

## Overs and Unders

For teachers who like an activity-based approach to the study of statistics and probability, Overs and Unders is a rich introductory activity that can be studied by students with various levels of mathematical maturity. It has been used in Queensland from upper primary school to Year 11 Maths A.

Recently the University of Queensland sponsored a 'Fun of Mathematics' night. Bill Simpson presented Over and Unders in a richer way than I had previously seen. In this article I will try to capture the flavour of Bill's presentation.

### Materials

Two dice; one large table; five poker chips for each player plus fifty or so chips for the bank; one white board marker. Optional: armband, croupier's eyeshade, bow-tie, hair slicked back.

### The activity

*Bill is at the front of the room, dressed like a croupier, behind a large table – not too large, as it is important that he can easily reach all parts of the table. On the table directly in front of Bill is a white board marker and piles of poker chips.*

[In the old days you could often find a game of Over and Unders at the local school fete, being run by one of the parents. It was illegal but this was generally ignored as the gambling activity raised money for the school.]

'Come on down, come on down. You've got to be in it to win it. If you don't speculate you can't accumulate.'

*The students gather around the table.*

'Ladies and gents, the name of the game is Over and Unders. Here is how you play.'

*Bill takes the white board marker and draws two lines on the table, dividing it into three sections. In one section he writes 'Under 7', in the middle section he writes, '7' and in the remaining section he writes 'Over 7'. Bill looks around furtively.*

'I hope the boss doesn't catch me doing this. Now I am going to toss two dice. There are three possible outcomes – the total is Under 7, the total equals 7 or the total is Over 7.'

*Bill rolls the dice a few times. The patter changes depending on the outcomes, e.g.*

'Under 7; and Under 7 again. It looks like Under 7 hot.' Under 7 again. There – an Over 7, that balances things up a bit.'

*Bill now explains how to bet.*

'Ladies and gents, boys and girls, you can bet on Under 7.' *Bill places a small stack of chips in the Under 7 section.*

'If the total of the two dice is under 7, ladies and gents, you win even money.' *He places a stack of equal height next to the first stack.*

If you bet on Over 7 and the total is over 7, ladies and gents, you win even money. *Bill repeats his actions in the Over 7s section.*

And if you bet on 7 and the total is 7, ladies and gents, I pay out not even money, not 2 to 1 but, ladies and gents, I pay 3 to 1 on a total of 7.' Bill places three stacks, each as high as the original one, next to the original stack.

'Just think about it – there are only three possibilities and on 7s I pay out 3 to 1. You walk away with four times your bet. Ladies and gents, I will probably lose my shirt on this, but that's the sort of generous guy I am.'

*Bill now places a few chips in each section, rolls the dice and demonstrates how he pays out, by placing a stack of chips from the bank next to the winning stack and then taking up the losing chips.*

*He then hands out five chips to each player.*

'OK, ladies and gents, place your bets, place your bets. You've got to be in it to win it. If you don't speculate you can't accumulate. You have to bet big to win big. Under 7 pays even money, over 7 pays even money and 7 pays 3 to 1.

*Students place one or more chips in various sections of the board.*

'If you don't bet big you can't win big. Any more bets? OK, the table is closed, no more bets.'

*Bill rolls the dice. They come up 4 and 2, say.*

'4 and 2, that's 6, that's under 7. Under 7 pays even money.'

*Bill places a stack of chips next to each winning bet and then clears away the losing bets.*

'OK, place your bets. If you don't speculate you can't accumulate...' etc

*The students play a couple of rounds.*

'I seem to be doing OK, so I'll tell you what. Being a magnanimous chap, I'm going to increase the payout on 7s to 4 to 1. That's right, ladies and gents, if 7s comes up, for every chip you bet on 7s you win 4 more chips if 7 is the total.

*The students play two or three more rounds.*

'Now, let's have a show of hands – who is well ahead? Who is about even? Who is losing? Who is broke?'

Usually the majority of students are broke, a few are about even and a few are well ahead. Bill points out how well the bank has fared, then collects up the chips and asks the students to return to their seats.

'Now that we've played a few rounds, and you have a feel for the game, I want you to think about a strategy that will give you the best chance of winning. You can talk it over with your mates if you wish. Once you have your strategy figured out I want you to write your strategy down.'

*The students are given a few minutes to do this. Common strategies are to wait for a run and then either bet on the same outcome ('the lucky streak theory') or on the opposite outcome ('the law of averages theory'). The class members should share some of the strategies.*

*'OK come on down, come on down. You've got to be in it to win it, if you don't speculate you can't accumulate, to win big you have to bet big.'*

*The above scenario is repeated. Usually it takes longer for the students to lose their chips as students are more cautious in their betting, but the final outcome is the same. The bank is the big winner. The students turn in their chips and return to their seats. Bill hands out a table of random numbers generated in a spreadsheet that uses the correct probability for each total of two dice. He explains how the table can be used to simulate the tossing of two dice.'*

*'What are the chances of winning each of the bets? We can use the Overs and Unders simulation sheet to find out.' Pick a starting spot and a direction at random. Select 50 numbers and record them as either Under, Over or Sevens. Then total up how many are in each category.'*

*Bill demonstrates how to do this on an overhead transparency. He then gives the students time to gather the data. The class can pool their results to find the experimental probability of each of the outcomes. Depending on the sophistication of the class, the theoretical probability can now be calculated and compared to the class results.*

*It is instructive to have students use the Overs and Unders simulation sheet to test their strategy. Here is an example.*

*'We can also use the table to see what happens after we get a run of three Under 7s. Move your finger along a randomly chosen row or column until you get three numbers under 7 in a row. Record the next number. Repeat this until you have recorded 20 numbers.'*

*The students carry out this simulation. The class results can be quickly collated on the board. The students will see that a run of Under 7s doesn't alter the chance of an Under 7 coming up next.*

*This activity closes with a discussion of who really wins these games, and what the almost certain outcome is for people who gamble often.*