

D.1 Symmetry

Draw a square with sides 3 cm long. Place a mirror carefully along one side of the square. You will see an image of the square in the mirror. Draw this image on the paper behind the mirror. Do this with the mirror on each side of the square in turn. Shade in the original square carefully with a pencil.

Now place the mirror on one side of one of the images and draw the image of the image. If you continued to draw images in this way, what pattern would you obtain?

Could a person tile a floor using only square tiles?

(If you have no mirror, you can cut a square from paper or card and draw round it. Now flip it over along one edge and draw round it in its new position. Do this carefully. Return the square to its original position and flip it over along another edge. Draw round the square. Continue in this way until you are sure of what the pattern is going to be.)

Choose another regular polygon instead of the square and repeat the process of reflecting it in the mirror and drawing its image on the paper behind the mirror. Can you find any polygon which will make a pattern on its own? Can you find a polygon which will combine with other polygons to give a pattern which will cover the paper?

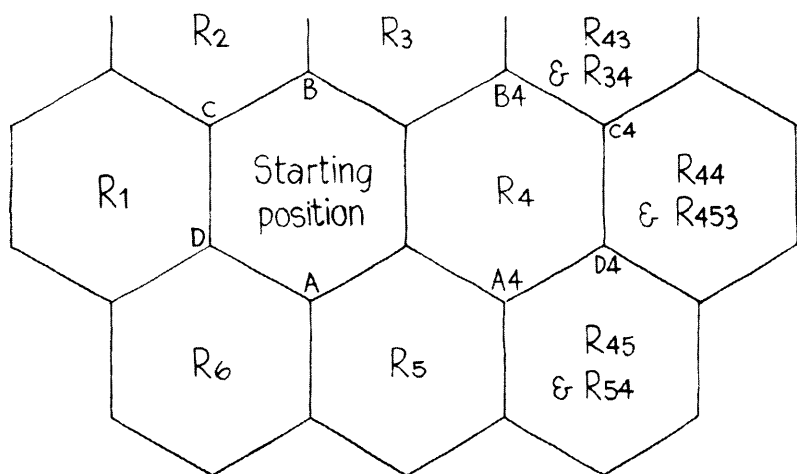
D.1

Notes for the teacher

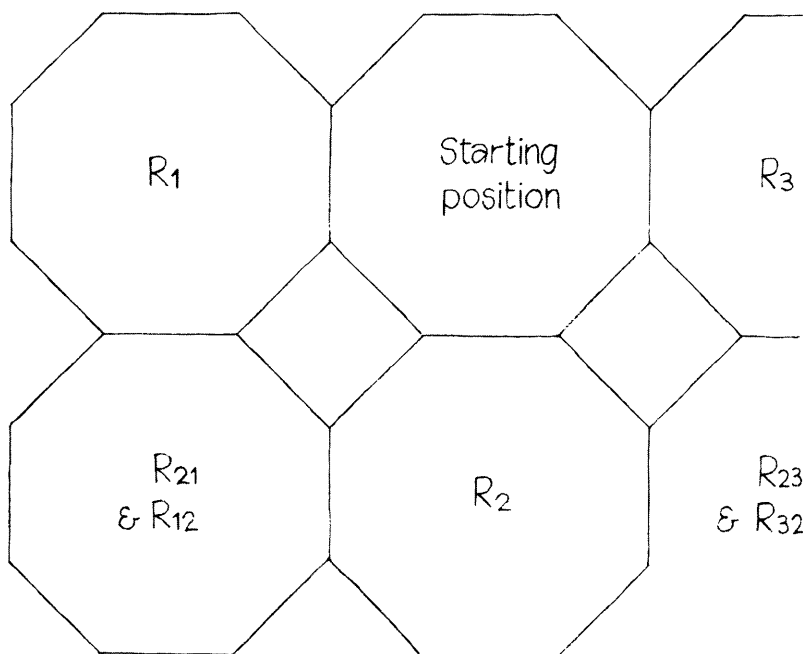
This exercise is designed to show that some polygons will tessellate to give a complete pattern while others will not. The three regular polygons which tessellate are the square, the equilateral triangle and the regular hexagon; one which does not is the regular octagon, although it will tessellate in combination with a square.

	Axis				
	R_{12} & R_{21}	R_2	R_{23} & R_{32}	R_{233} & R_{323} etc	Axis
	R_1	Starting position	R_3	R_{33}	
	R_{14} & R_{41}	R_4	R_{34} & R_{43}	R_{343} & R_{433} etc	

A tessellation of squares



A tessellation of regular hexagons



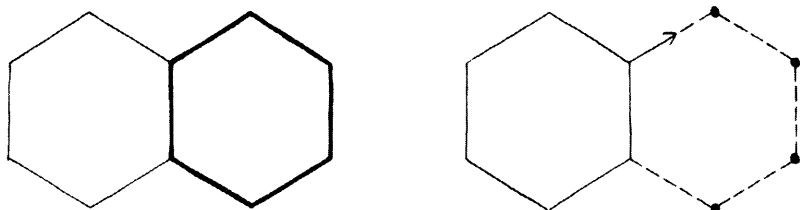
A regular octagon will not tessellate to give a complete pattern unless it combines with a square.

If a regular polygon will tessellate, a mirror placed on a side of the polygon will reflect the tessellation correctly no matter on which side of the polygon the mirror is placed. The pupil should be warned however to verify that corresponding points on the polygon and on its various images all lie on straight lines.

The pupil might be asked to mark on the tessellation some lines which represent axes of symmetry for the tessellation.

It is possible to find a rhombus which will tessellate by reflection in its sides. If a pupil finds this tessellation he should be asked to compare it with the tessellation of equilateral triangles, and also to try to make other shapes of rhombus tessellate in the same way.

It is not easy to reproduce a mirror image by drawing freehand behind a mirror. This difficulty may be overcome by drawing the original polygon on tracing paper. The tracing is turned over and fitted against the original, and the position of the vertices of the 'image' are marked with a sharp pencil or with the point of a pair of compasses. The vertices of the image are joined with pencil and ruler when the tracing paper is removed.



The same result is obtained if we use a polygon cut from paper or card and flip it over as the word-card suggests. We shall do better in this case not to draw round the edges of the card or paper but to mark carefully the positions of the vertices and to join these with pencil and ruler after the piece of card or paper has been removed.

The pupil should be asked whether every polygon which has an axis of symmetry will tessellate by reflection in its sides; whether every polygon which has an axis of symmetry will tessellate somehow; and whether any polygon which has no axis of symmetry will tessellate. (Examples of the last-mentioned are all triangles and all quadrilaterals.)