

Introduction to Sequences and Series

The aim of this activity is to emphasise the importance of understanding how the notation relates to the sequence.

Write two sequences on the board such as:

2 4 6 8 ...

2 4 8 16 ...

Ask students to say what features are the same and what is different about these two sequences.

Write their response on the board using mathematical notation e.g.

'both sequences start with 2' becomes $u_1 = 2$

'both sequences have 4 as the second term' becomes $u_2 = 4$.

'in one sequence 2 is added each time to get the next term and the other is multiplied by 2' gives the opportunity to write two inductive definitions. There is no need to mention APs and GPS at this stage as this is all about exploring the structure and notation of sequences.

'The first sequence is the 2 times table' becomes $u_n = 2n$ and so algebraic definitions are introduced.

Words such as 'divergent' can be introduced if students comment that 'both sequences go on forever and get bigger and bigger'.

Then repeat for two more sequences such as:

1 3 7 15 ...

15 11 7 3 ...

Continue with a range of sequences that have similarities and differences until students are familiar with the notation. Then give them 2 sequences with some similarities and differences written in notation and ask students to state what the notation tells them about the sequences.

E.g. 3 6 12 15

 2 7 12 17

Same

$$u_3 = 12$$

$$S_2 = 9$$

Different

$$u_1 = 3; \quad u_1 = 2$$

$$S_4 = 36; \quad S_4 = 38$$

This is also a good opportunity to introduce any notation that has not occurred so far e.g. sum of a series.

Another pair could be:

 2 5 11 23 ...

 1 3 7 15 ...

Same

$$u_{n+1} = 2u_n + 1$$

Divergent

Different

$$S_3 = 18; \quad S_3 = 11$$

$$u_1 = 2; \quad u_1 = 1$$

Repeat for other sequences - even going back to some of the sequences done earlier - ask students what the notation that was written down, said about the sequences.

Then give students a sequence and ask them to write down as much as they can about the sequence using sequence notation.

Give students an algebraic or an inductive definition and ask them to write down the first three terms, or the fifth term, etc. Giving an inductive definition without a starting term can highlight the need for a starting number when defining a sequence inductively.

Finally give the students all, or some, of the boxes on the sheet below. Students have to decide for which sequences $u_5 = 10$ is true and give justification for their decision.

For which sequence is $u_5 = 10$

$$u_{n+1} = 2u_n$$

$$u_4 = 5$$

$$u_{n+1} = \frac{1}{2}u_n - 6$$

$$u_3 = 76$$

$$u_n = 4n - 10$$

$$u_{n+1} = 4u_n - 10$$

$$u_6 = 30$$

$$u_n = 2(n - 1)$$

$$u_{n+1} = \frac{1}{2}u_n$$

$$u_1 = 160$$

$$u_n = n(n - 2)$$