THE MACHINES

THE TEACHING GUIDE

Experience 1

Related resources:

- Access videos of *Exploding Dots*™ lessons at: http://gdaymath.com/courses/exploding-dots/
- Be sure to review the *Getting Started* guide, available here.
- Printable student handouts for this experience are available here.

Experience 1: The Machines
Overview

Student Objectives

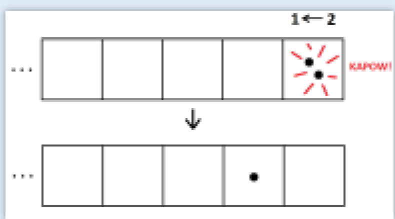
The introduction of a 1 ← 2 machine, followed by exploration of other machines, leads students to systems that convert numbers into curious codes. The introduction of a 1 ← 10 machine then suggests that these codes might not be as strange as one first suspects.

The Experience in a Nutshell

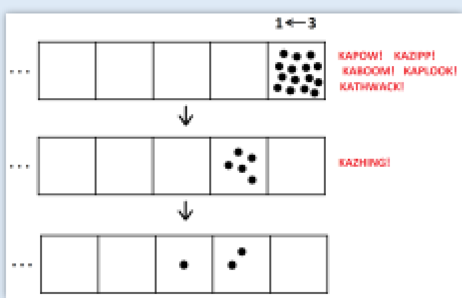
This introduction lays the groundwork for the meaningful math to come. Here students explore the *Exploding Dots* machines and become adept at converting numbers into different codes, finding the code for twenty-two, say, in a 1 ← 3, 1 ← 5, or a 1 ← 10 machine.

To illustrate:

In a 1 ← 2 machine, pairs of dots in any one box "explode" – that is disappear – to produce one dot, one box to their left. (This explains the machine name "1 ← 2" written this backwards way.) We see that two dots turn into one dot and zero dots in a 1 ← 2 machine. We say that 2 has 1 ← 2 machine code 10.



In a 1 ← 3 machine the number 15 has code 120.



And in a 1 ← 10 machine, the number 273 has code … 273!

Setting the Scene

View the welcome video from James to set the scene for this experience:
http://gdaymath.com/lessons/explodingdots/1-1-welcome/ [0:20 minutes]

The 1 ← 2 Machine

This is Core Lesson # 1, corresponding to Lesson 1.2 on

gdaymath.com/courses/exploding-dots/. James has a video of this lesson here:

http://gdaymath.com/lessons/explodingdots/1-2-1-leftarrow-2-machine/ [2:40 minutes]

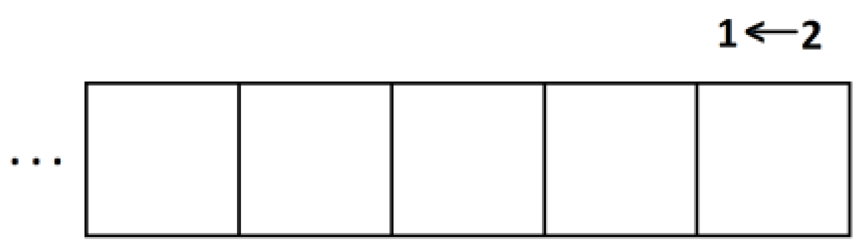
Here is the script James follows when he gives this lesson on a board. Of course, feel free to adapt this wording as suits you best. You will see in the video when and how James draws the diagrams and adds to them. Also, the sound effects he makes are fun.

Welcome to a journey.

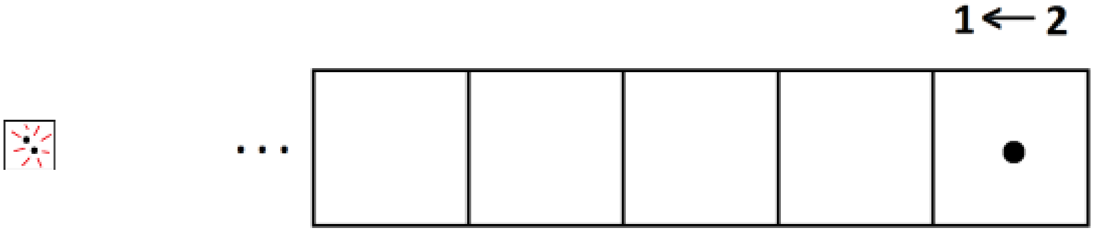
It is a mathematical journey based on a story from me, James, that isn't true.

When I was a child I invented a machine – not true - and this machine is nothing more than a row of boxes that extends as far to the left as I could ever desire.

I gave this machine of mine a name. I called it a "two-one machine" both written and read in a funny backwards way. (I knew no different as a child.)

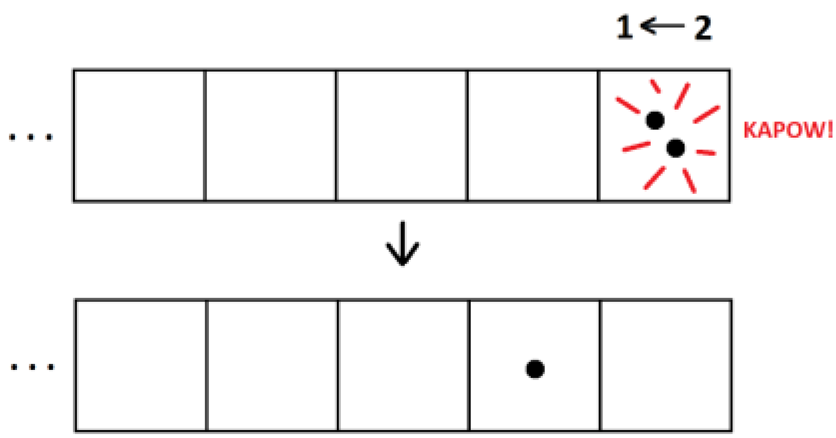


And what do you do with this machine? You put in dots. Dots always go into the rightmost box. Put in one dot, and, well, nothing happens: it stays there as one dot. Ho hum!



But put in a second dot – always in the rightmost box – and then something exciting happens.

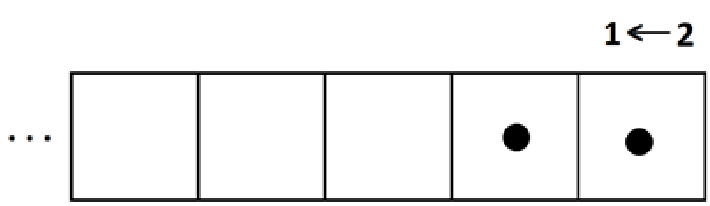
Whenever there are two dots in a box they explode and disappear - KAPOW! – to be replaced by one dot, one box to the left.



(Do you see now why I called this a "1 ← 2 machine" written in this funny way?)

We see that two dots placed into the machine yields one dot followed by zero dots.

Putting in a third dot – always the rightmost box – gives the picture one dot followed by one dot.

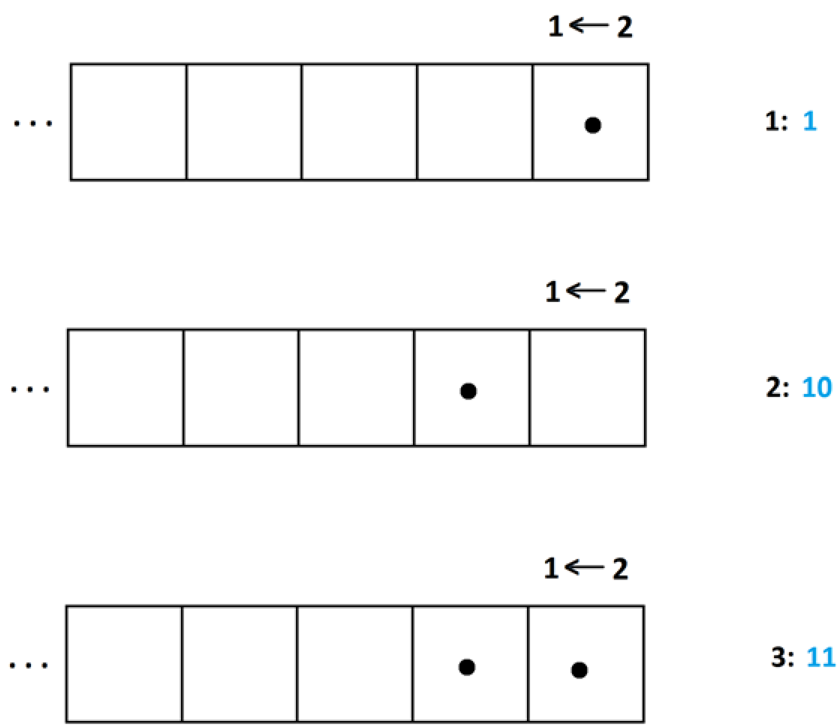


I realized that this machine, in my untrue story, was giving codes for numbers.

Just one dot placed in the machine, stayed as one dot. Let's say that that the 1 ← 2 machine code for the number one is 1.

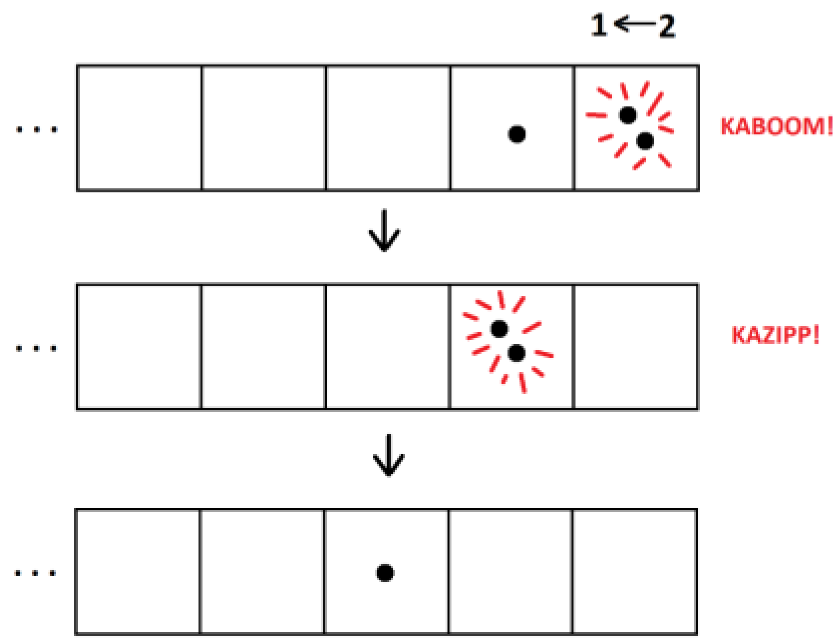
Two dots placed into the machine, one after the other, yielded one dot in a box followed by zero dots. Let's say that the 1 ← 2 machine code for the number two is 10.

Putting a third dot in the machine gives the code 11 for three.



What's the 1 ← 2 machine code for four?

Putting a fourth dot into the machine is particularly exciting: we are in for multiple explosions!



1: 1
2: 10
3: 11
4: 100

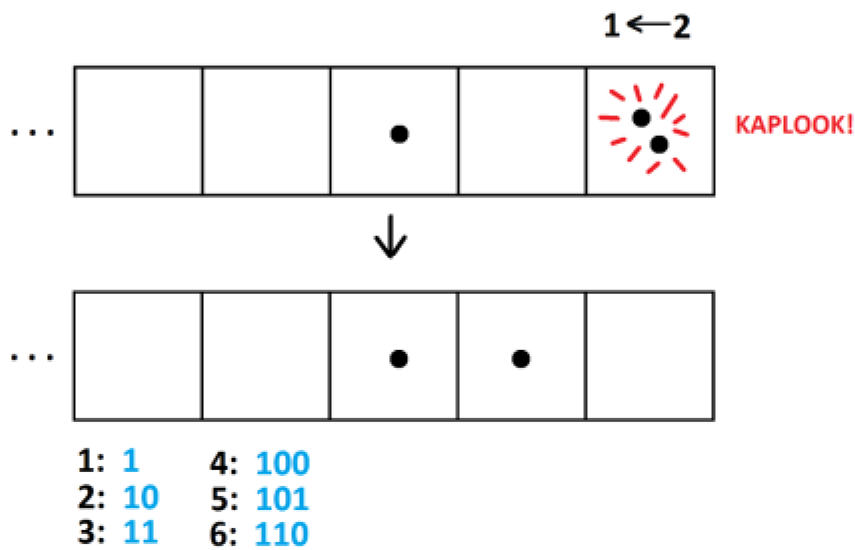
The 1 ← 2 code for four is 100.

At this point older students are usually yelling out loud “This is binary” or “This is base two”. Depending on what is appropriate I either pretend not to hear these cries, or I acknowledge them, thank the audience members for their cleverness, but say that I needed more time than them and that I still personally need to think through all this. (After all, I am a young child in this story.)

What will be the code for five?

Can you see it’s 101?

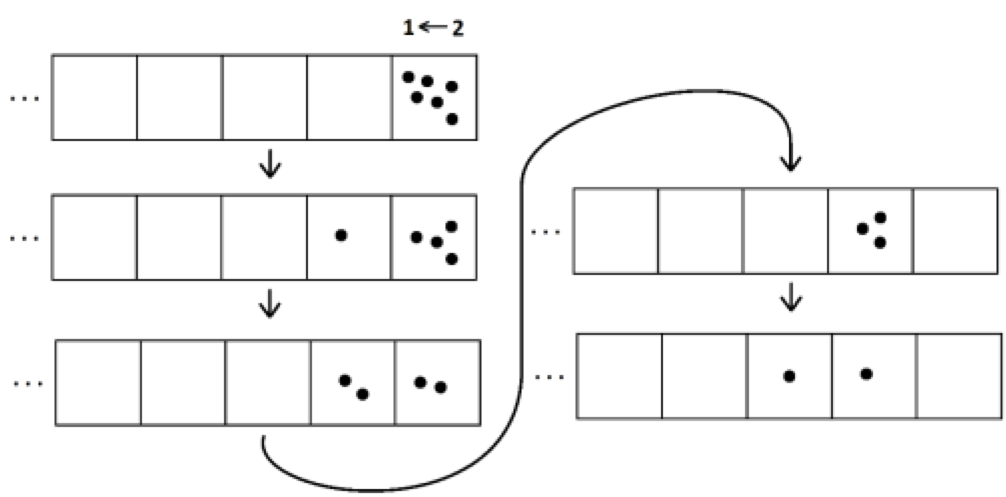
And the code for six? Adding one more dot to the code for five gives 110 for six.



If you have younger students, you might want to add this next extra thinking moment.

Actually, we can also get this code for six by clearing the machine and then putting in six dots all at once. Pairs of dots will explode in turn to each produce one dot, one box to their left.

Here is one possible series of explosions. Sound effects omitted!



Do you get the same final code of 110 if you perform explosions in a different order? (Try it!)

What is the 1 ← 2 machine code for the number thirteen?

(It turns out to be 1101. Can you get that answer?)

Some students might solve this by putting dots into the machine one at a time—7 dots, 8 dots, all the way up to 13 dots. Others might start with a blank machine and put 13 dots in the rightmost box.

It is not important to actually spend time on this question. If students balk at answering this question say "Yep. Good response. Let's just say it is tricky and move on!" And do move on! We'll be coming back to this very question later on, and students will pride themselves on seeing how to answer this question with ease at that time. Keep the feeling of this lesson light, and fun, and un-pressured.

There are hours of fun to be had playing with codes in a 1 ← 2 machine.

But then one day, I had an astounding flash of insight!

Handout A

Use the handout shown below for students who want practice questions from this lesson to mull on later at home. This is NOT homework; it is entirely optional. (See the document "Experience 1: Handouts" for a printable version.)

Exploding Dots

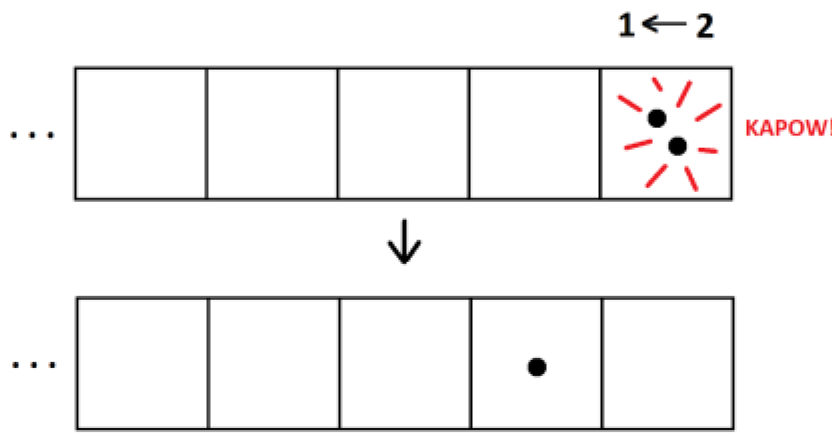
Experience 1: The Machines

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout A: The 1 ← 2 Machine

Here's the rule for a 1 ← 2 machine:

Whenever there are two dots in a box they explode and disappear - KAPOW! – to be replaced by one dot, one box to the left.



And here are some questions you might, or might not, want to try:

1. a) What is the 1 ← 2 machine code for the number thirteen?

(It turns out to be 1101. Can you get that answer?)

b) What is the code for fifty in this machine? (Whoa!)

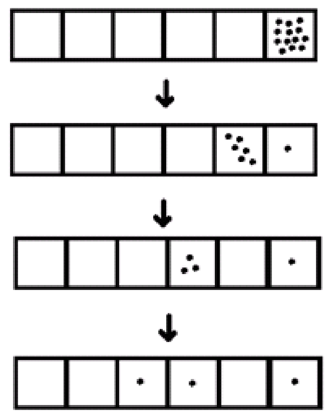
2. Could a number ever have code 100211 in a 1 ← 2 machine, assuming we always choose to explode dots if we can?

3. **CHALLENGE:** Which number has code 10011 in a 1 ← 2 machine?

Solutions to Handout A

1.

a) Here's how the code 1101 appears from thirteen dots.



b) The number fifty has code 110010.

2. Assuming we want to make the agreement that we'll always choose to explode dots if we can, then the code 100211 is not complete: the two dots in the third-to-last box can explode to give a final code of 101011.

3. This is the code for the number nineteen. (Well see next experience a swift way to see this.)

Other Machines

This is Core Lesson # 2, corresponding to Lesson 1.3 on

gdaymath.com/courses/exploding-dots/. James has a video of this lesson here:

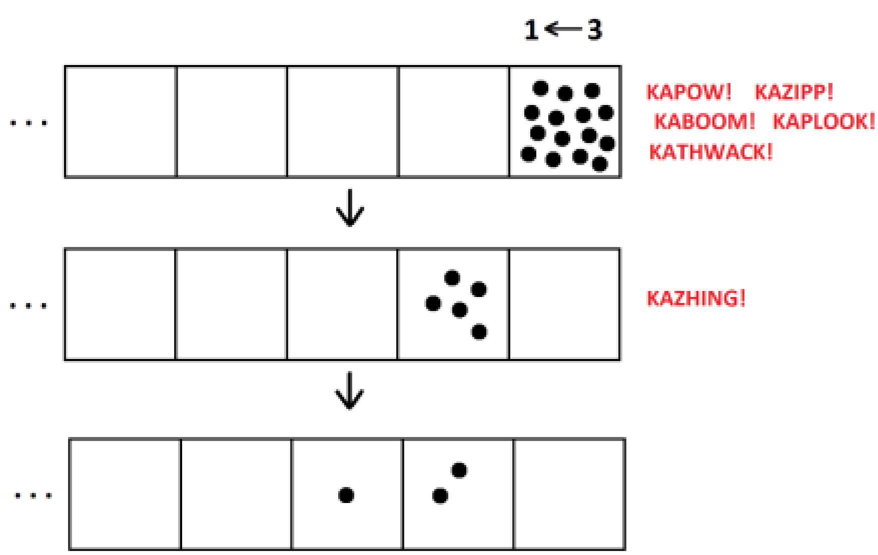
http://gdaymath.com/lessons/explodingdots/1-3-machines/ [2:02 minutes]

Instead of playing with a 1 ← 2 machine, I realized I could play with a 1 ← 3 machine (again written and read backwards, a "three-one "machine). Now whenever there are *three* dots in a box, they explode away to be replaced with one dot, one box to the left.

The video follows a script very similar to the script of a 1 ← 2 machine: work out the code for 1 dot, 2 dots, 3 dots, 4 dots, 5 dots, 6 dots, and then suddenly 13 dots.

Alternatively, one could take this approach.

Here's what happens to fifteen dots in a 1 ← 3 machine.



First there are five explosions in the first box, with each explosion making a dot in the second box to the left. Then three of those dots explode away. This leaves behind two dots and makes one new dot one place to the left. We thus see the code 120 for fifteen in a 1 ← 3 machine.

What is the 1 ←3 machine code for the number thirteen?

Again, if this students balk or struggle with this in any way just write "It's tricky" and suggest we move on! If some students are familiar with base-three codes and yell out the answer, that's fine. Just write up the answer with a question mark next to it and say "I need more time. I am going to have to think through this for myself still."

There are hours of fun to be had working out the codes for numbers in a 1 ← 3 machine.

But then I had another flash if insight. Instead of doing a 1 ← 3 machine, I realized I could do a 1 ← 4 machine, or a 1 ← 5 machine, or any numbered machine I like!

Handout B: *Other Machines*

Use the handout shown below for students who want practice questions from this lesson to mull on later at home. This is NOT homework; it is entirely optional. (See the document "Experience 1: Handouts" for a printable version.)

Exploding Dots

Experience 1: The Machines

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout B: *Other Machines*

Here are some more questions you might, or might not, choose to ponder.

1. a) Show that the code for four in a $1 \leftarrow 3$ machine is 11.

b) Show that the code for thirteen in a $1 \leftarrow 3$ machine is 111.

c) Show that the code for twenty in a $1 \leftarrow 3$ machine is 202.

2. Could a number have code 2041 in a $1 \leftarrow 3$ machine? If so, would the code be "stable"?

3. Which number has code 1022 in a $1 \leftarrow 3$ machine?

We can keep going!

4. What do you think rule is for a $1 \leftarrow 4$ machine?

What is the $1 \leftarrow 4$ code for the number thirteen?

5. What is the $1 \leftarrow 5$ code for the number thirteen?

6. What is the $1 \leftarrow 9$ code for the number thirteen?

7. What is the $1 \leftarrow 5$ code for the number twelve?

8. What is the $1 \leftarrow 9$ code for the number twenty?

9. a) What is the $1 \leftarrow 10$ code for the number thirteen?

b) What is the $1 \leftarrow 10$ code for the number thirty-seven?

c) What is the $1 \leftarrow 10$ code for the number 5846?

Solutions to Handout B

1. a) Do it! b) Do this one too! c) You're on a roll. Do this third one as well!

2. Again, if we agree to do all the explosions we can, then this code is not complete: three of the dots in the second-to-last box can explode to give 2111.

3. The number thirty-five has this code.

4. "Four dots in any one box explode and are replaced by one dot one place to the left." The number thirteen has code 31 in a $1 \leftarrow 4$ machine.

5. 23

6. 14

7. 22

8. 22 (Same code as the previous answer – but, of course, the interpretation of the code is different.)

9. a) 13 b) 37 c) 5846 (These are the codes we use for numbers in everyday life!)

The 1 ← 10 Machine

This is Core Lesson # 3, corresponding to Lesson 1.4 on

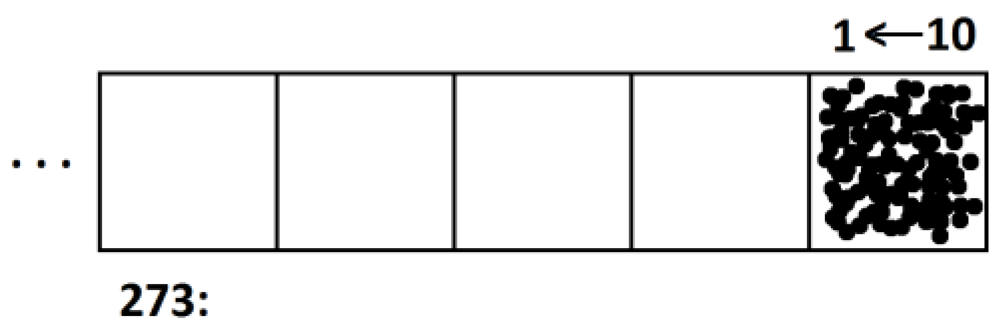
gdaymath.com/courses/exploding-dots/. James has a video of this lesson here:

http://gdaymath.com/lessons/explodingdots/1-4-1-leftarrow-10-machine/ [2:56 minutes]

Okay. Let's now go wild.

Let's go all the way up to a 1 ← 10 machine and put in 273 dots in a 1 ← 10 machine!

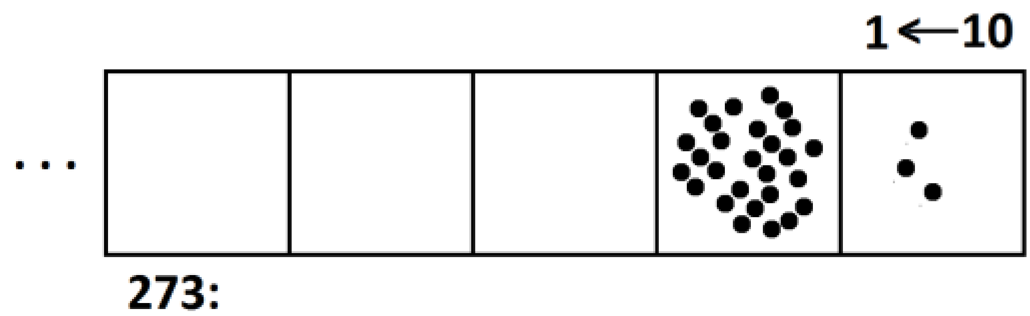
What is the secret 1 ← 10 code for the number 273?



I thought my way through this by asking a series of questions.

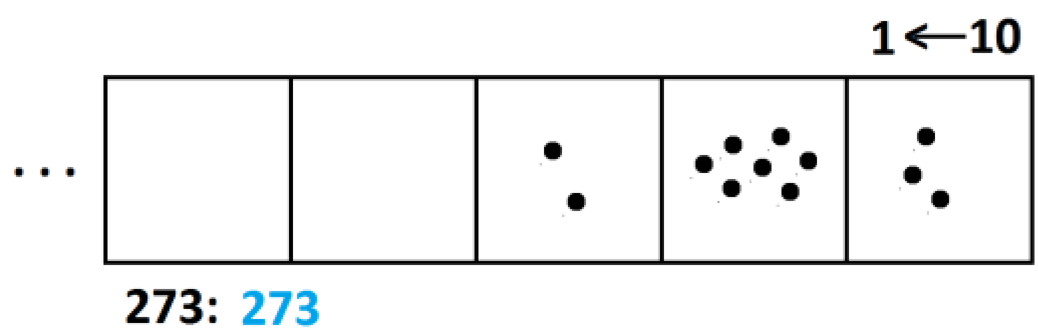
> *Will there be any explosions? Are there any groups of ten that will explode?* Certainly!
>
> *How many explosions will there be initially?* Twenty-seven.
>
> *Any dots left behind?* Yes. Three.

Okay. So, there are twenty-seven explosions, each making one dot one place to the left, leaving three dots behind.



Any more explosions? Yes. Two more.

Any dots left behind? Seven left behind.



The $1 \leftarrow 10$ code for two hundred seventy-three is…273. Whoa!

Something curious is going on!

What is the natural big question to ask?

At this point older students are crying out "base ten." I just say, "Okay. Of course you are right. So just be patient with me and let's make sure we're very very clear on how these place-value systems work" and move on to the next experience right now.

Handout C: *Wild Explorations*

Use the handout shown below for students who want some deep-thinking questions from this Experience to mull on later at home. This is NOT homework; it is entirely optional, but this could be a source for student projects. (See the document "Experience 1: Handouts" for a printable version.)

Exploding Dots

Experience 1: The Machines

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout C: *WILD EXPLORATIONS*

Here are some "big question" investigations you might want to explore, or just think about. All will become clear as the story unfolds in further chapters, but it could be fun to mull on these ideas now.

EXPLORATION 1: WHAT ARE THESE MACHINES DOING?

Can you figure out what these machines are actually doing?

Why is the code for two hundred and seventy-three in a 1 ← 10 machine, "273"? Are all the codes for numbers in a 1 ← 10 sure to be identical to how we normally write numbers?

If you can answer that question, can you then also make sense of all the codes for a 1 ← 2 machine? What does the code 1101 for the number thirteen mean?

Comment: Experience 2 answers these questions.

EXPLORATION 2: DOES THE ORDER IN WHICH ONE EXPLODES DOTS SEEM TO MATTER?

Put nineteen dots into the rightmost box of a 1 ← 2 machine and explode pairs of dots in a haphazard manner: explode a few pairs in the right most box, and then some in the second box, and then a few more in the rightmost box, and then some in the second box again, and so on. Do it again, this time changing the order in which you do explosions. And then again!

Does the same final code of 10011 **appear each, and every time?**