$

```
m.set_objective_expr()
```

$$\sum_{n=1}^{numOfRelax} \sum_{t=1}^{twoWeeksTime} \left(\sum_{i=firstPoint}^{lastPoint-1} d(n,t,i,i+1) \right.$$

```
m.add_constraint()
```

Расписание водителей

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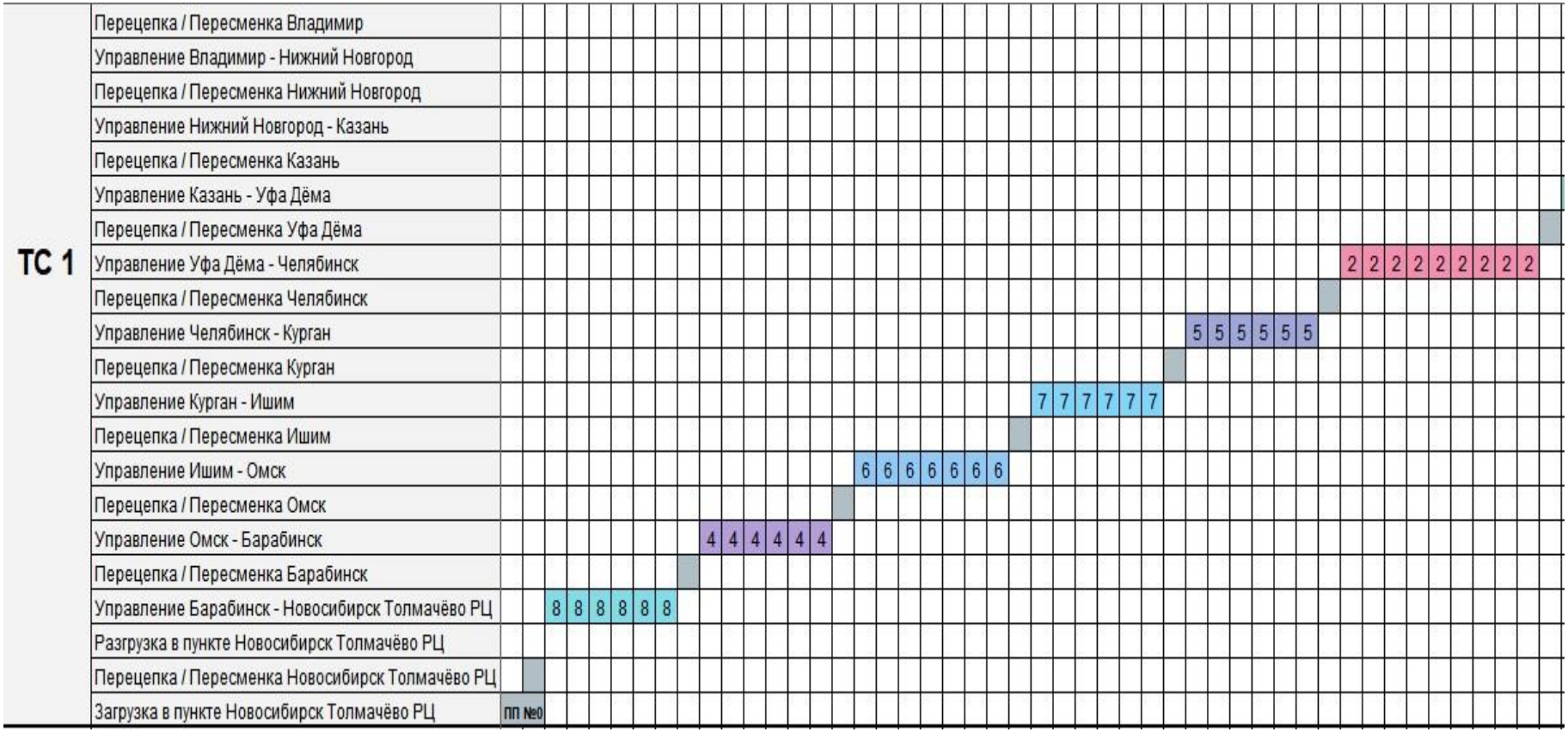

Расписание водителей

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Задача маршрутизации бензовозов

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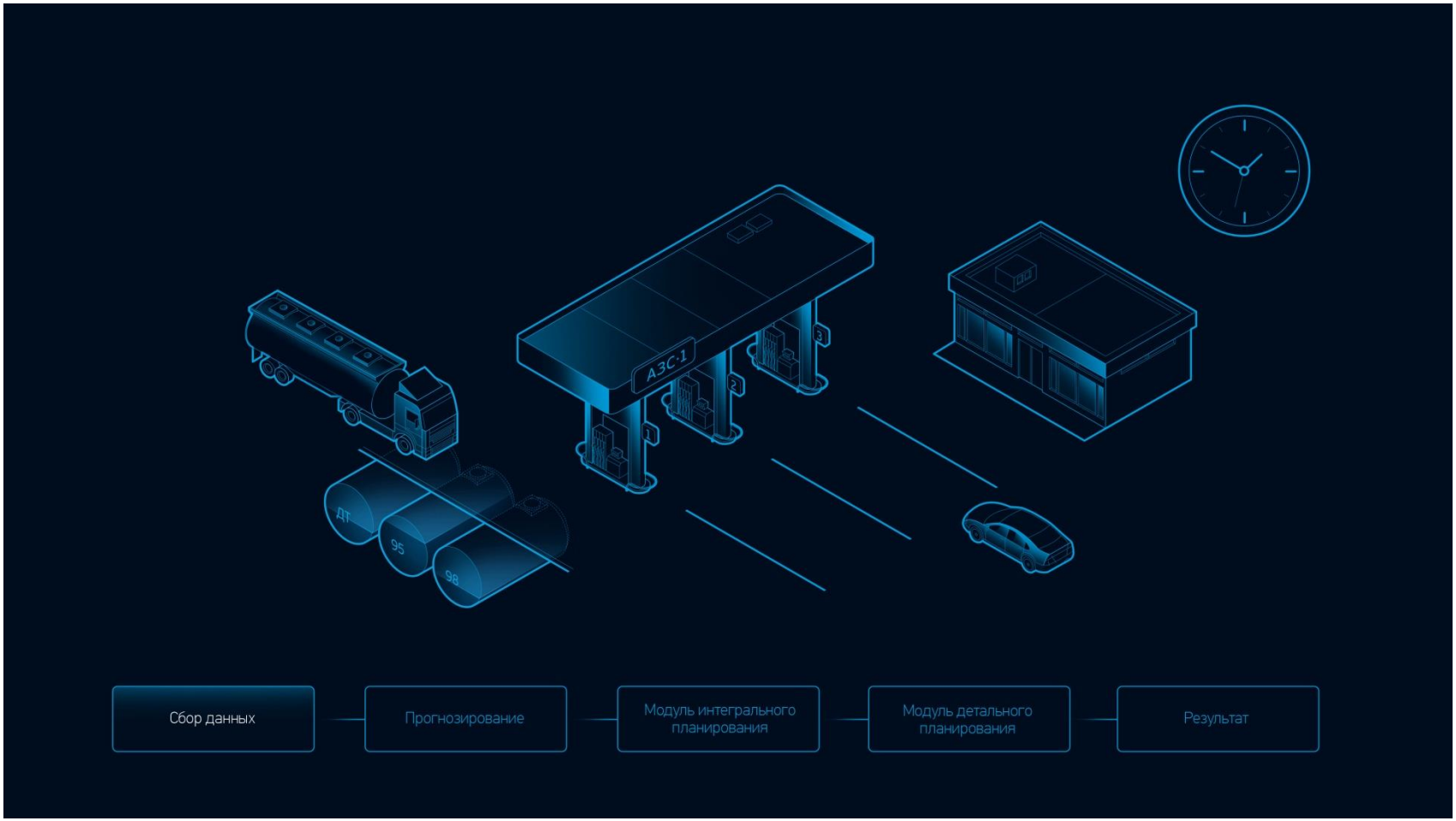

Учет секций бензовозов

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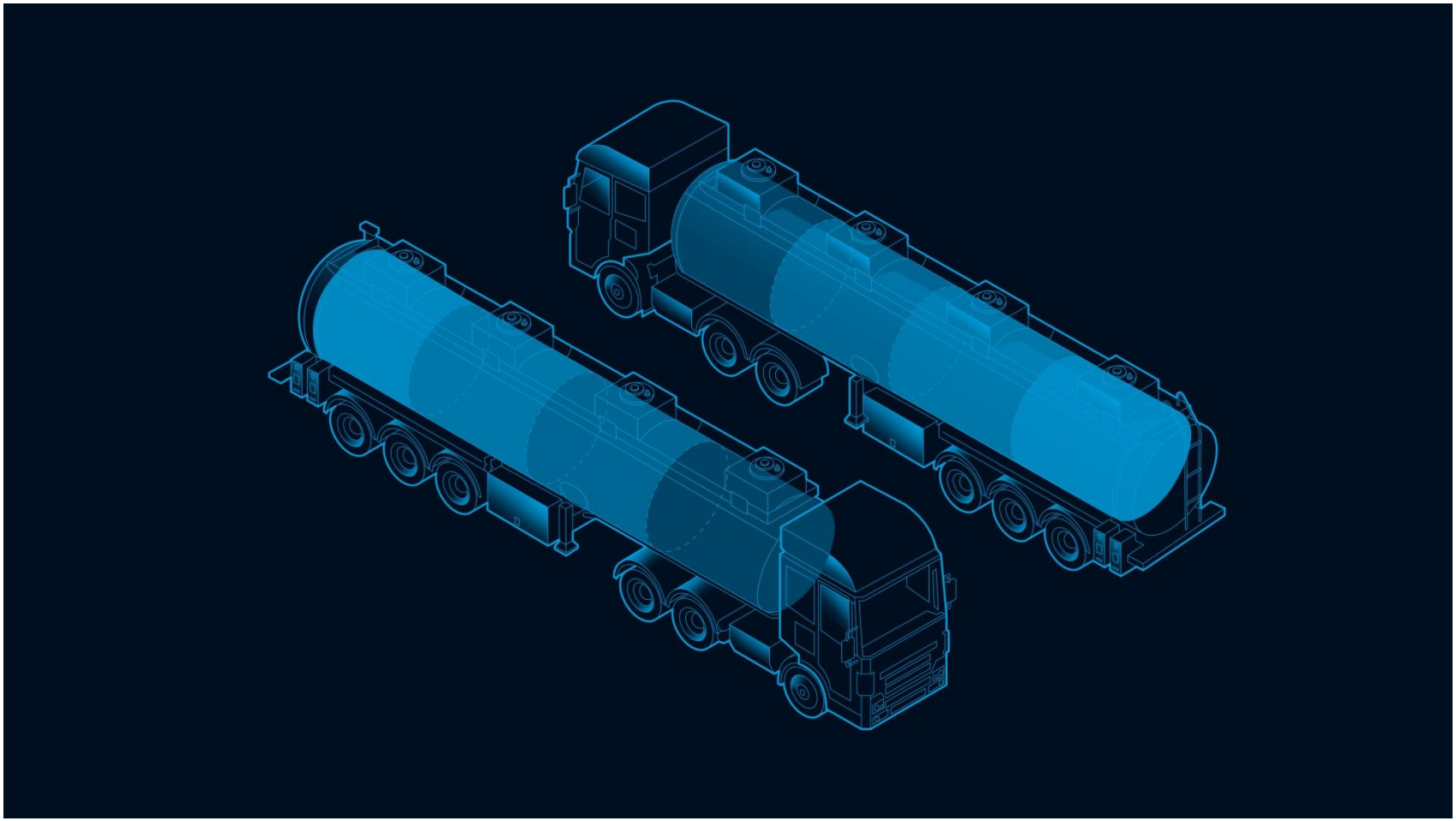

Учет нефтебазы

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Линейное программирование

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- LP – linear programming
- MIP – mixed integer programming

$$Bx_B = b - Nx_N$$

$$\begin{aligned} B &= LU \\ LUx_B &= b - Nx_N \\ Lw &= b - Nx_N \\ Ux_B &= w \end{aligned}$$

Проблемы

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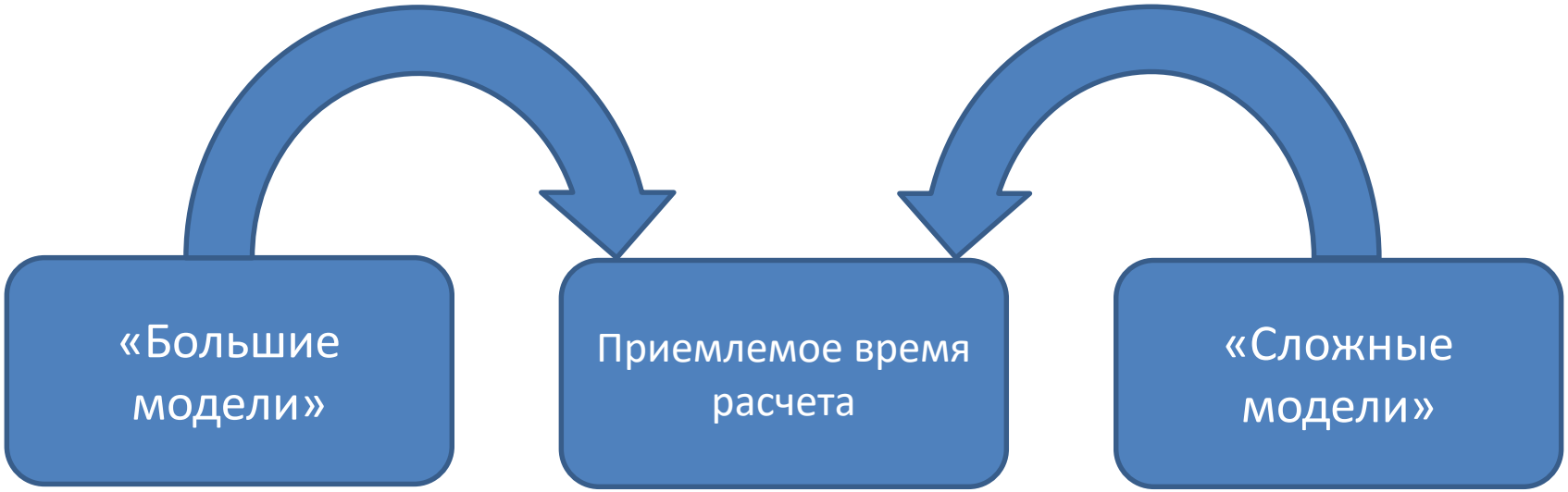

Решения

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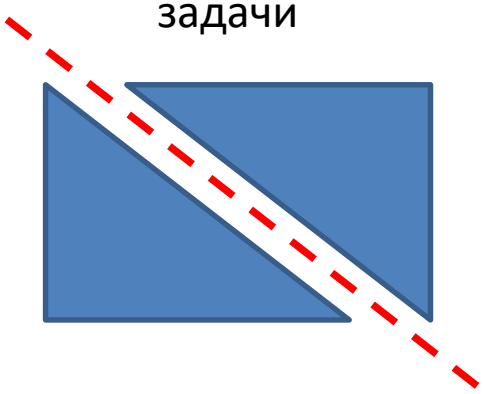
Субоптимальное решение

Декомпозиция задачи



Манипуляции с числовыми значениями

$10000x - 0{,}001y = 0$

$x \leq 0{,}001$

$10x - 0{,}001y = 0$

$x \leq 1$

**20 % усилий дают 80 % результата, а остальные 80 % усилий — лишь 20 % результата*

Проблемы

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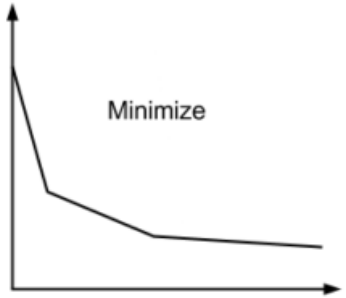
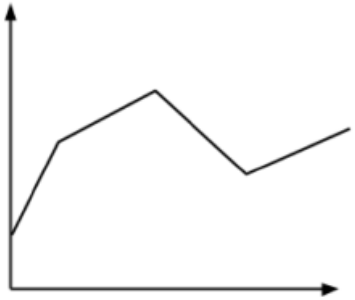
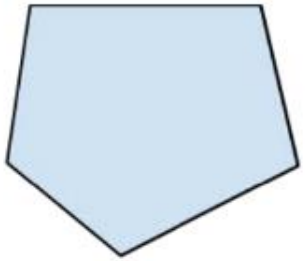
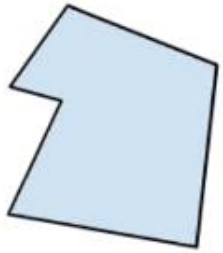


Convex



Non-convex





$$\min\left\{\max_i x_i\right\} \Rightarrow \begin{array}{l}\min z \\ z \geq x_i \ \forall i\end{array}$$

$$\min\left\{\min_i x_i\right\} \Rightarrow \begin{array}{l}\min z \\ z \geq x_i - M(1-y_i) \\ \sum_i y_i = 1 \\ y_i \in \{0,1\}\end{array}$$

Выбор ноды

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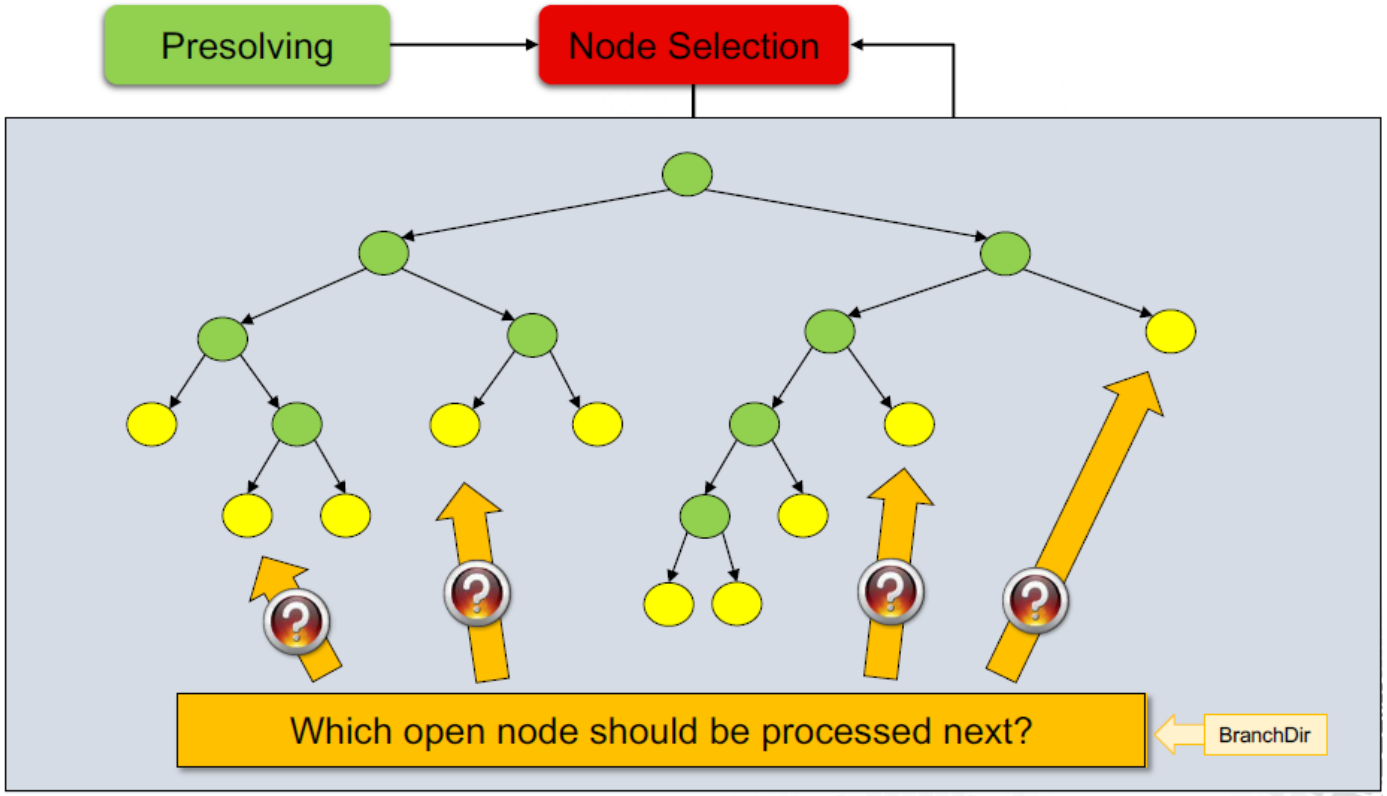

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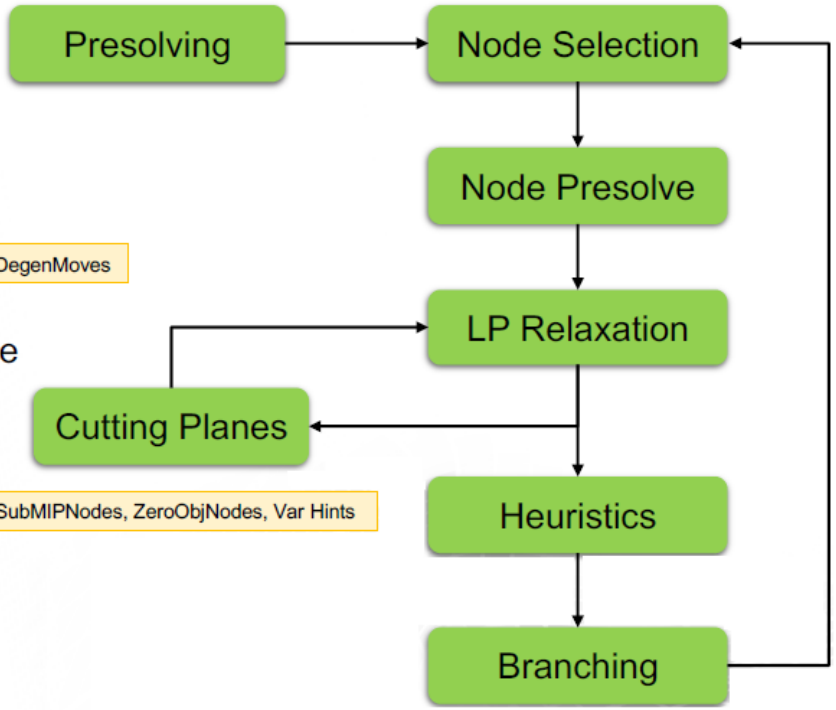
- Presolve ← Presolve, PrePasses, AggFill, Aggregate, DualReductions, PreSparsify, ...
  - Tighten formulation and reduce problem size
- Node selection ← BranchDir, Var Hints
  - Select next subproblem to process
- Node presolve ← Symmetry
  - Additional presolve for subproblem
- Solve continuous relaxations ← Method, NodeMethod, DegenMoves
  - Ignoring integrality
  - Gives a bound on the optimal integral objective
- Cutting planes ← Cuts, CutPasses, GomoryPasses, CliqueCuts, ...
  - Cut off relaxation solutions
- Primal heuristics ← Heuristics, MinRelNodes, PumpPasses, RINS, SubMIPNodes, ZeroObjNodes, Var Hints
  - Find integer feasible solutions
- Branching variable selection ← VarBranch
  - Crucial for limiting search tree size



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Спасибо за внимание!

Красильников Михаил

Ложкинс Алексейс