The Royal Academy of Engineering

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The Mathematics of Engineering Surveying (3)

C&BE

CONSTRUCTION AND THE BUILT ENVIRONMENT DIPLOMA

Scenario

As a new graduate you have gained employment as a graduate engineer working for a major contractor that employs 2000 staff and has an annual turnover of £600m. As part of your initial training period the company placed you in their engineering surveying department for a six-month period to gain experience of all aspects of engineering surveying. One of your first tasks was to work with a senior engineering surveyor to establish a framework of control survey points for a new £12m highway development consisting of a two mile by-pass around a small rural village that, for many years, has been blighted by heavy traffic passing through its narrow main street.

Having established the control framework you are now required to establish the position of a number of additional control points to be subsequently used to establish the road centre-line.

In this exercise you will carry out the geometric calculations that would enable you to determine the precise position of the new control points using the coordinates of the existing control survey points and survey measurements. The technique that you will use is referred to as a ***resection*** technique

Importance of Exemplar in Real Life

When surveying on the civil engineering or construction site it is often necessary to find the coordinates of new control points or points of detail. This is relatively simple if both the existing and the new point are accessible but often one of them is not and so other techniques are required. For example the new or existing survey points may be targets on walls, points on high buildings or points on land to which access is denied. When the existing points are accessible but the new point is not then the survey needs to employ an ***intersection*** technique. When the existing points are not accessible but the new point is then the survey needs to employ a ***resection*** technique.

Figures 1 and 2 show the road construction scheme where the control survey points will have been established along the approximate line of the road using a *traverse* technique. The pronounced circular curves of the road can be clearly seen and for each curve it will be necessary to establish the exact position of the curve's centre-line points so that the full curve alignment can be accurately established. To facilitate this it may be necessary to establish a number of additional control points nearer to the road centre-line than the originally established control framework. The techniques described in this exemplar might well be used for this purpose.

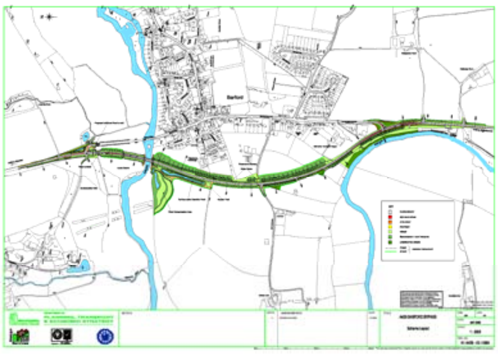

Figure 1: *Road alignment as seen on a map*

Figure 2 *Road alignment as seen from the air*

Background Theory

Two alternative *resection* methods are described below:

Method (a)

In figure 3 below the point P is the location of a new control station that is to be established using a ***resection*** technique from the three established traverse stations or points of known coordinates, A,B and C should be visible from P as shown in figure 3. In the figure, A, B and C are known fixed points, from which the distances AB and BC can be calculated. The angle φ can also be calculated from the coordinates of A,B and C and is hence a known quantity. The angles α and β will be measured by accurate sightings to the three traverse stations in the field and will hence be known, whilst the angle θ has to be calculated. Once θ has been calculated the coordinates of P can then be determined.

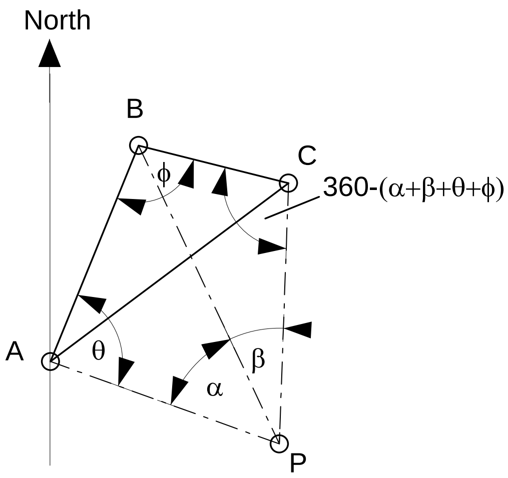

Figure 3: Establishing P by Resection - Method (a)

In the above figure the angle at C = $360-(\alpha+\beta+\theta+\phi)=(360-\alpha-\beta-\phi)-\theta=X-\theta$
where *X* contains site-measured (α,β) or readily calculated (ϕ) angles and θ is an unknown angle, to be determined.

From triangle ABP and using the sine rule: $PB = AB\dfrac{\sin\theta}{\sin\alpha}$

From triangle BCP and using the sine rule: $PB = BC\dfrac{\sin(X-\theta)}{\sin\beta}$

Therefore:

$$PB = AB\frac{\sin\theta}{\sin\alpha} = BC\frac{\sin(X-\theta)}{\sin\beta}$$
$$\therefore \frac{\sin(X-\theta)}{\sin\theta} = \frac{AB\sin\beta}{BC\sin\alpha} = K \qquad (1)$$

Where: $$K = \frac{AB\sin\beta}{BC\sin\alpha} \qquad (2)$$

K is fixed value as it can be calculated from the two known fixed distances (AB, BC) and two measured angles (α,β). Therefore from equation (1):

$$\frac{\sin(X-\theta)}{\sin\theta} = K$$
$$\therefore \frac{\sin X\cos\theta-\cos X\sin\theta}{\sin\theta} = K$$
$$\therefore \sin X\cot\theta-\cos X = K$$

Hence: $$\cot\theta = \frac{K+\cos X}{\sin X} \qquad (3)$$

Equation (3) can be used to calculate the angle θ. Once this angle is known then the geometry of the triangles can be solved to determine the distances AP, BP and CP. For example applying the sine rule to triangle ABP:

$$AP = AB\frac{\sin(180-\theta-\alpha)}{\sin\alpha} \qquad (4)$$

and, knowing the coordinates of station A, the coordinates of P can hence be calculated from the length of AP and the bearing of AP (see the exemplar "*The Mathematics of Engineering Surveying (1)* for this part of the calculation). Only one distance AP, BP or CP and their corresponding bearings are needed to calculate the coordinates of P but if one is used then the others can then be used as a check on the accuracy of the computation.

Method (b)

We will not give the proof for the geometric relationships given below but Method (b), known as *Tienstra's Method* provides an alternative and possibly neater solution to that given above. Figure 4 again shows the three points of known coordinate, A B and C, and the fourth point, P, the coordinates of which are to be established by resection. The three angles $x\,(B\hat{P}C)$, $y\,(C\hat{P}A)$ and $z\,(A\hat{P}B)$ are measured in the field and are measured in a *clockwise* direction as shown. Three separate diagrams are shown in figure 4 to take into account three possible positions of *P* in relation to A,B and C.

Once these angles are measured in the field the coordinates of *P* (E_p *and* N_p) can be calculated using the equations (5),(6) and (7) given below

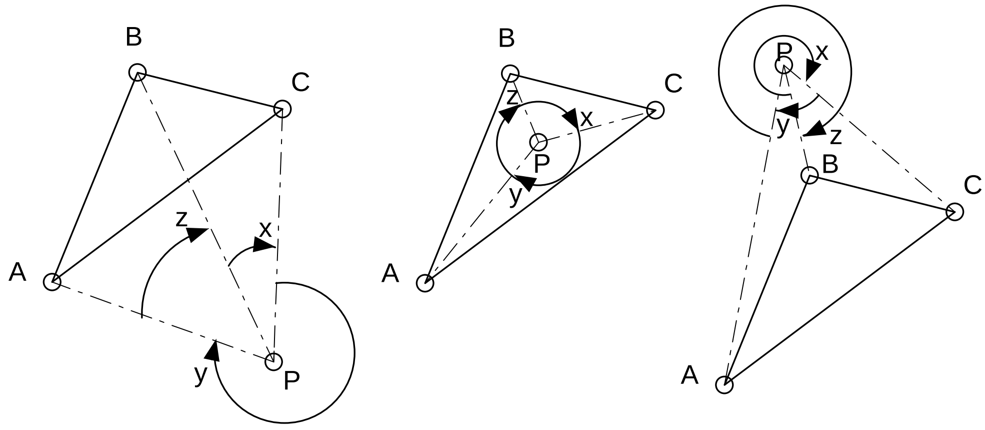

Figure 4: Establishing P by Resection - Method (b)

Calculate:

$$a = \tan^{-1}\left(\frac{E_C - E_A}{N_C - N_A}\right) - \tan^{-1}\left(\frac{E_B - E_A}{N_B - N_A}\right)$$

$$b = \tan^{-1}\left(\frac{E_A - E_B}{N_A - N_B}\right) - \tan^{-1}\left(\frac{E_C - E_B}{N_C - N_B}\right) \qquad (5)$$

$$c = \tan^{-1}\left(\frac{E_B - E_C}{N_B - N_C}\right) - \tan^{-1}\left(\frac{E_A - E_C}{N_A - N_C}\right)$$

then calculate:

$$K_1 = \frac{1}{\cot a - \cot x} \quad K_2 = \frac{1}{\cot b - \cot y} \quad K_3 = \frac{1}{\cot c - \cot z} \qquad (6)$$

hence, calculate the Easting and Northing:

$$E_P = \frac{K_1 E_A + K_2 E_B + K_3 E_C}{K_1 + K_2 + K_3} \quad \text{and} \quad N_P = \frac{K_1 N_A + K_2 N_B + K_3 N_C}{K_1 + K_2 + K_3} \qquad (7)$$

It should be noted that method (b) can not be used if the three known points A,B and C lie on a straight line and neither method can be used if all four points lie on the circumference of a circle.

Questions

Example [Method (a)]

The table below gives the coordinates of three of the traverse points established for a section of the new road. A further control station is to be established using the resection technique and by sighting on to stations A,B and C.

The angles measured in the field are $A\hat{P}B = 40^o 08' 24''$ and $B\hat{P}C = 57^o 36' 00''$. Calculate the angle θ, the length AP and hence the coordinates of this newly established control station. (refer to figure 3)

Example [Method (b)]

Using the same data as above calculate the coordinates of *P* using *Tienstra's Method*. The clockwise angles measured in the field are:

$x = B\hat{P}C = 57^o 36' 00''$, $y = C\hat{P}A = 262^o 15' 36''$, $z = A\hat{P}B = 40^o 08' 24''$

Station	**Easting**	**Northing**
	(metres)	**(metres)**
A	1000.000	2000.000
B	1078.331	2077.869
C	1172.191	2154.753

Where to find more

1. Schofield W, Breach M, *Engineering Surveying*, 6th edn, Oxford: Butterworth-Heinemann
2. Bird J, *Engineering Mathematics,* 5th edn, Elsevier, 2007 (ISBN 978-07506-8555-9)

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The Mathematics of Engineering Surveying (3)

INFORMATION FOR TEACHERS

Teachers will need to understand and explain the theory outlined above and have knowledge of:

- Some terminology relating to engineering surveying
- Geometry and trigonometry

Topics covered from Mathematics for Engineers

- Topic 1: Mathematical Models in Engineering
- Topic 3: Models of Oscillations
- Topic 5: Geometry

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 5.2: Calculate distances
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other units of the Advanced Diploma in Construction & The Built Environment

Unit 2 Site Surveying
Unit 3 Civil Engineering Construction
Unit 6 Setting Out Processes

Solution to the Questions

Method [a]: $\theta = 39^o 03' 36''$, $AP = 168.30m$, $E_P = 1167.446m$, $N_P = 2016.916m$
Method [b]: $E_P = 1167.446m$, $N_P = 2016.916m$

These exercises can be replicated with other sets of coordinates. However the geometry can be quite challenging and the learner should sketch out each problem, ideally on graph paper to visualise the problem and its solution.

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