

# Exploring Pure Maths

The MEW Group



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## Introduction

The introduction of investigations and problem solving into the assessment component of GCSE has led to more students of mathematics exploring mathematics ‘under their own steam’. It is therefore sensible to introduce investigational work into the sixth form classroom, so that students can develop their skills further. This pack is designed to provide short problems to stimulate discussion of mathematics amongst students and between students and their teachers; and to provide some longer investigations that search more deeply into the students’ understanding of the topics in GCE Advanced level mathematics.

Mathematics is a subject to explore in its own right as well as to use in problem solving. The algebra, calculus and trigonometry in GCE Advanced level are essentially mathematical methods and there are several resources available for teachers to show the use of these methods in ‘real problem solving situations’. However, investigations in pure mathematics can provide an enjoyable experience of ‘playing’ with mathematics. They need not be long and difficult proofs, indeed problems that might be thought of as recreational can often challenge and develop the students’ grasp of a topic.

Investigational work can be used to develop very important problem solving skills. A list of skills common to many investigations is given below:

- extract information
- gather information
- organise information
- summarise information
- form patterns
- extend and generalise
- test the generalisation
- explain the generalisation

It is difficult, if not impossible, to teach these skills formally. They are learnt by ‘doing’ investigations and not by ‘watching’. Investigations promote inventiveness of thought and help students to be logical and systematic in their approach to problem solving. Furthermore, work of this type gives students the opportunity to develop both oral and written skills.

### **The student material is in three parts:**

**Part 1** consists of 90 problems designed to test students’ understanding of some of the basic concepts of mathematics. We have called this section ‘*What and Why. . ?*’, to reflect the aim of the problems which is to help expose common misunderstandings and thus help students to resolve them. These short problems could be used by students individually, but we would suggest that they are better used if tackled as a group activity because this will encourage discussion. It is through discussing their misunderstandings that students can best resolve them. Each problem may take a few minutes to answer and several could be fitted into a lesson. The 90 problems can be classified into eight loose groupings identified in the table overleaf.

Problem numbers	Area of mathematics
1 — 9	Proofs and fallacies
10— 15	Trigonometry
16—37	Algebra and functions
38—61	Limits and calculus
62—66	Numerical methods
67—72	Vectors
73—85	Complex numbers
86—90	Matrices

Within each band of problems, the level of difficulty and amount of required background knowledge increases as the problem numbers increase.

**Part 2** contains 20 longer investigations. The first few of these investigations are suitable for use with GCSE level students as coursework. This was a deliberate policy when planning the pack. Investigations are not an activity to use at the end of term (or the end of year); they should be an integral part of the learning process. Furthermore students could be encouraged to solve these problems using higher level mathematical skills than were available to them at GCSE.

**Part 3** provides some photocopiable ‘hints and nudges’ for students. These should be given out if a group gets stuck while tackling an investigation. They provide a more structured route through an investigation for those who need it.

The pack is designed to provide material for the whole GCE Advanced level range. Most of the investigations can be tackled with different levels of mathematics so that most students can improve their problem solving skills irrespective of their ability level. For the ‘*What and Why.. ?*’ problems, pre-knowledge of the concept or skill will be required so that the problems can be used as revision or to test conceptual understanding.

Inevitably finding teaching time for using investigations in the sixth form is going to be difficult. Over the next few years GCE Advanced level syllabuses and assessment will develop to incorporate and build upon the good practice of GCSE. However, in the short term until these changes occur, we know that many teachers are looking for investigational work to continue to stimulate their students in the way that some coursework activities have stimulated them. One of the main burdens of investigations at GCSE is the marking of student work, and teachers are inevitably concerned about the burden continuing into Advanced level. However we believe that the educational benefits to students of problem solving and investigations are enormous and that these activities should be encouraged in the sixth form with or without any formal assessment. We hope that this pack will allow teachers and students to enjoy working together on problems and investigations that really challenge their understanding of fundamental mathematical ideas.

The MEW (Mathematics in the Everyday World) Group is a group of mathematics teachers who meet regularly to produce material for the various components of GCE Advanced level in Mathematics. There are many textbooks which contain the theory of pure mathematics, statistics, mechanics and decision mathematics, but few contain tests of conceptual understanding and investigations. One of the aims of the MEW Group is to provide extra resources for teachers of these subjects so as to fill the gap. *Exploring Pure Maths* is the

second publication of the group and is intended for all courses of mathematical methods at school, college, university or polytechnic at the GCE Advanced level of mathematics. It is part of a package with *Exploring Mechanics* and *Exploring Statistics*. We hope that you and your students enjoy using all three packs.

The authors are indebted to Sharon Ward for typing the draft manuscript, to our families for releasing us for weekends to develop the materials and to the publishers for encouraging us to continue after the publication of *Exploring Mechanics*.

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