exploding dots

GLOBALMATHPROJECT

buzzmath

MECHANIA

CHAPTER 1

EXPLODING DOTS
CHAPTER 1

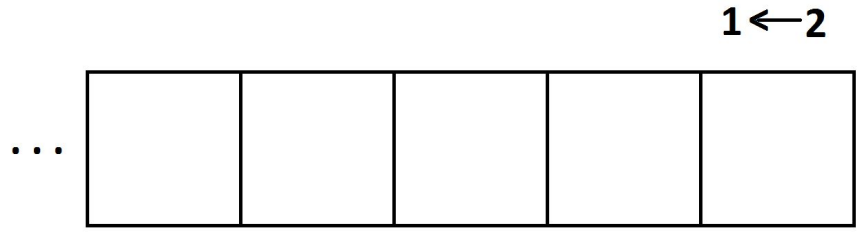
MACHINES

Welcome to a journey.

It is a mathematical journey based on a story from me, James, that isn't true.

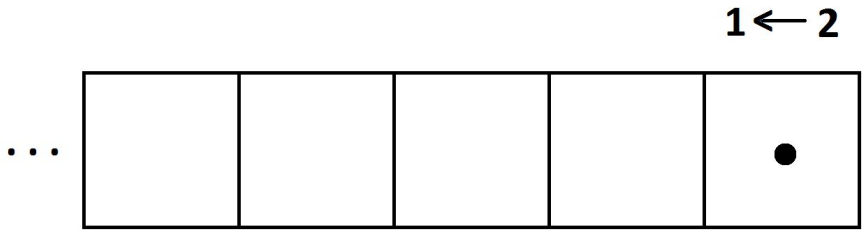
When I was a child I invented a machine – not true - and this machine is nothing more than a row of boxes that extends as far to the left as I could ever desire.

I gave this machine of mine a name. I called it a "two-one machine" both written and read in a funny backward way. (I knew no different as a child.)



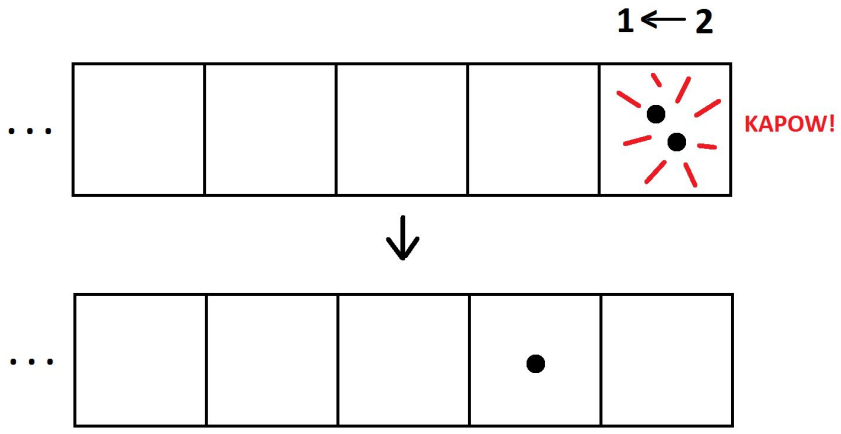
And what do you do with this machine? You put in dots. Dots always go into the rightmost box.

Put in one dot, and, well, nothing happens: it stays there as one dot. Ho hum!



But put in a second dot – always in the rightmost box – and then something exciting happens.

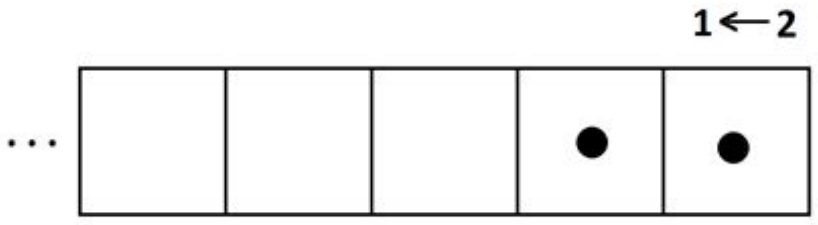
> *Whenever there are two dots in a box they explode and disappear - KAPOW! – to be replaced by one dot, one box to the left.*



(Do you see now why I called this a "$1 \leftarrow 2$ machine" written in this funny way?)

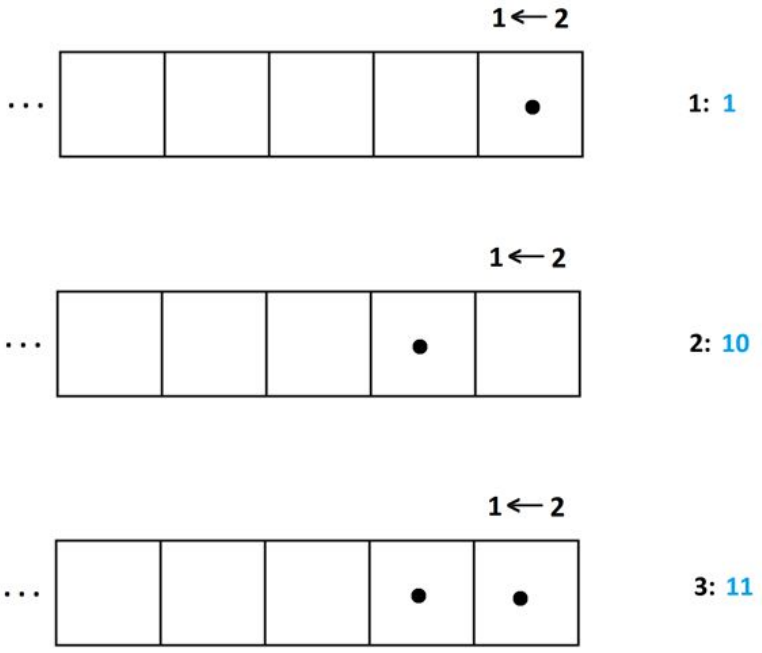
We see that two dots placed into the machine yields one dot followed by zero dots.

Putting in a third dot – always the rightmost box – gives the picture one dot followed by one dot.



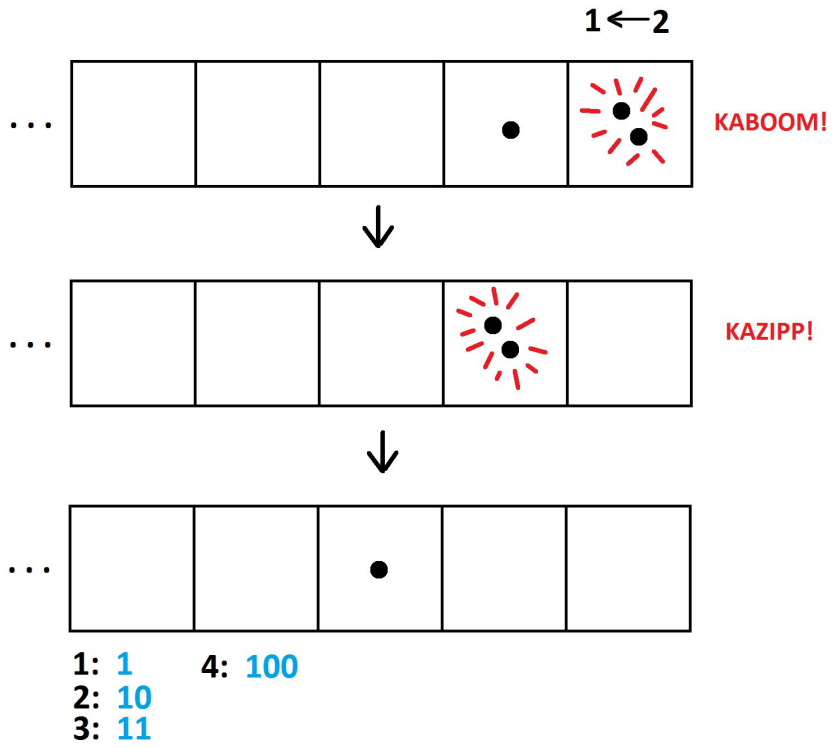
I realized that this machine, in my untrue story, was giving codes for numbers.

> Just one dot placed into in the machine, stayed as one dot. Let's say that that the $1 \leftarrow 2$ machine code for the number one is 1.
>
> Two dots placed into the machine, one after the other, yielded one dot in a box followed by zero dots. Let's say that the $1 \leftarrow 2$ machine code for the number two is 10.
>
> Putting a third dot in the machine gives the code 11 for three.



What's the $1 \leftarrow 2$ machine code for four?

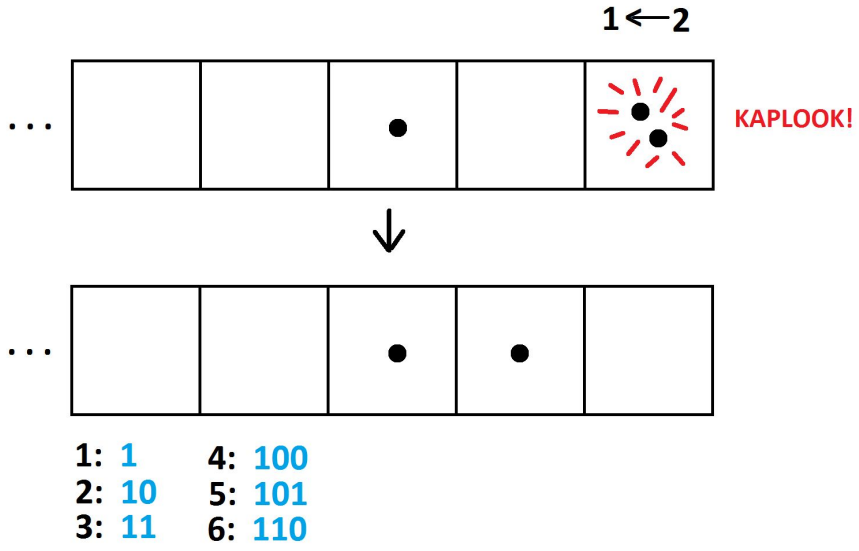
Putting a fourth dot into the machine is particularly exciting: we are in for multiple explosions!



The $1 \leftarrow 2$ code for four is 100.

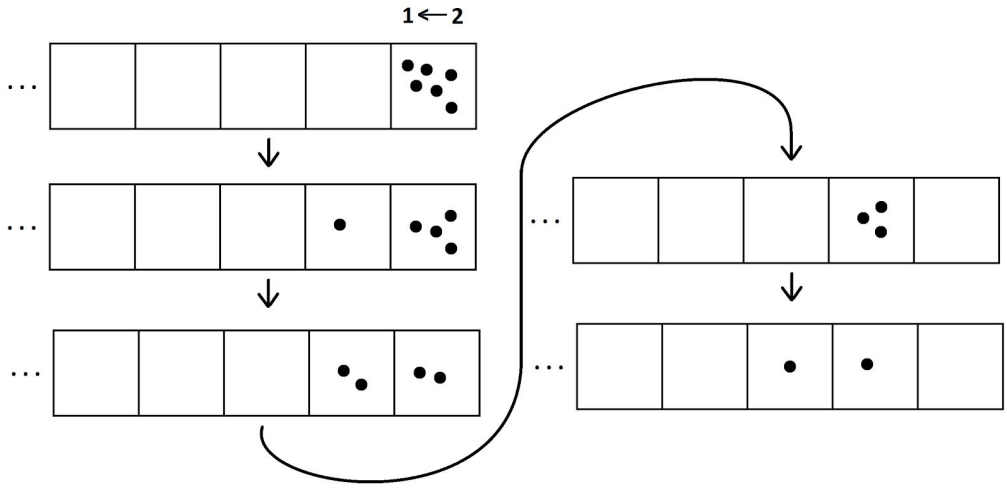
What will be the code for five? Can you see it's 101?

And the code for six? Adding one more dot to the code for five gives 110 for six.



Actually, we can also get this code for six by clearing the machine to then put in six dots all at once. Pairs of dots will explode in turn to each produce one dot, one box to their left.

Here is one possible series of explosions. Sound effects omitted!



Do you get the same final code of 110 if you perform explosions in a different order? (Try it!)

Here are some questions you might or might not want to try. My solutions to them appear at the end of this chapter.

1. a) What is the $1 \leftarrow 2$ machine code for the number thirteen?

(It turns out to be 1101. Can you get that answer?)

b) What is the code for fifty in this machine? (Whoa!)

2. Could a number ever have code 100211 in a $1 \leftarrow 2$ machine, assuming we always choose to explode dots if we can?

3. Which number has code 10011 in a $1 \leftarrow 2$ machine?

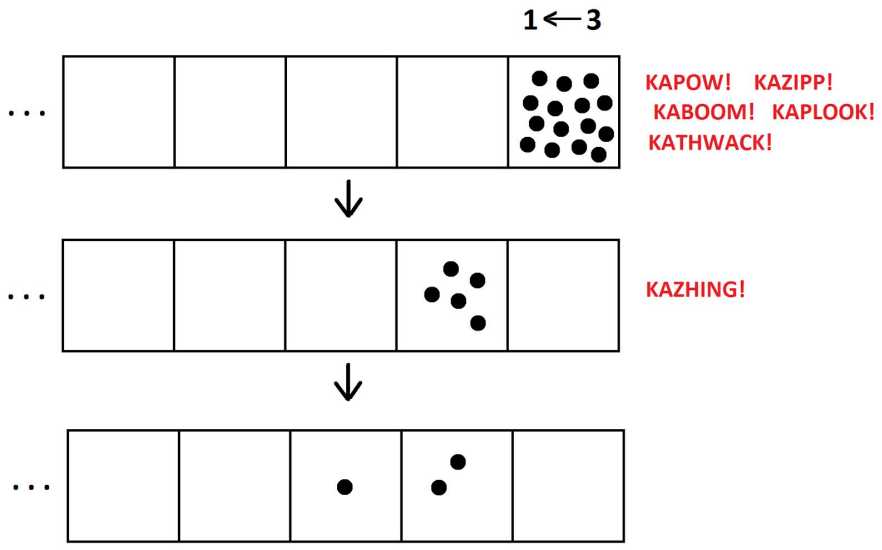
There are hours of fun to be had playing with codes in a $1 \leftarrow 2$ machine.

But then one day, I had an astounding flash of insight!

OTHER MACHINES

Instead of playing with a $1 \leftarrow 2$ machine, I realized I could play with a $1 \leftarrow 3$ machine (again written and read backwards, a "three-one "machine). Now whenever there are *three* dots in a box, they explode away to be replaced with one dot, one box to the left.

Here's what happens to fifteen dots in a $1 \leftarrow 3$ machine.



First there are five explosions in the first box, with each explosion making a dot in the second box to the left. Then three of those dots explode away. This leaves behind two dots and makes one new dot one place to the left. We thus see the code 120 for fifteen in a $1 \leftarrow 3$ machine.

Here are some more questions you might or might not choose to ponder.

4. a) Show that the code for four in a $1 \leftarrow 3$ machine is 11.

b) Show that the code for thirteen in a $1 \leftarrow 3$ machine is 111.

c) Show that the code for twenty in a $1 \leftarrow 3$ machine is 202.

5. Could a number have code 2041 in a $1 \leftarrow 3$ machine? If so, would the code be "stable"?

6. Which number has code 1022 in a $1 \leftarrow 3$ machine?

We can keep going!

7. What do you think rule is for a $1 \leftarrow 4$ machine?

What is the $1 \leftarrow 4$ code for the number thirteen?

8. What is the $1 \leftarrow 5$ code for the number thirteen?

9. What is the $1 \leftarrow 9$ code for the number thirteen?

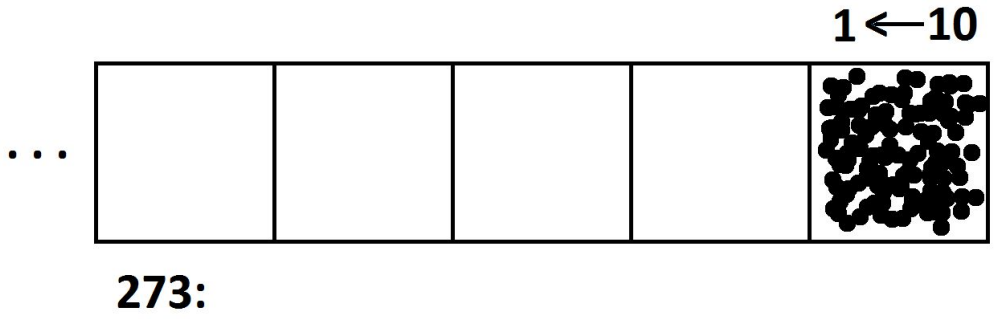
10. What is the $1 \leftarrow 5$ code for the number twelve?

11. What is the $1 \leftarrow 9$ code for the number twenty?

Okay. Let's now go wild.

Let's go all the way up to a $1 \leftarrow 10$ machine and put in 273 dots in a $1 \leftarrow 10$ machine!

What is the secret $1 \leftarrow 10$ code for the number 273 ?



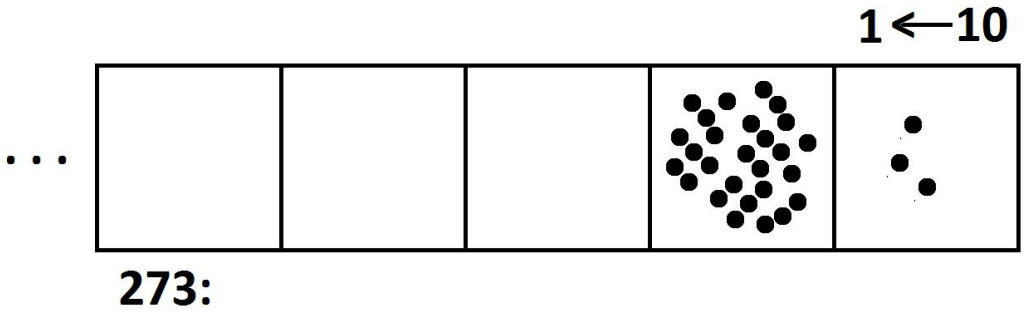
I thought my way through this by asking a series of questions.

Will there be any explosions? Are there any groups of ten that will explode? Certainly!

How many explosions will there be initially? Twenty-seven.

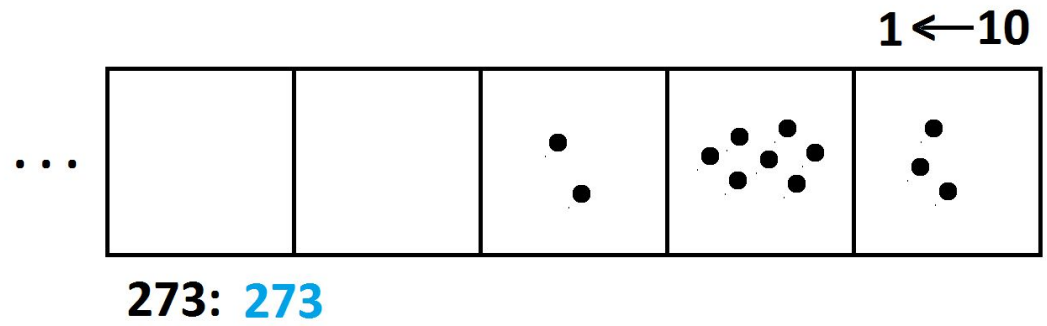
Any dots left behind? Yes. Three.

Okay. So there are twenty-seven explosions, each making one dot one place to the left, leaving three dots behind.



Any more explosions? Yes. Two more.

Any dots left behind? Seven left behind.



The $1 \leftarrow 10$ code for two hundred seventy three is... 273 . Whoa!

12. a) What is the $1 \leftarrow 10$ code for the number thirteen?

b) What is the $1 \leftarrow 10$ code for the number thirty-seven?

c) What is the $1 \leftarrow 10$ code for the number 5846 ?

Something curious is going on!

What is the natural big question to ask?

WILD EXPLORATIONS

Here are some "big question" investigations you might want to explore, or just think about. All will become clear as the story unfolds in further chapters, but it could be fun to mull on these ideas now.

EXPLORATION 1: WHAT ARE THESE MACHINES DOING?

Can you figure out what these machines are actually doing?

Why is the code for two hundred and seventy three in a $1 \leftarrow 10$ machine, "273"? Are all the codes for numbers in a $1 \leftarrow 10$ sure to be identical to how we normally write numbers?

If you can answer that question, can you then also make sense of all the codes for a $1 \leftarrow 2$ machine?

What does the code 1101 for the number thirteen mean?

Comment: Chapter 2 answers these questions.

EXPLORATION 2: DOES THE ORDER IN WHICH ONE EXPLODES DOTS SEEM TO MATTER?

Put nineteen dots into the rightmost box of a $1 \leftarrow 2$ machine and explode pairs of dots in a haphazard manner: explode a few pairs in the right most box, and then some in the second box, and then a few more in the rightmost box, and then some in the second box again, and so on. Do it again, this time changing the order in which you do explosions. And then again!

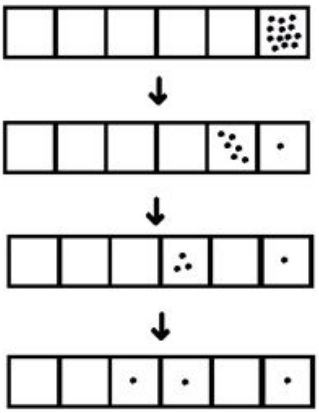
Does the same final code of 10011 appear each and every time?

SOLUTIONS

As promised, here are my solutions to the questions posed.

1.

a) Here's how the code 1101 appears from thirteen dots.



b) The number fifty has code 110010 .

2. Assuming we want to make the agreement that we'll always choose to explode dots if we can, then the code 100211 is not complete: the two dots in the third-to-last box can explode to give a final code of 101011 .

3. This is the code for the number nineteen. (Well see next lesson a swift way to see this.)

4. a) Do it! b) Do this one too! c) You're on a roll. Do this third one as well!

5. Again, if we agree to do all the explosions we can, then this code is not complete: three of the dots in the second-to-last box can explode to give 2111 .

6. The number thirty-five has this code.

7. "Four dots in any one box explode and are replaced by one dot one place to the left." The number thirteen has code 31 in a $1 \leftarrow 4$ machine.

8. 23

9. 14

10. 22

11. 22 (Same code as the previous answer – but, of course, the interpretation of the code is different.)

12. a) 13 b) 37 c) 5846 (These are the codes we use for numbers in everyday life!)