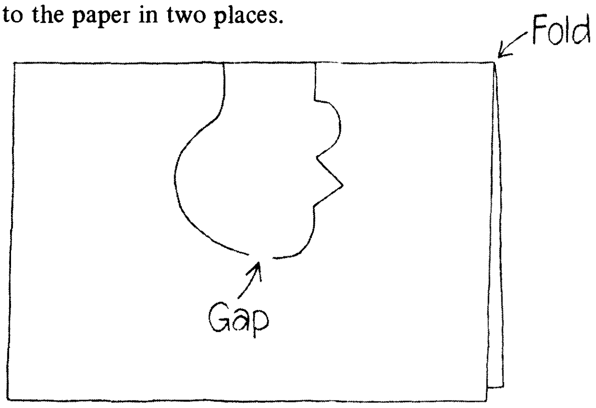
E.1 Symmetry

Fold a sheet of paper in two. Starting from the fold, draw any simple shape you like but leave a small gap in its outline on the side furthest away from the fold. Cut round the outline but be careful not to cut where you have left the gap. The shape will remain attached to the paper in two places.



Before you open the paper, write down what you would have expected to find if you had cut the shape out completely.

Now push the shape down *inside* the paper as far as it will go, still keeping the paper folded. What do you expect to see when you open the paper?

Open out the paper until the two halves are at right angles. Look at the shape you have cut out. How many copies of the shape can you see?

By placing a mirror behind the hole in the paper, try to make the shape appear to fit the hole.

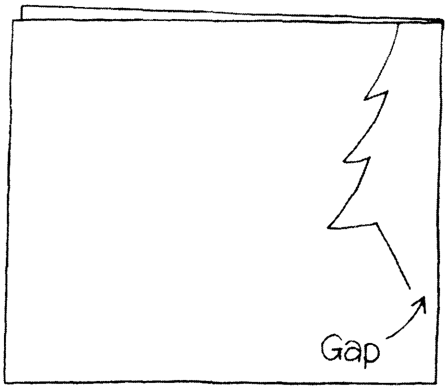
Do the shape and paper together have any kind of symmetry? Do they have an 'axis' of symmetry? Can you place a small strip of card anywhere inside the shape and the paper so that the card acts in the same way as an axis of symmetry would?

Repeat this experiment with other shapes partly cut from a folded sheet of paper. Will the results always be symmetrical?

What is the simplest shape which can be cut in this way?

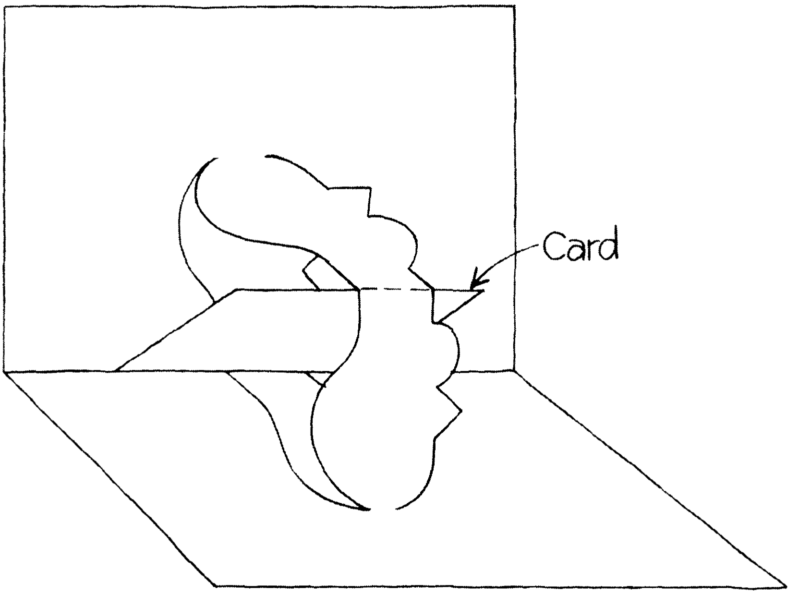
E.2 Symmetry

Repeat the work you did in card E.1 but this time make a second fold in the paper at right angles to the first before you draw and partly cut out the shape. This time you should begin to cut at the first fold and stop just before you reach the second fold. What differences do you find between the shapes you obtain from two folds and the shapes you have obtained from a single fold?



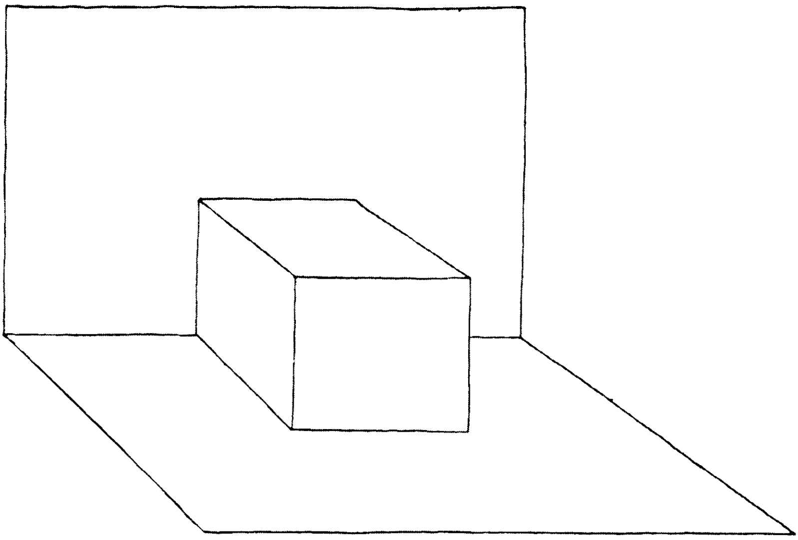
E.1 and E.2
Notes for the teacher

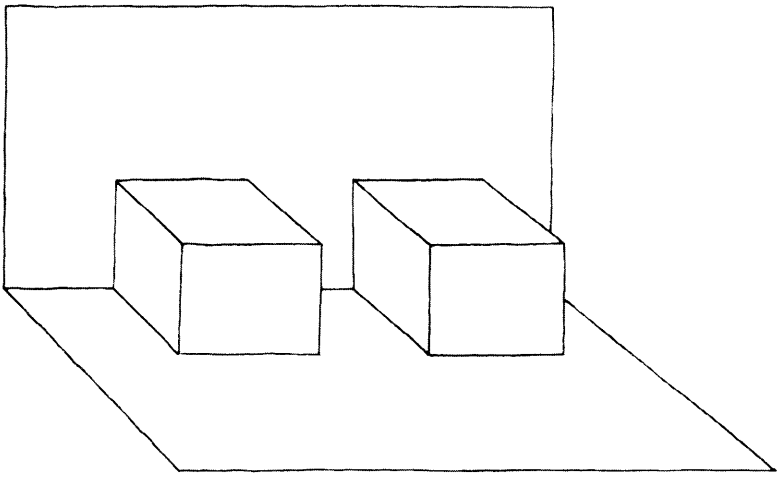
These cards introduce children to the working of the 'pop-up' principle which they may have seen put to work in birthday or Christmas cards. These pop-ups offer a useful entry into the study of symmetry in three dimensions, and of the symmetry of solid figures generally. In three dimensions we talk about 'planes' of symmetry instead of axes of symmetry whenever reflection is involved. The small piece of card can be used to represent a plane of symmetry. Each pop-up shape and its hole will have at least one plane of symmetry; the folds in the paper will lie in this plane.



Shapes which have been cut from paper which has been folded twice will have at least two planes of symmetry, and again the position of these planes will be marked by the position of the folds.

Sometimes the shapes will have more than two planes of symmetry; three planes of symmetry can easily be found in shapes cut from paper which has been folded three times, but shapes cut from paper with two folds in it may also have planes of symmetry which lie obliquely to the horizontal and vertical planes. An example of such a shape is a cube. This can be made with a single cut when the paper is folded twice or with two cuts when the paper is folded once. Can any other recognisable shapes (cuboids, octahedra, etc.,) be made by straight cuts from a single or double fold?

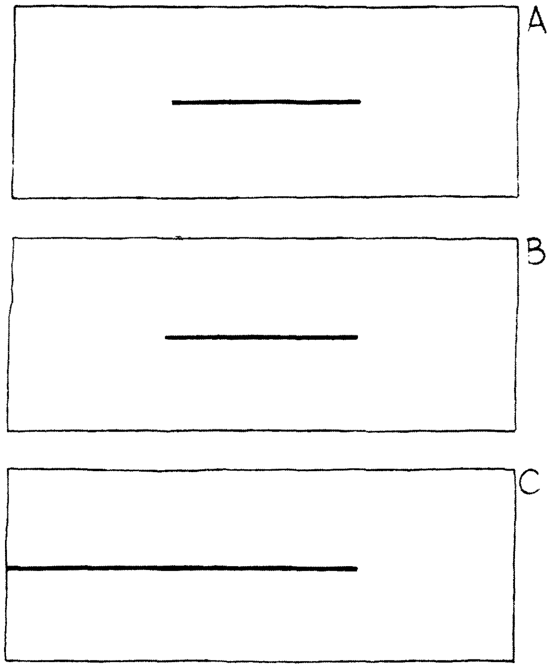




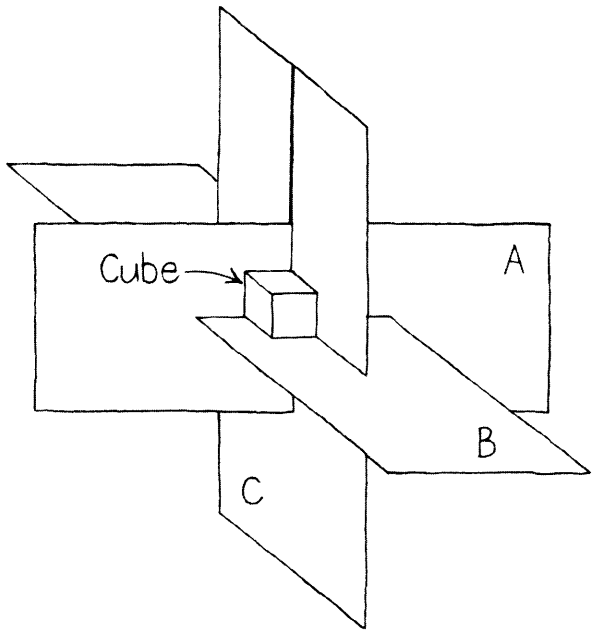
E.3 Symmetry

a Draw the 'net' of a cube on paper. On each square of the net mark and join the midpoints of opposite sides. Now make the cube from the net, with the lines which you have drawn showing on the outside. Could you use these lines to guide you in cutting through the cube so that the two halves you obtain would be *symmetrically* equal? In how many ways could you do this? Are these the only lines by which a cube can be cut symmetrically in half?

b Take three postcards and cut slots in them as shown in the diagram. In cards A and B the slot should be as long as the width of the card. Card C is cut in the same way but one end of the slot is continued as far as the edge of the card. The slots must be cut symmetrically in the cards; that is they must be parallel to the edges and must pass through the centre of the card.



Colour each card differently; or you may be able to cut your cards from sheets of card of different colours.



Slide card B through the slot in A; then slide C on either side of A and through the slot in B. Fit some small wooden cubes into the 'corners' made where the three cards meet. How many cubes do you need in order to make a larger cube? Lift four cubes off one card; can you explain what we mean by a 'plane of symmetry'?

E.4 Symmetry

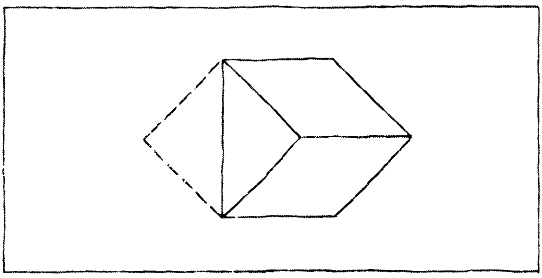
Make an open model or 'skeleton' of a cube from drinking straws and pipe-cleaners. Add to this skeleton a diagonal on each of two opposite faces, making sure that the two diagonals are in the same *plane*. Test that they are in the same plane by resting a sheet of card or paper across them. Would you agree that this plane is a plane of symmetry for the cube? Can you find other planes of symmetry of the same type?

E.3 and E.4

Notes for the teacher

The pupil who remembers that the diagonals of a square are axes of symmetry of the square will not find it too difficult to see that these diagonals may show the positions of planes of symmetry of a cube. These cards carry on, therefore, the work begun in cards E.1 and E.2.

The diagonal planes of symmetry of a cube can also be seen in the imagination if a cube is pushed half way through a hole. The hole can be cut in card or paper, and should be of such a size that the square can be pushed through it obliquely.

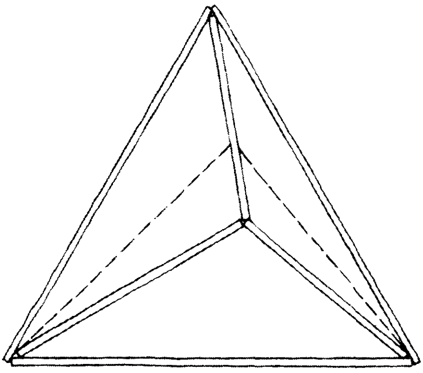


The methods described in the cards of finding the planes of symmetry of a cube can be used to find the planes of symmetry of other solids, such as a cuboid or an octahedron.

Note: If the slot is to take the thickness of a piece of card, it must be made with *two* cuts close together. Otherwise a great deal of distortion will occur in the finished model.

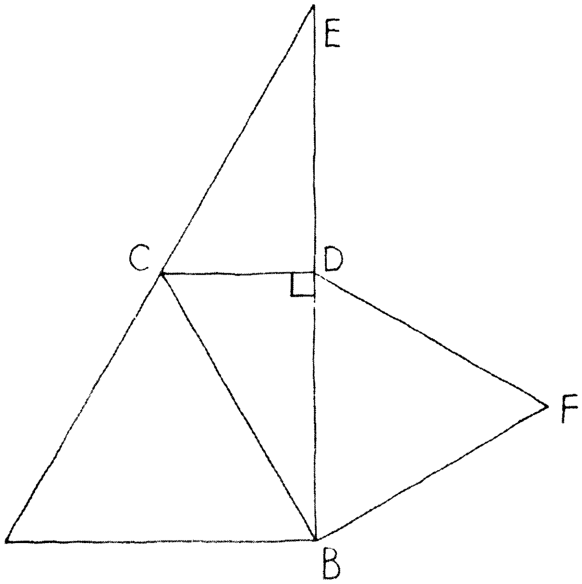
E.5 Symmetry

a Make a skeleton of a regular tetrahedron, using drinking straws and pipe-cleaners. (A tetrahedron has four faces, each of which is a triangle. In a regular tetrahedron, each face is an equilateral triangle.)



Using pieces of straw or coloured wool, join the midpoint of one edge of the skeleton to the two vertices at either end of the opposite edge. Would you agree that the triangle formed by the two lines and the opposite edge are part of a plane of symmetry of the tetrahedron? How many other planes of symmetry can you find using this method?

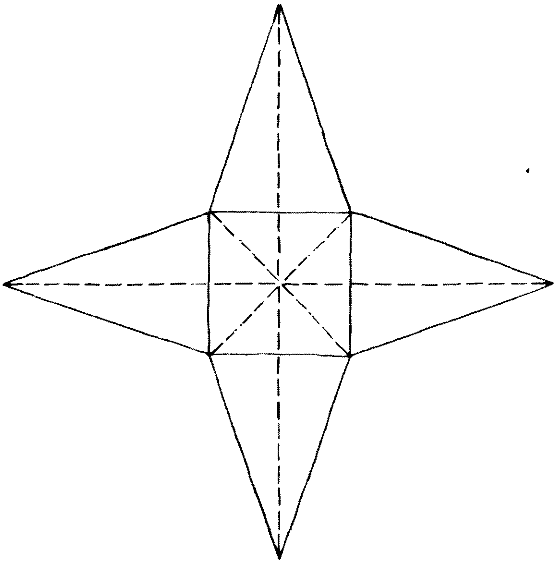
b Make a model of a tetrahedron from paper. Make two models of half a tetrahedron from paper: the net of the half tetrahedron is shown on the right.



Your teacher will show you the net of a whole tetrahedron. Fit the two halves together and by comparing this model with the whole tetrahedron, try to decide how many planes of symmetry a regular tetrahedron has. Incidentally you will have to think hard about how to make up the half tetrahedra if they are to fit together to form a whole tetrahedron. (The measurements of the net of the half tetrahedron can be worked out if you think that ABC is an equilateral triangle; and that BCD and CDE are each half an equilateral triangle; and that certain lines have to be equal in length if they are to be stuck together. If you make the half tetrahedra the same size as the straw skeleton, you will be able to fit the half tetrahedra inside the skeleton and check the position of the planes of symmetry.

E.6 Symmetry

Below you will see the net of a square pyramid. Make this pyramid from paper or from drinking straws and pipe cleaners, using any scale you like. Can you use the dotted lines which have been drawn on the net to forecast how many planes of symmetry your pyramid will have.



E.5 and E.6

Notes for the teacher

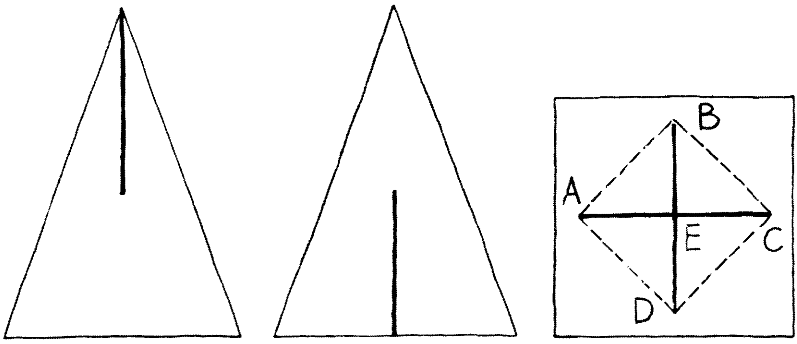
Pupils can be asked to make pyramids on other shaped bases – triangles, regular pentagons, regular hexagons, and so on – and to find their planes of symmetry.

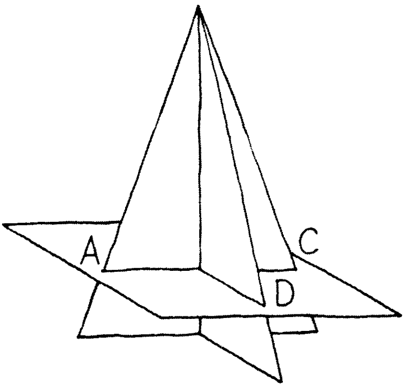
Time can be saved, especially for children who are slow at making models, if wooden or plastic models of regular and semi-regular solids are made available in the class room. The teacher might make some of his own from paper or some may be made from paper by other children as a separate assignment. It is not easy to find all the planes of symmetry of a solid such as a regular tetrahedron: using straws and wool, it is more likely that the pupil will discover that a regular tetrahedron does not have twelve planes of symmetry, a result easily got by abstract reasoning, but only six, or one through each edge.

It is not difficult to make dissections of certain solid figures which can help us study their planes of symmetry. An example is given below of how to make a dissection of a square pyramid; the method can be adapted for other shapes.

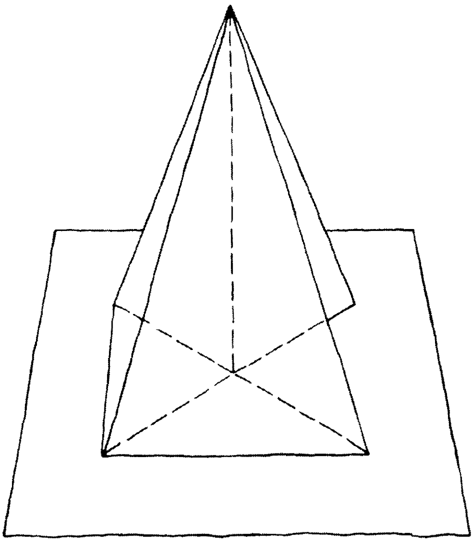
Two congruent isosceles triangles are cut from card and on each is drawn the bisector of the vertical angle. On each triangle a cut is made along this line as far as its mid-point, but on one triangle the cut starts at the base, on the other it starts at the vertex. The two triangles can now be slid together along the cuts so that they intersect each other at right angles. Each cut must be wide enough to take the thickness of the card.

This assembly is now pushed through two slots cut at right angles to each other in a square piece of card: in the diagram, ABCD is the square base of the pyramid. The slots in this base are slightly shorter than the bases of the two triangles.





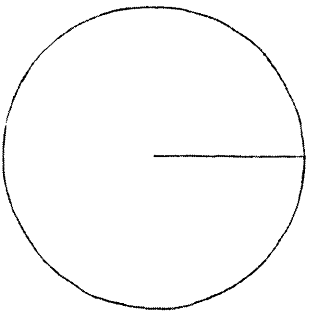
The two triangles are treated as being part of the two planes containing the diagonals of the base of the pyramid. The complete pyramid can be built up from four quarters; the net of one of these appears on the right. The triangle PQR is right-angled and isosceles, and RT is equal to the vertical height of the pyramid. Both PS and QS are equal to the slant height of the pyramid measured along its slant edges. Each net when folded and stuck will represent a quarter of the pyramid. Two such quarters can be pushed into the spaces between the triangles to show a plane of symmetry passing through one diagonal of the base (diagram below).

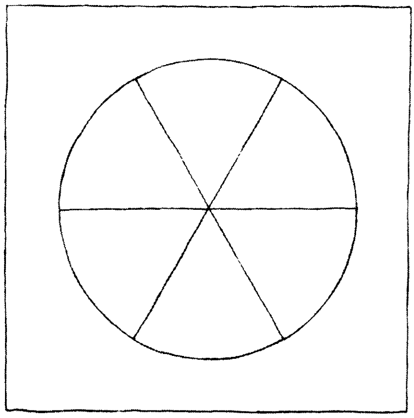


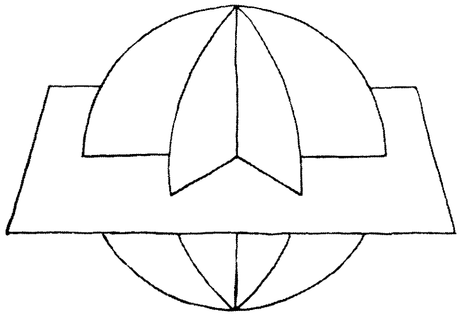
Plane of symmetry

It will help the child in copying the net of a solid if lines of equal length on the net are drawn using the same colour. Such nets might be prepared by the teacher and given to children to copy at his discretion.

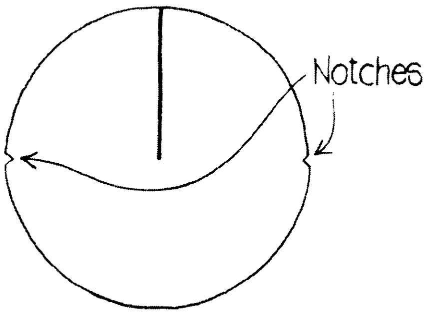
Some of the planes of symmetry of a sphere can be shown in a similar way. Three circular discs of equal radius are cut from card and are slit along the line of their diameters, each slit being a little longer than half the diameter. The discs are slid together to intersect one another and they are then pushed through three slots in another piece of card. These slots are made along the diameters of a circle whose radius is the same as that of the discs; they should intersect at an angle of about sixty degrees.

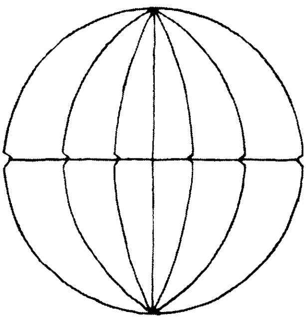






Alternatively, a number of circular discs of equal radius are cut from card and are made to intersect in the same way. Sellotape can be used to fix them together permanently. Two small notches are cut on the edge of each disc, at the extremities of the diameter at right angles to the diameter along which the disc has been slit. When the discs are 'opened out', a light elastic band can be fitted into the notches to show a further plane of symmetry. (Do the notches have to be made on the diameter at right angles to the slits, or would *any* diameter serve? How many planes of symmetry does a sphere have?)





E.7 Symmetry

Study carefully a regular prism. Does it have any planes of symmetry? Is there any connection between the number of planes of symmetry of the prism and the number of axes of symmetry of its cross-section?

Make a model of a simple prism to show its planes of symmetry.

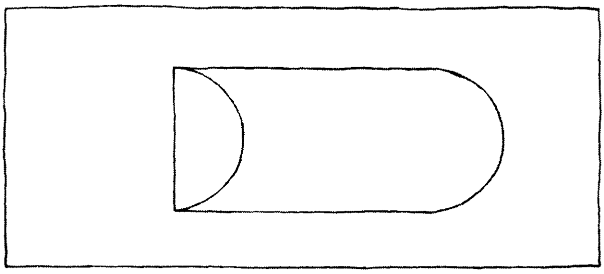
E.7

Notes for the teacher

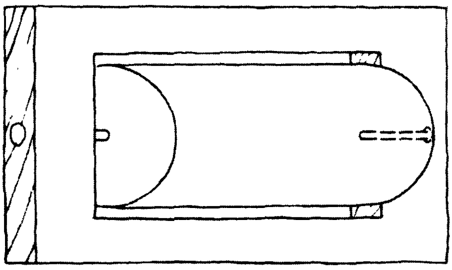
If the pupil tabulates for several prisms the number of planes of symmetry each possesses and the number of axes of symmetry of the cross-section (taken parallel to the ends of the prism) he should find a simple relationship between the two numbers. He may on the other hand find too simple a relationship; the numbers are not in fact the same. Every axis of symmetry certainly indicates the position of a plane of symmetry but there is an additional plane of symmetry midway between the two end faces of the prism and parallel to them.

Examples of prisms can be found among food cartons and children can make a collection of these; or they may make their own collection of prisms from thin card: the nets are simple to draw.

The planes of symmetry of a cylinder can be shown by cutting a rectangular hole in a piece of card, in which the cylinder will just fit, the dimensions of the hole being equal to the height of the cylinder and to the diameter of its cross-section.



If a piece of wood is substituted for the card and if nails are driven through the wood to penetrate the centres of the ends of any tin which can be emptied without removing an end, e.g., a soft drink tin, a more permanent model can be made.



The method of cutting a hole in card to fit a solid object, thereby showing a plane of symmetry of the object is a general one and its use is not restricted to solids such as prisms which may have several planes of symmetry. We may for example show the single plane of symmetry of a model car by this method.