B.1 Symmetry

a Fold a piece of paper in two and mark the fold well by passing your thumb nail along it. Pierce a small hole through the folded paper using a pin or the point of a pair of compasses, and open the paper out. You will see two holes in it.

How are the holes related
i to each other?
ii to the mark of the fold on the paper?
(Hint: join the holes with a pencil line.)

If you pierce other holes in the paper, will they be related in the same way?

b Fold a piece of paper in two, and then in two again, making the folds parallel. Pierce a small hole through the folded paper and open the paper out. How many holes are there in the paper?

How are the holes related
i to one another?
ii to the folds in the paper?
(It may help you to number the holes and to mark the lines of the folds with letters of the alphabet.)

c Fold a piece of paper several times, making the folds parallel. Pierce a hole through the folded paper and open the paper out. Are the holes related to one another and to the folds in the same way as they were in **a** and **b** above? Are there interesting differences in their relationships?

Use diagrams to illustrate your answers.

B.2 Symmetry

a Fold a piece of paper in two and then make a second fold at right angles to the first. Pierce a small hole through the folded paper. Before you unfold the paper, write down how many holes you expect to see when you do unfold it. Write down also the shape of the figure you will make if you join the holes in a clockwise direction with pencil lines. Then unfold the paper and check that you were right. How are the holes related to one another and to the folds in the paper? (It may help you if you number the holes and mark the folds with letters of the alphabet.

b If the pencil lines joining the holes form a square, can you say why? If they do not form a square, can you suggest where the hole must be pierced if we want the lines to form a square?

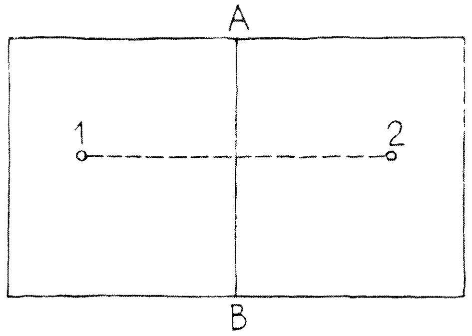
c Fold the paper three times, making each fold at right angles to the previous fold. Pierce a hole through the folded paper and write down
i the number of holes you expect to see when you open the paper
ii how you think they will be related to one another and to the folds in the paper.

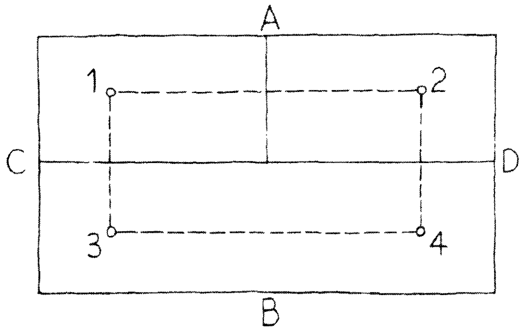
d Repeat this exercise with four folds in the paper.

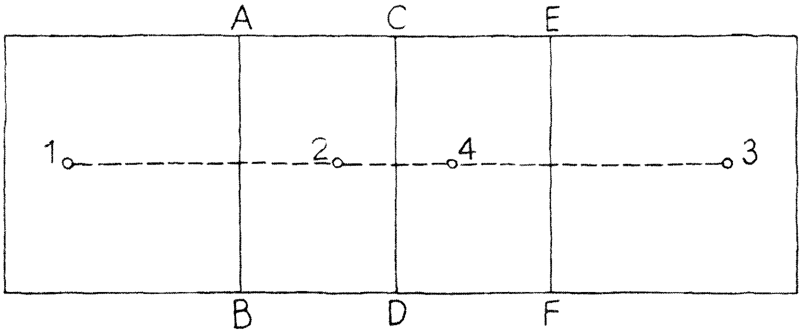
B.1 and B.2
Notes for the teacher

Both these cards are intended to show the pupil that:

a holes made in folded paper will lie on a number of straight lines;
b holes will be equidistant from certain axes of symmetry; and
c the lines joining holes made in this way will be at right angles to certain axes of symmetry.







If one fold is made, the relationship between the holes and the fold is a simple one.

If more than one fold is made, the holes will be symmetrically placed about more than one axis.

If the folds are parallel, not only will the holes be symmetrically placed about more than one axis but they will all lie on the same straight line.

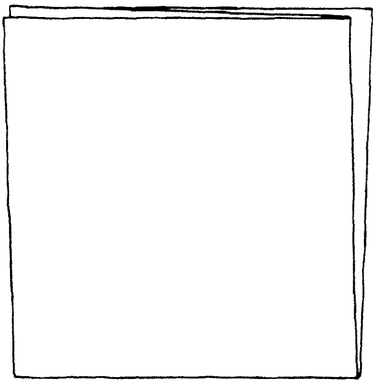
Making more folds will obviously result in making more holes.

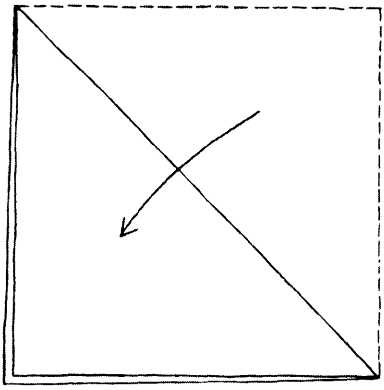
In the diagrams, holes 1 and 2 are symmetrically placed about AB; 1 and 3, and 2 and 4, are symmetrically placed about CD; and 3 and 4 are symmetrically placed about AB in the middle diagram and about EF in the lowest diagram.

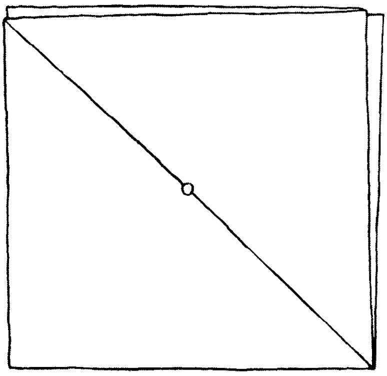
The number of holes made will be the same when the folds are made at right angles as when they are parallel to one another; only the arrangement of the holes will be different. If two folds are made in the paper at right angles, the four holes will lie at the vertices of a rectangle, though they may possibly lie at the vertices of that particular rectangle to which we give the name of square. If the two folds are made at an angle which is not a right angle, where will the holes lie?

Two holes may be taken to represent a point and its image by reflection in an axis of symmetry. The distance of the point from the axis of symmetry will be the same as the distance of its image from the axis of symmetry. However, when the point is reflected in more than one axis of symmetry, the distances between the point and its images need not be equal. In particular, the distances between a point and its two images obtained by placing the point randomly near the junction of two mirrors set at right angles to each other are unlikely to be equal. If they are not equal, the point and its images (including the double image by reflection in both axes) will lie at the vertices of a rectangle. When the distances are equal, the point and its images lie at the vertices of a square.

All we need to do if we want the holes in our paper to lie at the vertices of a square is to fold the paper twice with the folds at right angles, and then by bringing the folds together to make a third fold at an angle of 45° with each of the other two. The hole must be pierced somewhere along this third fold.

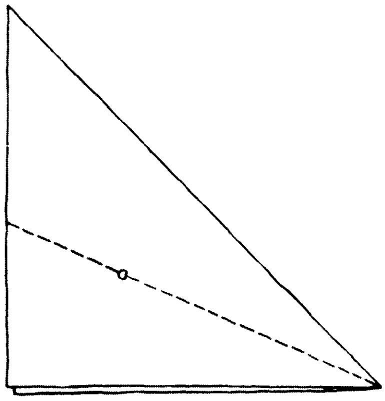






Folding the paper at right angles three times will give a rectangle with eight holes on its perimeter; folding it four times at right angles will give two rectangles, one inside the other.

None of the ways of folding so far discussed involves an oblique fold, except where we obtained a square by piercing the hole *on the line of the fold.* If we make two folds at right angles and a third fold at an angle of 45° to the other folds, and then pierce a hole in the paper, the holes will lie at the vertices of an octagon. If we want a regular octagon, we have to pierce a hole on the bisector of the angle between the folds, just as we had to do when we wanted a square. The angle bisector can of course be found by making a fourth fold.



B.3 Symmetry

For this card you will need plain paper and a piece of carbon paper.

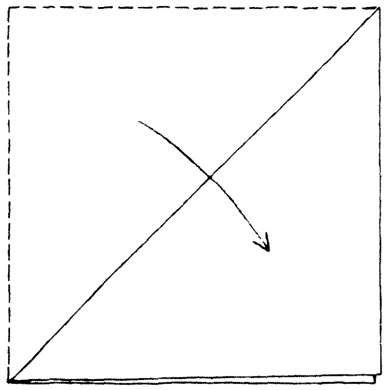
a Place a sheet of plain paper and a sheet of carbon paper together, with the ink side of the carbon paper towards the plain paper. Fold the two sheets in two together, and draw a short line, which need not be straight, on the plain paper. Open the paper and look on the back. You should see two lines. How are they related to each other and to the fold? If you can, explain these relationships.

Draw other lines in the same way, some at a distance from the fold and others meeting it. Draw one (straight) line which meets the fold at right angles. Draw another (straight) line which meets the fold at an angle which is not a right angle. How are the two lines which you see in each case on the back of the paper related to each other and to the fold?

b Fold the paper and the carbon paper as you did in **a** and draw a triangle on the paper. You will find two triangles on the back of the plain paper. Study them carefully and write a description of how they are related to each other and to the fold.

B.4 Symmetry

Repeat all that you did in card B.3 but this time fold the paper twice, with the second fold at right angles to the first. Draw a straight line on the folded paper from one fold to the other. What shape do you expect to find when you open out the paper?



Experiment with a third fold which bisects the angle between the other two folds. Can you now draw lines which will form a square? a rectangle? an octagon? when you unfold the paper?

Experiment also with regular and irregular shapes drawn on the folded paper.

B.3 and B.4

Notes for the teacher

Both these cards help to build up an awareness of the fact that when two shapes are symmetrical about an axis, corresponding points in a shape and in its image by reflection are equidistant from the axis of symmetry, represented in this case by the lines of the fold. This will be more noticeable if objects are drawn which themselves have little symmetry, e.g. cars, areoplanes, animals. Introducing an oblique line of reflection shows that axes of symmetry need not be horizontal or vertical, and that lines joining corresponding points in a shape and its image by reflection will be perpendicular to the axis of symmetry, no matter at what angle the latter is inclined to the horizontal.