Understanding the Rocket Engine Performance in BLOODHOUND SSC

The BLOODHOUND Engineering Project

Mechanical Engineering

This exemplar requires some knowledge of Physics and Chemistry together with Maths.

INTRODUCTION

Figure 1: The BLOODHOUND SSC

The BLOODHOUND Project is an iconic engineering and education adventure that is pushing technology to its limit, providing us with a once in a lifetime opportunity to inspire the next generation of scientists and engineers. BLOODHOUND SSC (Super Sonic Car) has been designed to set a new land speed record of 1,000 miles per hour (mph). BLOODHOUND SSC is powered by a EUROJET EJ200 jet engine and a hybrid rocket engine.

A rocket is basically an engine which carries both a fuel and oxidiser (it does not require any oxygen from the air). Hybrid rockets use a solid fuel and a liquid oxidiser. The fuel is contained within the combustion chamber and the liquid oxidiser is injected at the top of the chamber. Therefore a hybrid can be easily shutdown by turning off the supply of liquid oxidiser, making them more suitable for use in a land speed record car. Hybrids use the same oxidisers as liquid propellant systems; fuels can include synthetic rubbers and plastics (such as PVC).

IMPORTANCE OF EXEMPLAR IN REAL LIFE

Thrust is a reaction force described quantitatively by Newton's Second and Third Laws. When a system expels or accelerates mass in one direction, the accelerated mass will exert an equal but opposite force on that system. A rocket engine produces thrust by expelling hot gas at a very high velocity.

Calculating thrust and specific impulse are fundamental to the design of any rocket engine. Achieving the optimum oxidiser:fuel (O:F) ratio and understanding how the chamber pressure and the combustion process affect specific impulse is the key to achieving an efficient system.

SCENARIO

Figure 2: Static tests of the 15.2cm (6-inch) hybrid chamber

The two pictures above in figure 2 show static tests of the 15.2cm (6-inch) diameter hybrid rocket chamber that is being used to develop the hybrid rocket for BLOODHOUND SSC.

The hybrid rocket in BLOODHOUND SSC uses HTP (a concentrated hydrogen peroxide H_2O_2, basically water with an extra oxygen atom) as the oxidiser and a synthetic rubber Hydroxyl-Terminated Polybutadiene (HTPB) as the primary fuel. The hybrid combustion chamber for BLOODHOUND SSC has a catalyst pack to decompose the HTP, the decomposition products then enter the fuel grain. The fuel automatically ignites in contact with the decomposition products, generating pressure. The combustion products then enter the nozzle which converts the high pressure, low velocity gas into high velocity low pressure gas generating up to 122 kN (27,500 lbs) of thrust. The chamber contains 181 kg of fuel which can run for up to 20 seconds. The chamber is 45.7cm in diameter and 4.27m long. Development work has been conducted on a 15.2 cm (6-inch) diameter chamber. Several firings have been conducted to test various aspects of chamber design, fuel grain configuration and catalyst material.

One of the challenges is to find optimum specific impulse of the rocket while reducing the harshness of the environment (in terms of high heat fluxes) experienced by key components of the motor (specifically the nozzle and throat). Calculating the heat transfer rate to rocket motor components is frequently dealt with using CFD analysis and is not discussed here. Also, there are so many dynamic variables in a rocket combustion chamber that we need to simplify the equations in order to get an approximate mathematical answer. To try and calculate all the dynamic variables requires extremely complex mathematics.

Therefore, in this exemplar, we will look at some of the fundamental aspects of calculating rocket performance and examine some of the more complex aspects which are key to achieving an efficient hybrid rocket for BLOODHOUND SSC.

MATHEMATICAL MODEL

A rocket is propelled forward by a thrust force equal in magnitude, but opposite in direction, to the time-rate of momentum change of the exhaust gas accelerated from the combustion chamber through the rocket engine nozzle (shown in figure 2).

Mathematically, it can be expressed as:

$$T = \frac{dm}{dt} v \quad \ldots (1)$$

where

T = Thrust

$\frac{dm}{dt}$ = Mass flow rate of exhaust

v = Speed of the exhaust gases measured relative to the rocket

Please note that equation (1) is true only for a perfectly expanded nozzle.

In case of BLOODHOUND SSC, this thrust is expressed as follows:

$$T = \frac{dm}{dt} v_e + (P_e - P_a) A_e \quad \ldots (2)$$

where

v_e = Exit velocity of the exhaust gases

P_e = Rocket engine nozzle exit pressure

P_a = Ambient pressure

A_e = Exit plane area

The second expression in equation (2) shows that the thrust produced by a rocket varies with altitude because of the dependence on ambient pressure. Thus, rocket performance is usually defined by two limits T_{sl} (thrust at sea level) and T_v (thrust in vacuum). The customary index of performance is specific impulse, I_{sp} defined by:

$$I_{sp} = \frac{T}{g \times \frac{dm}{dt}} \quad \ldots (3)$$

where g is the acceleration due to gravity which varies with altitude in the same way that T does. Using specific impulse to characterise the rocket performance is analogous to using miles per gallon or specific horsepower to characterise a car.

In order to address the question of O:F, we need to keep some other parameters constant. The mathematical analysis is simplest if we assume that the motor designers expand the rocket engine nozzle in such a way that the static pressure at exit (P_e) closely matches the static pressure at the operating altitude (P_a). Hence, the second term in equation (2) is usually small. Thus, ignoring that term, we get:

$$T = \frac{dm}{dt} v_e \quad \ldots (4)$$

Combining equation (3) and (4), we get:

$$I_{sp} = \frac{v_e}{g} \text{ or } I_{sp} \propto v_e \ldots (5)$$

This shows that I_{sp} is strongly dependent on the exit velocity v_e which can be calculated from the following expression:

$$v_e = \sqrt{\frac{2\gamma}{(\gamma - 1)} R T_c \left[1 - \left(\frac{P_e}{P_c} \right)^{(\gamma-1)/\gamma} \right]} \quad \ldots (6)$$

where

γ = Ratio of specific heats

R = Specific gas constant

T_c = Combustion chamber temperature

P_c = Combustion chamber pressure

(Please refer to the isentropic expansion explained in Reference 3, 4 or 5 under the title "Where to find more" in order to get better understanding of equation (6) above. Proof of this equation is beyond the scope of this exemplar.)

For a particular fuel:oxidiser combination, γ is a slowly varying function of O:F and hence can be considered as constant for simplicity. The ratio P_e/P_c can be considered as a constant here to get a fair approximation of the specific impulse. However, variable ratio P_e/P_c requires more complicated analysis and is not covered here.

Thus, from equation (6), we see that:

$$v_e \propto \sqrt{RT_c} \ \ldots (7)$$

We know that the specific gas constant R is defined as the ratio of the universal gas constant R_u and the mean molecular weight M_m, i.e.:

$$R = \frac{R_u}{M_m}$$

Hence,

$$R \propto \frac{1}{M_m} \ \ldots (8)$$

where M_m is the mean molecular weight of the combustion products in this case.

Combining equation (7) and (8), we can say that:

$$v_e \propto \sqrt{\frac{T_c}{M_m}} \ \ldots (9)$$

Finally, observing equation (5) and (9) together, it is clear that the optimum I_{sp} can only be achieved when the ratio of the combustion temperature T_c to the mean molecular weight M_m of the combustion products is maximised. In particular, this reasoning explains why many rockets produce afterburning plumes. At first glance, the combustion of H_2 or CO in the plume would appear to point the inefficiency of the rocket engine since burning these materials within the rocket engine would result in the release of more chemical energy. However, the mean molecular weight of a hydrogen oxygen mixture is considerably lower than for pure water vapour. As such, we find that an optimum I_{sp} is achieved at a value of O:F (oxidiser to fuel ratio) that is significantly lower than the stoichiometric ratio *(please refer to Reference 7)* for complete combustion. This leads to the presence of combustible products in the plume. These combustibles are here by design to enhance I_{sp}, i.e. they should not be regarded as unburnt fuel. I_{sp} is the main driver of hybrid rocket design rather than combustion efficiency. Hybrid rockets are often run oxidiser rich (O:F larger than stoichiometric) since a large flow rate of oxidiser is required to liberate fuel vapour from the fuel grain. As a consequence, the I_{sp} from a hybrid is usually lower than a high efficiency liquid engine (cryogenic H2/LOX).

CONCLUSION

In this exemplar, we have looked at fundamental rocket performance parameters such as calculating thrust and specific impulse and also made an attempt to explain why rocket exhaust plumes frequently afterburn upon mixing with the ambient air producing bright flames in the exhaust gas. This is because optimum specific impulse is frequently achieved when light flammable elements remain present in the exhaust plume i.e. not all the fuel is burnt down to its final heavier state (usually H_2O or CO_2).

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. *Mechanics of Fluids*, B. S. Massey, need more details.
4. *Rocket Exhaust Plume Phenomenology*, Frederick S. Simmons, 2000, Published by the AIAA
5. *Rocket Propulsion Elements*, George P. Sutton , Donald M. Ross, Oscar Biblarz; 7th Edition
6. http://www.bloodhoundssc.com
7. http://moodle.student.cnwl.ac.uk/moodledata_shared/CDX%20eTextbook/dswmedia/fuelSys/gasoline/fund/stoichiometricratio.html

Daniel Jubb, The Falcon Project Ltd.

He says:

"*I was born in Manchester in 1984. I have been interested in rockets from an early age. When I was 10, I received a model rocket kit. I launched hundreds of model rockets and then decided that I wanted to make much larger rockets to try to reach higher altitudes.*

The Falcon Project was founded in 1995. Falcon now designs and manufactures custom solid, liquid and hybrid propellant rocket systems at facilities in the US and UK, with applications ranging from mine disposal and target drones to high altitude research rockets. We also established research programmes into liquid and hybrid propellant rocket engines.

I have always been interested in science and mathematics and how they are applied to engineering and rocket design.

I became involved with the BLOODHOUND Project in November 2005. The Falcon Project Ltd designed and manufactured the hybrid rocket for BLOODHOUND SSC. "

Dr. Merrifield, Fluid Gravity Engineering Ltd.

He says:

"*As an aerothermal engineer, an understanding of mathematics is a necessity. Applied mathematics forms the basis of the predictive methodology used in this industry and enables the quantitative assessment of a number of important areas pertinent to the design of space vehicles e.g: rocket performance, heat shield heating and exposure to shear stresses during atmospheric entry, aerodynamics and vehicle trajectory, materials response to heating and/or mechanical load ... to name only a few. Fluid Gravity Engineering Ltd worked in Partnership with The Falcon Project Ltd to develop a computational hybrid combustion model to support the development work on the hybrid rocket for BLOODHOUND SSC."*

INFORMATION FOR TEACHERS

The teachers should have some knowledge of

- The terms used in aerodynamics and rocket engineering (thrust, specific impulse, etc.)
- The terms used in Physics and Chemistry (e.g. specific gas constant, universal gas constant, mean molecular weight, pressure, temperature, etc.)
- Simple calculations and manipulating algebraic expressions

TOPICS COVERED FROM “MATHEMATICS FOR ENGINEERING”

- Topic 1: Mathematical Models in Engineering

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-2: Applications of Computer Aided Designing
- Unit-4: Instrumentation and Control Engineering
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science