Your numbers divided

What's the mystery?	You are able to predict numbers that exactly divide a series of random numbers chosen by students.
Domain(s)	Mathematics.
Subdomain keywords	Divisors, prime numbers, prime factorisation.
Age group	15 to 16 years old.
Expected time for the mystery	Approximate time for teacher preparation: ten min. Approximate time in classroom: one hour.
Safety/supervision	None Disclaimer: the authors of this teaching material will not be held responsible for any injury or damage to persons or properties that might occur in its use.
Preparation and list of materials	Calculators or student mobile phones if allowed in class.
Learning objectives	Students will understand prime numbers and prime factors of numbers.

Guidance notes for teachers

This classroom-tested teaching plan uses the four innovations of the TEMI project, as detailed in the *Teaching the TEMI Way* book (TTTW). You should read this companion book to get the most from your teaching.
The TEMI techniques used in this teaching plan are: 1) productive science mysteries (TTTW section 1), 2) the 5E model for engaged learning (TTTW section 2), 3) the use of presentation skills (showmanship) to engage your students (TTTW section 3), and 4) the apprenticeship model for learning through gradual release of responsibility (GRR) (TTTW section 4). You might also wish to use the hypothesiser lifeline sheet (available on the TEMI website) to help your students document their ideas and discoveries as they work.

Engage: capture students' attention.

You ask three students to use their calculators (or mobile phones if allowed) to enter a random number. You state you will be able to predict numbers that will exactly divide their chosen random number. Ask them to enter any three-digit number on the calculator and to keep it hidden from you.

Pretend to get premonitions from each of them, saying that with just three digits it's too easy; to make it harder, they should each use a larger number. So they enter the same three-digit number again to give a six-digit number. For example, if they entered 345 initially, then their new number would be 345345.

You are then able to instantly tell each of them a different small number that will exactly divide their personal six-digit number: the first number is divisible by 7, the second by 11, and the third by 13. They each perform the division and show that you are right: there is no remainder.

For final part of the trick, you say you will instantly calculate a six-digit number that is exactly divisible by the three small numbers you already gave: the three numbers derived from their initial free choices. You tell them this six-digit number and again the calculator shows you have been able to correctly calculate the number in your head.

The secret of the trick is that the three small numbers you call out are always 7, 11, and 13. The rest of the trick works by itself.

Explore: collect data from experiments.

You can repeat the trick again; however, this time, get the class to write down all the numbers you use. Then get them to look for patterns in this numerical data: what can be seen?

They should notice that in the numbers used:

1. The personal three-digit numbers need to be repeated (e.g. 432432).

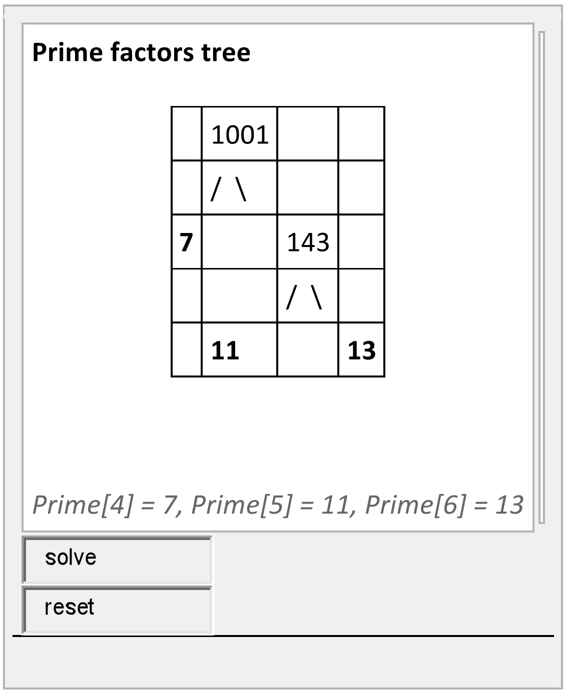
2. They should notice the factors are always 7, 11, and 13, even if you call them out in a different order.
3. They should notice the final six-digit number you give is a repeated three-digit number.

Explain: what's the science behind the mystery?

The trick relies on the fact that entering any three-digit number followed by the same three digits again is exactly the same as multiplying the original three-digit number by 1001. For example, 345345 is 345 x 1001.

The small numbers you use in your predictions and 7, 11, and 13 are the prime factors of 1001. Remember that in number theory, the **prime factors** of a positive integer are the **prime** numbers that divide that integer exactly. The **unique prime factorisation theorem** states that every integer greater than one is either a prime number itself or is the product of prime numbers and that this product is **unique**.

Therefore, only 7, 11, or 13 will divide exactly into any of the students' duplicated personal numbers. The final part of the trick showing your mathematical powers simply involves you giving any six-digit number that is a repetition of any three-digit number (e.g. 765765). This will of course be divisible by 7, 11, and 13 due to the same mathematical principal.



Prime factors tree image from the prime factor calculator website:
http://www.calculatorsoup.com/calculators/math/prime-factors.php

The data collected when you perform the trick again, along with hints about the relevance of primes to the trick working and leading them to the revelation that multiplication by 1001 is identical to repeating a three-digit number, will support them in discovering the principle behind the trick.

Extend: what other related areas can be explored?

Will the trick work with a single digit repeated three times? The answer is yes, as 333333 is 333 x 1001
Can students find the prime factors of the following numbers?

9 (answer: 3).
39 (answer: 3 and 13).

Determining the prime factors of a number is an example of a technique frequently used to ensure cryptographic security in encryption for online shopping.

Evaluate: check the level of student scientific understanding.

You can evaluate their understanding of prime numbers and prime factors by posing the following questions:

- Will the trick work for a single-digit number (e.g. initial number 3, duplicated number 33)?
- The answer is no: 33 is 3 multiplied by 11 and 11 is a prime number with no prime factors.

- Will the trick work for a two-digit duplicated number (e.g. 3434)?
- The answer is no: 3434 is 34 multiplied by 101 and 101 is a prime number with no prime factors.

You might want to have the students try and calculate the prime factors of some numbers by hand; see, http://www.calculatorsoup.com/calculators/math/prime-factors.php

Showmanship: tips on how to teach and present this mystery.

This trick is easy to do and works by itself: you just need to remember the instructions and the numbers 7, 11, and 13. Chose a presentation you are happy with and practise it a few times in private before you perform. Ensure that the students know to enter the same three digits twice by giving them an example. Do try and make this digit duplication look like it is you trying to make the trick harder rather than something to make the trick work. When predicting the numbers, 7,11, and 13, look strained and get the students each to confirm that you are correct.

The final section with a six-digit number should be done slowly, again making it look like it is taking effort to calculate the final figure from the numbers you gave earlier. The more you can make it seem that you could have predicted any three numbers based on the personal numbers the students entered, the stronger the effect.

Letting students use the calculator on their own phones, if allowed, can add a personal touch and remove any suspicion about trick school calculators.

When teaching the maths, getting to the 1001 multiplication revelation may need to be signposted more for the weaker students through suitable hints.

GRR: teaching skills using Gradual Release of Responsibility.

Demonstrated enquiry (level 0): teacher-as-model. You show how to carry out an enquiry process, which students then copy. Explain your hypothesis and tests by 'talking aloud'. Students then record your thinking onto their hypothesiser lifeline sheet. This includes indicating the numbers you use, thus presenting the data they need to collect and analyse.
Structured enquiry (level 1): 'we do it'. Students then use their hypothesiser lifeline sheet to record their own alternative ideas about why the trick works and to record their tests and conclusions regarding other possible explanations by running a series of experiments themselves. Here they look at whether single and double digit numbers work and the importance of prime numbers to the solutions.
Solving the mystery: students are led towards the explanation by using ideas about prime factorisation and number patterns when multiplying by 1001.

Resources

Videos showing this trick, among others, being performed and explained to teach basic mathematics can be found at:
www.mathematicalmagic.com

An online prime factor calculator can be found at:
http://www.calculatorsoup.com/calculators/math/prime-factors.php
This also shows how to manually calculate prime factors.

THE STUDENT WORKSHEET CAN BE COPIED AND USED IN THE CLASSROOM.
Note that, in some cases, answers to earlier questions may be found later in the student worksheet.

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Engage: what's interesting?

Tasks	How can your teacher make these predictions about the numbers you all freely chose and how are they able to calculate the final six-digit number in their head so quickly?

Explore: what's happening?

Tasks	What do you notice about the numbers used? Write these down: these will be the data you can use to help explain the trick. When the trick is repeated, what new clues (data) can you discover: write down your information and ideas about how the trick works.

Explain: what's causing it?

What's interesting and important about the numbers 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31?

What's interesting and important about the numbers 345345, 987987, 123123, and numbers like these? Is there a pattern you can see?

Starting from, say, 456, who can get to 456456 in a different way than by just repeating the initial three-digit number?

Extend: what's similar?

Would the trick work if you entered three digits identical to your first number? If so, why?

FP7-Science-in-Society-2012-1, Grant Agreement N. 321403

Evaluate: what's my understanding?

Tasks	Would the trick work if we start with a two-digit number? If not, why not?