Further Pure Mathematics 1

Complex Numbers

Section 1: Introduction to complex numbers

Multiple Choice Test

1) (3 + 4j) – (2 – j) =

(a) 5 + 5j
(b) 1 + 5j
(c) 5 + 3j
(d) 1 + 3j
(e) I don't know

2) [(1 – 2j) + (1 + j)](-3 + j) =

(a) -5 + 5j
(b) -7 – j
(c) -7 + 5j
(d) 5 – 5j
(e) I don't know

3) The roots of the equation

$z^2 + 6z + 10 = 0$

are

(a) 3 + j, 3 – j
(b) 3 + 2j, 3 – 2j
(c) -3 + 2j, -3 – 2j
(d) -3 + j, -3 – j
(e) I don't know

4) Given that $p + q\text{j} = \dfrac{1}{12 - 5\text{j}}$, the values of p and q are given by

(a) $p = \dfrac{12}{169}, q = \dfrac{5}{169}$
(b) $p = \dfrac{12}{169}, q = -\dfrac{5}{169}$
(c) $p = \dfrac{12}{119}, q = \dfrac{5}{119}$
(d) $p = \dfrac{12}{119}, q = -\dfrac{5}{119}$
(e) I don't know

5) Which of the following complex numbers is not equal to the others?

(a) 2 – 3j
(b) $\dfrac{13}{2 + 3\text{j}}$
(c) $\dfrac{13}{2 - 3\text{j}}$
(d) $\dfrac{3 + 2\text{j}}{\text{j}}$
(e) I don't know

6) $z = \frac{3+4\text{j}}{2-3\text{j}}$. The complex number w which satisfies the equation $zw = 1$ is

(a) $w = \frac{6+17\text{j}}{25}$

(b) $w = \frac{-6-17\text{j}}{25}$

(c) $w = \frac{-6-17\text{j}}{5}$

(d) $w = \frac{6+17\text{j}}{5}$

(e) I don't know

7) The solution of the equation

$$(3 - \text{j})(z + 4 - 2\text{j}) = 10 + 20\text{j}$$

is

(a) $z = 1 - 3\text{j}$

(b) $z = -3 + 9\text{j}$

(c) $z = 46 - 52\text{j}$

(d) $z = 5 + 5\text{j}$

(e) I don't know

8) Which of the following is NOT true?

(a) $\text{j}^4 = 1$

(b) $\frac{1}{\text{j}^3} - \text{j} = 0$

(c) $\frac{1}{\text{j}} + \text{j}^3 = 0$

(d) $\frac{1}{\text{j}^2} = \text{j}^2$

(e) I don't know

9) The values of a and b (with $a > 0$) which satisfy

$$(a + b\text{j})^2 = 5 + 12\text{j}$$

are

(a) $a = 3$, $b = -2$

(b) $a = 3$, $b = 2$

(c) $a = 2$, $b = 3$

(d) $a = 2$, $b = -3$

(e) I don't know

10) Which of the following statements is NOT true?

(a) $\frac{z}{z^*}$ is real

(b) $z + z^*$ is real

(c) zz^* is real

(d) $z - z^*$ is pure imaginary

(e) I don't know

Further Pure Mathematics 1

Solutions to Multiple Choice Test

1) The correct answer is (b)

$$\begin{aligned} 3+4j-(2-j) &= 3+4j-2+j \\ &= (3-2)+(4j+j) \\ &= 1+5j \end{aligned}$$

2) The correct answer is (a)

$$\begin{aligned} \left[(1-2j)+(1+j)\right](-3+j) &= (2-j)(-3+j) \\ &= -6+3j+2j-j^2 \\ &= -6+5j+1 \\ &= -5+5j \end{aligned}$$

3) The correct answer is (d)

$$\begin{aligned} z &= \frac{-6\pm\sqrt{36-4\times1\times10}}{2} \\ &= \frac{-6\pm\sqrt{-4}}{2} \\ &= \frac{-6\pm2j}{2} \\ &= -3\pm j \end{aligned}$$

4) The correct answer is (a)

$$\begin{aligned} p+qj &= \frac{1}{12-5j} \\ &= \frac{12+5j}{(12-5j)(12+5j)} \\ &= \frac{12+5j}{144+25} = \frac{12}{169}+\frac{5}{169}j \end{aligned}$$

So $p=\frac{12}{169}$, $q=\frac{5}{169}$.

5) The correct answer is (c)

$$\frac{13}{2-3j} = \frac{13(2+3j)}{(2-3j)(2+3j)} = \frac{26+39j}{13} = 2+3j$$

$$\frac{13}{2+3j} = \frac{13(2-3j)}{(2+3j)(2-3j)} = \frac{26-39j}{13} = 2-3j$$

$$\frac{3+2j}{j} = \frac{(3+2j)j}{-1} = \frac{3j-2}{-1} = 2-3j$$

So $\frac{13}{2-3j}$ is not equal to the others.

6) The correct answer is (b)

$$\begin{aligned} w &= \frac{2-3j}{3+4j} \\ &= \frac{(2-3j)(3-4j)}{(3+4j)(3-4j)} \\ &= \frac{6-17j-12}{25} \\ &= \frac{-6-17j}{25} \end{aligned}$$

7) The correct answer is (b)

$$(3-j)(z+4-2j) = 10+20j$$

$$\begin{aligned} z+4-2j &= \frac{10+20j}{3-j} \\ &= \frac{(10+20j)(3+j)}{(3-j)(3+j)} \\ &= \frac{30+70j-20}{10} \\ &= 1+7j \end{aligned}$$

$$\begin{aligned} z &= 1+7j-4+2j \\ &= -3+9j \end{aligned}$$

8) The correct answer is (c)

$$i^4 = (i^2)^2 = (-1)^2 = 1$$

$$\frac{1}{j^3} - j = \frac{j}{j^4} - j = \frac{j}{1} - j = 0$$

$$\frac{1}{j^2} = \frac{j^2}{j^4} = \frac{j^2}{1} = j^2$$

$$\frac{1}{j} + j^3 = \frac{j^3}{j^4} + j^3 = \frac{j^3}{1} + j^3 = 2j^3 = -2j$$

9) The correct answer is (b)

$$(a + bj)^2 = 5 + 12j$$

$$a^2 + 2abj - b^2 = 5 + 12j$$

Equating imaginary parts: $2ab = 12$

$$b = \frac{6}{a}$$

Equating real parts: $a^2 - b^2 = 5$

$$a^2 - \frac{36}{a^2} = 5$$

$$a^4 - 36 = 5a^2$$

$$a^4 - 5a^2 - 36 = 0$$

$$(a^2 - 9)(a^2 + 4)$$

Since a is real and positive, $a = 3$, and therefore $b = 2$.

10) The correct answer is (a)

Let $z = x + jy$, so $z^* = x - jy$

$z + z^* = x + jy + x - jy = 2x$ so $z + z^*$ is real

$z - z^* = x + jy - (x - jy) = 2jy$ so $z - z^*$ is pure imaginary

$zz^* = (x + jy)(x - jy) = x^2 + y^2$ so zz^* is real

$\frac{z}{z^*} = \frac{z^2}{zz^*}$ zz^* is real but z^2 is not, so $\frac{z}{z^*}$ is complex.