Further Pure Mathematics 1

Complex Numbers

Section 1: Introduction to complex numbers

Exercise

1. Find the roots of the following equations:
   (i) $z^2 + 25 = 0$ (ii) $4z^2 + 9 = 0$
   (iii) $z^2 - 2z + 2 = 0$ (iv) $4z^2 + 4z + 5 = 0$

2. In each of the following cases find
   a) $z_1 + z_2$ b) $z_1 - z_2$ c) z_1z_2 d) z_1^*
   e) z_2^* f) $z_1^* + z_2^*$ g) $z_1^* - z_2^*$ h) $z_1^*z_2^*$

   (i) $z_1 = 2 + 3j$; $z_2 = 1 - 2j$
   (ii) $z_1 = -2j$; $z_2 = 3 + j$
   What do you notice about the results?

3. Given that $z = (a + j)^4$ where a is real, find values for a such that
   (i) z is real,
   (ii) z is wholly imaginary.

4. Given that $a + bj$ is the conjugate of $(a + bj)^2$ find **all** possible pairs of values for a and b.

5. Simplify and write in the form $a + bj$:
   (i) $\dfrac{1}{3+2j} + \dfrac{1}{3-2j}$
   (ii) $3 + j + \dfrac{4}{3-j}$
   (iii) $\dfrac{3}{1-j} - \dfrac{2j}{2+j}$

6. Find values for a and b given that:
   (i) $(a + bj)(2 + j) \equiv a - 3j$
   (ii) $(a + j)(4 - bj) \equiv 3b + 2aj$

7. By writing $(a + bj)^2 = 3 - 4j$, find values for a and b and hence find the square roots of $3 - 4j$.

8. Find the values of p and q given that one root of the equation $z^2 + pz + q = 0$ is:
   (i) $2 - j$ (ii) $1 - 3j$
   (iii) $2j$ (iv) $5 - 3j$

9. Given that $\dfrac{5}{a+bj} + \dfrac{2}{1+3j} = 1$, where a and b are real, find the values of a and b.