

AIDAMS Mathematics

30-Question Practice Exam

Duration: 90 minutes

Choose exactly one answer for each question. Each question is worth one point, for a total of 30 points. Angles are measured in radians, and $\mathbb{N} = \{0, 1, 2, \dots\}$.

Contents

1	Notations for sets and set operations	2
2	Combinatorics and counting	2
3	Vectors, lines, and planes in space	3
4	Dot products, orthogonality, and distances	3
5	Parametric lines and Cartesian planes	4
6	Sequences	4
7	Limits of functions	5
8	Differentiation and convexity	5
9	Continuity and solving equations	6
10	Exponential and natural logarithm	6
11	Sine and cosine	7
12	Antiderivatives and differential equations	7
13	Integral calculus	8
14	Probability	8
	Detailed corrections	10

1 Notations for sets and set operations

Question 1. Subsets shared by two power sets

Correction

Let $A = \{a, b\}$ and $B = \{b, c\}$, where a, b, c are distinct. What is $\mathcal{P}(A) \cap \mathcal{P}(B)$?

- A. $\{b\}$.
- B. $\{\emptyset, \{b\}\}$.
- C. $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$.
- D. $\emptyset$.

Question 2. One witness or a witness for each input?

Correction

Let $E = \{-1, 0, 1\}$. Consider the statements

$$S : \forall x \in E, \exists y \in E, x + y = 0, \quad T : \exists y \in E, \forall x \in E, x + y = 0.$$

Which assessment is correct?

- A. Both S and T are true.
- B. Both S and T are false.
- C. S is false and T is true.
- D. S is true and T is false.

2 Combinatorics and counting

Question 3. A restricted beginning of a binary word

Correction

How many words in $\{0, 1\}^8$ contain exactly three ones and have different first and second symbols?

- A. $2\binom{6}{2} = 30$.
- B. $\binom{6}{2} = 15$.
- C. $\binom{8}{3} = 56$.
- D. $2\binom{6}{3} = 40$.

Question 4. A committee and its chair

Correction

A club has nine members, including distinct members a and b . An outcome consists of a four-member committee and a chair chosen from that committee. Members a and b may not both be on the committee. How many outcomes are possible?

- A. $4\binom{9}{4} = 504$.
- B. $\binom{9}{4} - \binom{7}{2} = 105$.
- C. $4\left(\binom{9}{4} - \binom{7}{2}\right) = 420$.
- D. $4\binom{9}{4} - \binom{7}{2} = 483$.

3 Vectors, lines, and planes in space

Question 5. When does a third vector complete a basis?

Correction

Let $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be a basis of space, and set

$$\mathbf{u} = \mathbf{e}_1 + 2\mathbf{e}_2, \quad \mathbf{v} = \mathbf{e}_1 - \mathbf{e}_2, \quad \mathbf{w}_\lambda = \mathbf{e}_1 + \lambda\mathbf{e}_2 + (\lambda - 2)\mathbf{e}_3.$$

For which real values of λ is $(\mathbf{u}, \mathbf{v}, \mathbf{w}_\lambda)$ a basis of space?

- A. Every $\lambda \in \mathbb{R}$.
- B. Exactly $\lambda \neq 2$.
- C. Exactly $\lambda = 2$.
- D. Exactly $\lambda \neq 0$.

Question 6. From a displacement to a point

Correction

In a coordinate frame, let $A = (-1, 2, 1)$, $B = (1, 3, 0)$, and $C = (0, 0, 2)$. The point M satisfies

$$\overrightarrow{AM} = 2\overrightarrow{AB} - \overrightarrow{AC}.$$

What are the coordinates of M ?

- A. $(2, 6, -2)$.
- B. $(3, 4, -3)$.
- C. $(4, 2, 0)$.
- D. $(2, 6, 0)$.

4 Dot products, orthogonality, and distances

Question 7. The other diagonal of a parallelogram

Correction

A parallelogram has adjacent side vectors $\mathbf{u}, \mathbf{v}$ with $\|\mathbf{u}\| = 3$ and $\|\mathbf{v}\| = 4$. Its diagonal $\mathbf{u} + \mathbf{v}$ has length $\sqrt{37}$. What is the length of the diagonal $\mathbf{u} - \mathbf{v}$?

- A. 5.
- B. $\sqrt{37}$.
- C. $\sqrt{19}$.
- D. $\sqrt{13}$.

Question 8. The foot of a perpendicular on a line

Correction

In an orthonormal frame, let $A = (0, 1, 0)$, $M = (2, 2, 3)$, and

$$\ell = \{A + t(1, 2, 0) : t \in \mathbb{R}\}.$$

Which point is the orthogonal projection of M onto ℓ ?

- A. $(2, 5, 0)$.
- B. $(0, 1, 0)$.
- C. $(4/5, 13/5, 0)$.
- D. $(2, 2, 0)$.

5 Parametric lines and Cartesian planes

Question 9. Intersecting a line with a plane

Correction

Consider the line and plane

$$\ell = \{(1+t, -1+2t, 2-t) : t \in \mathbb{R}\}, \quad P : 2x - y + z = 7.$$

What is their intersection point?

- A. $(-1, -5, 4)$.
- B. $(3, 3, 0)$.
- C. $(0, -3, 3)$.
- D. $(-1, -3, 4)$.

Question 10. A parameter in a system of planes

Correction

For $\lambda \in \mathbb{R}$, consider the three planes

$$P : x - y + 2z = 3, \quad Q_\lambda : 2x - 2y + 4z = \lambda, \quad R : x + y = 1.$$

Which description of their common solutions is correct?

- A. A single point when $\lambda = 6$, and no solutions otherwise.
- B. A line when $\lambda = 6$, and no solutions otherwise.
- C. A line for every real λ .
- D. The whole plane P when $\lambda = 6$, and no solutions otherwise.

6 Sequences

Question 11. An invariant interval and a limiting value

Correction

Let $(u_n)_{n \geq 0}$ satisfy $u_0 = 1$ and

$$u_{n+1} = \frac{4u_n}{1+u_n} \quad (n \geq 0).$$

Which description is correct?

- A. The sequence decreases to 0.
- B. The sequence increases to $+\infty$.
- C. The sequence increases to 3.
- D. The sequence is constant at 3 because 3 is a fixed point.

Question 12. Convergence and a rescaled error

Correction

A real sequence satisfies $|u_n + 2| \leq 2/n$ for every integer $n \geq 1$. Define $v_n = n(u_n + 2)$. Which statement must hold?

- A. $u_n \rightarrow -2$ and $v_n \rightarrow 0$.
- B. The sequence (u_n) is monotone.
- C. The sequence (v_n) is unbounded.
- D. $u_n \rightarrow -2$, while (v_n) is bounded but need not converge.

7 Limits of functions

Question 13. Cancellation at negative infinity

Correction

What is

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 - 6x + x})?$$

- A. 0.
- B. 3.
- C. -3 .
- D. $+\infty$.

Question 14. A leading coefficient and a finer limit

Correction

For $a \in \mathbb{R}$, define

$$f_a(x) = \frac{ax^2 - 2x}{3x^2 + 4x}, \quad x \in \mathbb{R} \setminus \{0, -4/3\}.$$

Suppose $\lim_{x \rightarrow +\infty} f_a(x) = 2$. Which pair gives a and $\lim_{x \rightarrow +\infty} x(f_a(x) - 2)$?

- A. $a = 2$ and the second limit is $-10/3$.
- B. $a = 6$ and the second limit is 0.
- C. $a = 6$ and the second limit is $-10/3$.
- D. $a = 6$ and the second limit is -10 .

8 Differentiation and convexity

Question 15. Zeros of a second derivative

Correction

Let $f(x) = x^6 - 5x^4$ for $x \in \mathbb{R}$. Which set consists exactly of the abscissas of its inflection points?

- A. $\{0\}$.
- B. $\{-\sqrt{2}, 0, \sqrt{2}\}$.
- C. $\emptyset$.
- D. $\{-\sqrt{2}, \sqrt{2}\}$.

Question 16. An endpoint can share the maximum

Correction

Let $f(x) = x^3 - 12x$ on $[-3, 4]$. Which set consists exactly of all points where f attains its absolute maximum?

- A. $\{-2, 4\}$.
- B. $\{-2\}$.
- C. $\{-3, 2\}$.
- D. $\{2, 4\}$.

9 Continuity and solving equations

Question 17. Existence, location, and uniqueness

Correction

Consider the equation $x^3 + 2x - 2 = 0$ on $\mathbb{R}$. Which assertion is correct?

- A. The equation has no real root.
- B. The equation has at least two roots in $[0, 1]$.
- C. The equation has exactly one real root, which belongs to $]3/4, 4/5[$.
- D. The equation has exactly one real root, which belongs to $]0, 1/2[$.

Question 18. Filling a removable discontinuity

Correction

For $c \in \mathbb{R}$, define $f : [-5, +\infty[\rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{\sqrt{x+5}-3}{x-4}, & x \neq 4, \\ c, & x = 4. \end{cases}$$

Which value of c makes f continuous at 4?

- A. $c = 0$.
- B. $c = 1/3$.
- C. $c = 6$.
- D. $c = 1/6$.

10 Exponential and natural logarithm

Question 19. A logarithmic equation and its domain

Correction

What is the set of real solutions of

$$\ln(x-2) + \ln(x+2) = \ln 5?$$

- A. $\{-3, 3\}$.
- B. $\{3\}$.
- C. $\{-3\}$.
- D. $\{\sqrt{5}\}$.

Question 20. Two allowed ranges for an exponential

Correction

What is the set of real solutions of

$$e^{2x} - 5e^x + 6 \geq 0?$$

- A. $] -\infty, \ln 2] \cup [\ln 3, +\infty[$.
- B. $[\ln 2, \ln 3]$.
- C. $] -\infty, 2] \cup [3, +\infty[$.
- D. $] -\infty, \ln 2[\cup] \ln 3, +\infty[$.

11 Sine and cosine

Question 21. A double-angle equation on one full turn

Correction

What is the solution set of $\cos(2x) = -\sin x$ on $[0, 2\pi]$?

- A. $\{\pi/6, 5\pi/6, 3\pi/2\}$.
- B. $\{7\pi/6, 11\pi/6\}$.
- C. $\{\pi/2, 7\pi/6, 11\pi/6\}$.
- D. $\{\pi/2, 3\pi/2\}$.

Question 22. An interior trigonometric maximum

Correction

Let $F(x) = 2\cos x - \cos(2x)$ for $x \in [0, \pi]$. Which statement gives its global maximum and where it is attained?

- A. The maximum is 2, attained at $x = \pi/2$.
- B. The maximum is $3/2$, attained only at $x = \pi/3$.
- C. The maximum is 3, attained at $x = 0$.
- D. The maximum is 1, attained only at $x = 0$.

12 Antiderivatives and differential equations

Question 23. A normalized logarithmic antiderivative

Correction

On $I =]-2, 2[$, let $f(x) = 2x/(x^2 - 4)$. Which expression is an antiderivative F of f throughout I and satisfies $F(0) = 3$?

- A. $F(x) = \ln(x^2 - 4) - \ln 4 + 3$.
- B. $F(x) = -\ln(4 - x^2) + \ln 4 + 3$.
- C. $F(x) = \ln(4 - x^2) + 3$.
- D. $F(x) = \ln(4 - x^2) - \ln 4 + 3$.

Question 24. An initial value prescribed away from zero

Correction

Which formula gives, for every $y_0 \in \mathbb{R}$, the solution on $\mathbb{R}$ of $y' = -4y + 12$ with $y(1) = y_0$?

- A. $y(x) = 3 + (y_0 - 3)e^{-4(x-1)}$.
- B. $y(x) = 3 + (y_0 - 3)e^{4(x-1)}$.
- C. $y(x) = 12 + (y_0 - 12)e^{-4(x-1)}$.
- D. $y(x) = 3 + y_0e^{-4(x-1)}$.

Question 25. The first time above a target level

Correction

A differentiable function $y : \mathbb{R} \rightarrow \mathbb{R}$ satisfies $y' = -2y + 10$ and $y(0) = 1$. For exactly which nonnegative times does $y(x) \geq 9/2$ hold?

- A. $x \in [(\ln 2)/2, +\infty[$.
- B. $x \in [0, 3(\ln 2)/2]$.
- C. $x \in [3(\ln 2)/2, +\infty[$.
- D. $x \in [3 \ln 2, +\infty[$.

13 Integral calculus

Question 26. Area when the upper curve changes

Correction

What is the total area between the graphs $y = x^2$ and $y = 2x$ for $0 \leq x \leq 3$?

- A. 0.
- B. $8/3$.
- C. $4/3$.
- D. $9/2$.

Question 27. A logarithmic endpoint in integration by parts

Correction

Evaluate $\int_0^{\ln 2} x e^x dx$.

- A. $2 \ln 2$.
- B. $2 \ln 2 - 2$.
- C. $\ln 2 - 1/2$.
- D. $2 \ln 2 - 1$.

14 Probability

Question 28. Tracing a defective component

Correction

A component comes from factory A with probability $2/5$ and from factory B with probability $3/5$. The defect probabilities are $1/20$ at A and $1/10$ at B . Given that a randomly selected component is defective, what is the probability that it came from A ?

- A. $1/4$.
- B. $2/5$.
- C. $1/20$.
- D. $3/4$.

Question 29. Strict and inclusive binomial thresholds

Correction

Let X count successes in six independent trials, each with success probability $1/2$. Let $k_>$ be the least integer in $\{0, \dots, 6\}$ such that $\mathbb{P}(X > k_>) \leq 7/64$. Let $k_\geq$ be the least integer in the same set such that $\mathbb{P}(X \geq k_\geq) \leq 7/64$. What is $(k_>, k_\geq)$?

- A. $(5, 4)$.
- B. $(4, 4)$.
- C. $(5, 5)$.
- D. $(4, 5)$.

Question 30. Scaling a sample mean and certifying a risk

Correction

The finite-valued variables $X_1, \dots, X_n$ are independent and identically distributed, with common mean 3 and variance 8. Set $M_n = (X_1 + \dots + X_n)/n$ and $Z_n = 2M_n - 1$. Let $n_{\min}$ be the least positive integer for which Chebyshev's bound certifies $\mathbb{P}(|M_n - 3| \geq 1/2) \leq 1/10$. Which triple $(\mathbb{E}[Z_n], \text{Var}(Z_n), n_{\min})$ is correct?

- A. $(5, 16/n, 320)$.
- B. $(5, 32/n, 320)$.
- C. $(5, 32/n, 321)$.
- D. $(3, 8/n, 80)$.

Detailed corrections

Answer key

Question	1	2	3	4	5	6	7	8	9	10
Answer	B	D	A	C	B	A	D	C	A	B
Question	11	12	13	14	15	16	17	18	19	20
Answer	C	D	B	C	D	A	C	D	B	A
Question	21	22	23	24	25	26	27	28	29	30
Answer	C	B	D	A	C	B	D	A	D	B

Notations for sets and set operations

Question 1 — Correct answer: B.

[Back to question](#)

A member of $\mathcal{P}(A) \cap \mathcal{P}(B)$ must be a subset of both A and B . Every element of that subset must therefore belong to $A \cap B = \{b\}$. Conversely, any subset of $\{b\}$ is a subset of both A and B . Hence

$$\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(\{b\}) = \{\emptyset, \{b\}\}.$$

This is B. Option A confuses the common element b with the subsets that form the elements of a power set. Option C lists the union of the two power sets and includes subsets that are not shared, such as $\{a\}$. Option D overlooks that the empty set is a subset of every set.

Question 2 — Correct answer: D.

[Back to question](#)

In S , the choice of y is made after the value of x is known, so it may depend on x . Choosing $y = -x$ works: the values 1, 0, and -1 all belong to E . Thus S is true. Statement T , however, requires one fixed value of y to work for every x . Taking $x = 0$ would force $y = 0$, whereas taking $x = 1$ would force $y = -1$. These requirements are incompatible, so T is false and D is correct. The other choices either reject the valid input-dependent witnesses for S or treat them as though they supplied a single witness for T .

Combinatorics and counting

Question 3 — Correct answer: A.

[Back to question](#)

The first two symbols must be either 01 or 10, giving two disjoint cases. Each case already contributes exactly one of the three required ones. Among the six remaining positions, choose the two positions that contain the other ones; their order is irrelevant because choosing the positions determines the word completely. Thus the count is

$$2\binom{6}{2} = 2\frac{6 \cdot 5}{2} = 30,$$

which is A. Option B counts only one of the two possible beginnings. Option C counts all words with three ones and ignores the restriction on the first two symbols. Option D puts three further ones in the remaining positions, producing four ones in total.

Question 4 — Correct answer: C.

[Back to question](#)

First choose the committee, without assigning a chair. There are $\binom{9}{4} = 126$ four-member committees in total. A forbidden committee contains both a and b , so its other two members are chosen from the seven remaining people, giving $\binom{7}{2} = 21$ forbidden committees. Hence there are $126 - 21 = 105$ admissible committees. Each has exactly four possible chairs, so the required number is

$$4\left(\binom{9}{4} - \binom{7}{2}\right) = 4 \cdot 105 = 420.$$

Therefore C is correct. Option A retains the forbidden committees, and B omits the choice of chair. Option D subtracts only one outcome per forbidden committee, although every such committee produces four forbidden choices of chair.

Vectors, lines, and planes in space

Question 5 — Correct answer: B.

[Back to question](#)

If $a\mathbf{u} + b\mathbf{v} + c\mathbf{w}_\lambda = \mathbf{0}$, comparison in the original basis gives

$$a + b + c = 0, \quad 2a - b + \lambda c = 0, \quad (\lambda - 2)c = 0.$$

When $\lambda \neq 2$, the last equation yields $c = 0$. The first two then give $b = -a$ and $3a = 0$, hence $a = b = c = 0$. The vectors are therefore independent and form a basis. When $\lambda = 2$, however, $\mathbf{w}_2 = \mathbf{u}$, so they cannot form a basis. Thus B describes exactly the admissible values. Option A includes this dependent case, whereas C selects only that case. Option D wrongly excludes $\lambda = 0$, for which the third component is -2 and independence holds, while wrongly admitting $\lambda = 2$.

Question 6 — Correct answer: A.[Back to question](#)

Subtract the coordinates of the starting point A to obtain the two displacements:

$$\overrightarrow{AB} = (2, 1, -1), \quad \overrightarrow{AC} = (1, -2, 1).$$

The required linear combination is consequently

$$\overrightarrow{AM} = 2(2, 1, -1) - (1, -2, 1) = (3, 4, -3).$$

These are vector coordinates, so add them to the coordinates of A : $M = (-1, 2, 1) + (3, 4, -3) = (2, 6, -2)$. This gives A. Option B stops at the displacement and does not account for the starting point. Option C results from adding $\overrightarrow{AC}$ instead of subtracting it. Option D mishandles the third component: it must be $1 + 2(-1) - 1 = -2$, not 0. No orthonormality assumption is needed for coordinate addition and scalar multiplication.

Dot products, orthogonality, and distances**Question 7 — Correct answer: D.**[Back to question](#)

Expand the squared length of the known diagonal using the dot product:

$$37 = \|\mathbf{u} + \mathbf{v}\|^2 = 3^2 + 4^2 + 2\mathbf{u} \cdot \mathbf{v}.$$

Thus $\mathbf{u} \cdot \mathbf{v} = 6$. For the other diagonal, the cross term changes sign, so

$$\|\mathbf{u} - \mathbf{v}\|^2 = 9 + 16 - 2 \cdot 6 = 13.$$

Its length is the positive square root $\sqrt{13}$, proving D. Option A drops the cross term and would require orthogonal sides, contrary to the nonzero dot product. Option B assumes that the two diagonals have equal lengths, which is not true here. Option C subtracts the dot product only once instead of the required factor of two.

Question 8 — Correct answer: C.[Back to question](#)

Any candidate projection on ℓ must have the form $H = (t, 1 + 2t, 0)$. Its displacement to M is $\overrightarrow{HM} = (2 - t, 1 - 2t, 3)$, which must be orthogonal to the line's direction $(1, 2, 0)$. Therefore

$$(2 - t) + 2(1 - 2t) = 0, \quad 4 - 5t = 0, \quad t = \frac{4}{5}.$$

This gives $H = (4/5, 13/5, 0)$, so C is correct. It lies on the line and satisfies the orthogonality condition, hence is the unique nearest point. Options A and B lie on the line but their displacements to M are not orthogonal to its direction. Option D simply drops the third coordinate and does not even lie on ℓ .

Parametric lines and Cartesian planes

Question 9 — Correct answer: A.

[Back to question](#)

Substitute all three coordinates of the line into the plane equation, using the same parameter throughout:

$$2(1+t) - (-1+2t) + (2-t) = 7 \iff 5-t = 7.$$

The unique solution is $t = -2$, which gives $(x, y, z) = (-1, -5, 4)$. Hence A is correct. Uniqueness also follows because the direction $(1, 2, -1)$ has nonzero dot product -1 with the plane normal $(2, -1, 1)$. Options B and C are points of the line at parameters 2 and -1 , respectively, but give $2x - y + z = 3$ and 6 , rather than 7 . Option D fails the line's second coordinate equation when the first and third coordinates force $t = -2$.

Question 10 — Correct answer: B.

[Back to question](#)

The left-hand side defining Q_λ is twice the left-hand side defining P . Thus any common point of these two planes must satisfy $\lambda = 2 \cdot 3 = 6$; for every other value they are parallel and disjoint. When $\lambda = 6$, the equation for Q_λ adds no restriction. Solve the remaining equations by setting $z = t$:

$$x - y = 3 - 2t, \quad x + y = 1 \implies (x, y, z) = (2 - t, t - 1, t).$$

As t ranges over $\mathbb{R}$, this describes a line with nonzero direction $(-1, 1, 1)$, establishing B. Option A overlooks the free parameter, C ignores the necessary consistency condition, and D ignores the additional restriction imposed by R .

Sequences

Question 11 — Correct answer: C.

[Back to question](#)

First prove that $0 < u_n < 3$ for every n . This holds at $n = 0$. If $0 < u < 3$, then $4u/(1+u) > 0$ and $4u/(1+u) < 3$ is equivalent to $u < 3$, so the interval is invariant. Moreover,

$$u_{n+1} - u_n = \frac{u_n(3 - u_n)}{1 + u_n} > 0.$$

The sequence is increasing and bounded above, hence converges to a finite limit L . Since $u_n \geq 1$, we have $L \geq 1$. Continuity gives $L = 4L/(1+L)$, or $L(L-3) = 0$. The lower bound excludes 0, leaving $L = 3$. Thus A reverses the variation, B ignores the proved upper bound, and D confuses a limiting equilibrium with an initial value: $u_0 = 1$ and $u_1 = 2$, so the sequence is not constant.

Question 12 — Correct answer: D.[Back to question](#)

The bound gives

$$-2 - \frac{2}{n} \leq u_n \leq -2 + \frac{2}{n}.$$

Both bounding sequences tend to -2 , so the squeeze theorem gives $u_n \rightarrow -2$. Multiplying the original absolute-value inequality by $n > 0$ yields $|v_n| \leq 2$, proving boundedness. There is no corresponding guarantee of convergence for (v_n) . For example, take $u_n = -2 + 2(-1)^n/n$. It satisfies the stated bound, whereas $v_n = 2(-1)^n$ alternates between -2 and 2 . This example also shows that (u_n) need not be monotone: $u_1 = -4$, $u_2 = -1$, and $u_3 = -8/3$. Consequently A wrongly strengthens an error bound to a vanishing rescaled error, B adds unsupported monotonicity, and C contradicts the uniform bound $|v_n| \leq 2$.

Limits of functions**Question 13 — Correct answer: B.**[Back to question](#)

For negative x , the square root is defined because $x^2 - 6x > 0$. The expression initially has the indeterminate form $+\infty - \infty$. Multiplication by the conjugate gives

$$\sqrt{x^2 - 6x} + x = \frac{-6x}{\sqrt{x^2 - 6x} - x} = \frac{6}{\sqrt{1 - 6/x} + 1}, \quad x < 0.$$

In the last step, divide numerator and denominator by $-x > 0$ and use $\sqrt{x^2} = |x| = -x$. As $x \rightarrow -\infty$, the denominator tends to $1 + 1 = 2$, so the limit is 3 . A discards the contribution of the linear term under the square root before resolving cancellation. C loses the sign coming from $|x| = -x$. D treats the difference of two divergent quantities as though its value were determined without simplification.

Question 14 — Correct answer: C.[Back to question](#)

Divide numerator and denominator by x^2 :

$$f_a(x) = \frac{a - 2/x}{3 + 4/x} \rightarrow \frac{a}{3}.$$

The required limit 2 therefore forces $a = 6$. For this value, subtract 2 before taking the second limit:

$$f_6(x) - 2 = \frac{6x^2 - 2x - 2(3x^2 + 4x)}{3x^2 + 4x} = \frac{-10}{3x + 4}.$$

Thus $x(f_6(x) - 2) = -10x/(3x + 4) \rightarrow -10/3$. The excluded denominator zeros do not affect either limit at $+\infty$. A assigns the desired quotient limit directly to its leading numerator coefficient. B assumes that multiplying a quantity tending to zero by a divergent factor preserves a zero limit. D omits the leading coefficient 3 of the denominator.

Differentiation and convexity

Question 15 — Correct answer: D.

[Back to question](#)

Differentiate twice and factor:

$$f'(x) = 6x^5 - 20x^3, \quad f''(x) = 30x^4 - 60x^2 = 30x^2(x^2 - 2).$$

For $x \neq 0$, the factor $30x^2$ is positive, so the sign of f'' is the sign of $x^2 - 2$. Hence f is convex on $] -\infty, -\sqrt{2}]$ and $[\sqrt{2}, +\infty[$, and concave on $[-\sqrt{2}, \sqrt{2}]$. The second derivative changes sign at both $-\sqrt{2}$ and $\sqrt{2}$, giving exactly those inflection abscissas. At 0, although $f''(0) = 0$, the second derivative is negative on both sides sufficiently close to 0, so convexity does not change. A and B mistake a zero of f'' for a sufficient inflection criterion. C overlooks the sign changes at the two nonzero zeros.

Question 16 — Correct answer: A.

[Back to question](#)

Since f is continuous on the closed bounded interval $[-3, 4]$, it attains an absolute maximum. Its derivative is

$$f'(x) = 3x^2 - 12 = 3(x - 2)(x + 2).$$

The interior critical points are -2 and 2 . For an absolute extremum, compare their values with both endpoint values:

x	-3	-2	2	4
$f(x)$	9	16	-16	16

The largest value is 16, attained at -2 and 4 . The derivative sign also shows that f increases before -2 , decreases between -2 and 2 , then increases after 2 ; there are no other maximizing points. B omits the endpoint that shares the maximum. C lists points with strictly smaller values. D includes 2, which is instead a minimum.

Continuity and solving equations

Question 17 — Correct answer: C.

[Back to question](#)

Set $g(x) = x^3 + 2x - 2$. This polynomial is continuous on $\mathbb{R}$, and $g'(x) = 3x^2 + 2 > 0$ everywhere. It is therefore strictly increasing, so it has at most one real root. At the proposed bracket endpoints,

$$g(3/4) = \frac{27}{64} + \frac{3}{2} - 2 = -\frac{5}{64} < 0, \quad g(4/5) = \frac{64}{125} + \frac{8}{5} - 2 = \frac{14}{125} > 0.$$

The intermediate value theorem gives a root strictly between $3/4$ and $4/5$. Combining existence with strict increase proves uniqueness on the whole real line. A ignores the opposite signs, B contradicts strict monotonicity, and D locates the root in a disjoint interval. Continuity supplies existence here; the derivative argument supplies uniqueness.

Question 18 — Correct answer: D.[Back to question](#)

Continuity at 4 requires the limiting value of the quotient to equal the assigned value c . For $x \neq 4$, rationalize:

$$\frac{\sqrt{x+5}-3}{x-4} = \frac{(x+5)-9}{(x-4)(\sqrt{x+5}+3)} = \frac{1}{\sqrt{x+5}+3}.$$

Cancellation is valid on the punctured domain used to compute the limit. The last denominator tends to $3+3=6$ as $x \rightarrow 4$, so the limit is $1/6$ and c must have exactly that value. A substitutes the zero numerator limit into an unresolved $0/0$ form. B drops one of the two terms in the rationalized denominator. C gives the denominator's limit instead of its reciprocal. The original formula's missing value is removable because the finite limit exists from both sides within the domain.

Exponential and natural logarithm

Question 19 — Correct answer: B.[Back to question](#)

Both logarithm arguments must be strictly positive: $x-2 > 0$ and $x+2 > 0$. Their combined domain is therefore $x > 2$. On this domain the product rule for logarithms gives

$$\ln(x-2) + \ln(x+2) = \ln((x-2)(x+2)) = \ln(x^2-4).$$

Because the logarithm is injective, the equation is equivalent, on the original domain, to $x^2-4=5$. Its algebraic candidates are $x=-3$ and $x=3$, but only 3 satisfies $x > 2$. Direct substitution also gives $\ln 1 + \ln 5 = \ln 5$ there. A retains both squared-equation candidates without checking the logarithm domains. C keeps only the inadmissible negative candidate. D omits the constant term -4 in the product.

Question 20 — Correct answer: A.[Back to question](#)

Set $t = e^x > 0$. The inequality becomes

$$t^2 - 5t + 6 = (t-2)(t-3) \geq 0.$$

The product is nonnegative outside the interval between its two roots, giving $t \leq 2$ or $t \geq 3$. Intersecting with $t > 0$ yields $0 < t \leq 2$ or $t \geq 3$. Since the exponential and logarithm are increasing inverse functions, these conditions become $x \leq \ln 2$ or $x \geq \ln 3$. Both finite endpoints are included because equality is allowed and the original expression vanishes there. B selects the interval where the product is nonpositive. C solves for t but fails to convert back to x . D removes the equality cases even though the inequality is non-strict. Thus A is the complete solution set.

Sine and cosine

Question 21 — Correct answer: C.

[Back to question](#)

The addition formula and $\cos^2 x + \sin^2 x = 1$ give $\cos(2x) = 1 - 2\sin^2 x$. Setting $s = \sin x$ therefore transforms the equation into

$$2s^2 - s - 1 = (2s + 1)(s - 1) = 0.$$

Thus either $\sin x = 1$ or $\sin x = -1/2$. On $[0, 2\pi]$, the first equation has the single solution $\pi/2$, and the second has the two solutions $7\pi/6$ and $11\pi/6$. Neither endpoint solves the original equation, since its left side is 1 and its right side is 0 there. This gives exactly C. Option A reverses the sign of the sine roots; B loses the root $\sin x = 1$; D includes $3\pi/2$, where the equation would require $-1 = 1$.

Question 22 — Correct answer: B.

[Back to question](#)

Put $c = \cos x$. As x ranges over $[0, \pi]$, c ranges over $[-1, 1]$ and takes every value there exactly once. The double-angle formula gives

$$F(x) = 2c - (2c^2 - 1) = \frac{3}{2} - 2\left(c - \frac{1}{2}\right)^2.$$

The square is nonnegative, so the maximum is $3/2$ and requires $c = 1/2$. This value is attainable, at the unique angle $x = \pi/3$ in the stated interval. Hence B is correct. Direct checks give $F(\pi/2) = 1$, refuting A, and $F(0) = 1$, refuting C. The latter value is also below the interior maximum, so D is false. Maximizing the two cosine terms independently would ignore their common argument.

Antiderivatives and differential equations

Question 23 — Correct answer: D.

[Back to question](#)

The numerator is the derivative of $x^2 - 4$, so a logarithmic antiderivative is $\ln|x^2 - 4|$. Throughout $I =]-2, 2[$, $x^2 - 4 < 0$, hence $|x^2 - 4| = 4 - x^2 > 0$. All antiderivatives on this interval have the form $F(x) = \ln(4 - x^2) + C$. Imposing $F(0) = 3$ gives $\ln 4 + C = 3$, so $C = 3 - \ln 4$ and D follows. In particular,

$$F'(x) = \frac{-2x}{4 - x^2} = \frac{2x}{x^2 - 4}.$$

Option A is undefined as a real logarithm on the entire interval. Option B meets the initial value but changes the derivative's sign. Option C has the correct derivative but gives $F(0) = \ln 4 + 3$, so it fails the prescribed normalization.

Question 24 — Correct answer: A.[Back to question](#)

The constant solution is $y = 3$, since $-4 \cdot 3 + 12 = 0$. Set $u = y - 3$; then $u' = -4u$ and $u(1) = y_0 - 3$. The homogeneous solution is $u(x) = (y_0 - 3)e^{-4(x-1)}$, yielding A. Indeed its value at $x = 1$ is y_0 , and

$$y'(x) = -4(y_0 - 3)e^{-4(x-1)} = -4y(x) + 12.$$

The homogeneous solution formula also proves uniqueness after the initial value fixes its multiplicative constant. Option B has the wrong exponential sign; it only works in the equilibrium case $y_0 = 3$. Option C solves $y' = -4y + 48$, with the wrong equilibrium. Option D gives $y(1) = 3 + y_0$. Consequently only A works for every real initial value.

Question 25 — Correct answer: C.[Back to question](#)

The equilibrium is 5. Writing $y(x) = 5 + Ce^{-2x}$ and applying $y(0) = 1$ gives $C = -4$, hence $y(x) = 5 - 4e^{-2x}$. The required inequality is therefore

$$5 - 4e^{-2x} \geq \frac{9}{2} \iff e^{-2x} \leq \frac{1}{8} \iff -2x \leq -\ln 8 \iff x \geq \frac{3 \ln 2}{2}.$$

The direction reverses when dividing by -2 . Equality is allowed, so the threshold time belongs to the interval. Also $y'(x) = 8e^{-2x} > 0$, confirming that the solution stays above the target after first reaching it. Option A reaches only $y = 3$; B reverses the interval direction; D omits the division by 2 and starts the interval too late.

Integral calculus**Question 26 — Correct answer: B.**[Back to question](#)

The curves meet where $x^2 = 2x$, namely at $x = 0$ and $x = 2$. The line is above the parabola on $[0, 2]$, while the parabola is above the line on $[2, 3]$. Geometric area is therefore

$$\begin{aligned} \mathcal{A} &= \int_0^2 (2x - x^2) dx + \int_2^3 (x^2 - 2x) dx \\ &= \left[x^2 - \frac{x^3}{3} \right]_0^2 + \left[\frac{x^3}{3} - x^2 \right]_2^3 = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}. \end{aligned}$$

Thus B is correct. The single signed integral $\int_0^3 (2x - x^2) dx = 0$ explains A: it cancels the two positive areas instead of adding them. Option C counts only one region. Option D is the area under $y = x$ on $[0, 3]$ and does not measure the separation of the two stated curves.

Question 27 — Correct answer: D.[Back to question](#)

Choose $u = x$ and $v' = e^x$ in integration by parts, so $u' = 1$ and $v = e^x$. Then

$$\begin{aligned}\int_0^{\ln 2} x e^x dx &= [x e^x]_0^{\ln 2} - \int_0^{\ln 2} e^x dx \\ &= 2 \ln 2 - (2 - 1) = 2 \ln 2 - 1.\end{aligned}$$

Equivalently, the antiderivative $(x - 1)e^x$ has values $2 \ln 2 - 2$ at the upper endpoint and -1 at the lower endpoint. Their difference gives the same result. Option A omits the remaining integral after integration by parts. Option B omits the lower-endpoint contribution. Option C is half the correct value, introducing an unjustified factor $1/2$ even though the exponential is e^x . The positive answer also agrees with the integrand being positive inside the interval.

Probability

Question 28 — Correct answer: A.[Back to question](#)

Let D denote the event that the component is defective. The two factories form a partition, so total probability gives

$$\mathbb{P}(D) = \frac{2}{5} \frac{1}{20} + \frac{3}{5} \frac{1}{10} = \frac{1}{50} + \frac{3}{50} = \frac{2}{25}.$$

The joint probability of coming from A and being defective is $\mathbb{P}(A \cap D) = (2/5)(1/20) = 1/50$. Conditioning on the observed defect therefore yields

$$\mathbb{P}(A \mid D) = \frac{1/50}{2/25} = \frac{1}{4}.$$

This proves A. Option B uses the original factory share before observing the defect. Option C is the forward conditional $\mathbb{P}(D \mid A)$, which is a different probability. Option D is $\mathbb{P}(B \mid D)$, the complementary posterior. Factory A 's lower defect rate explains why its posterior share is below its original share.

Question 29 — Correct answer: D.[Back to question](#)

Independence and the common success probability imply $X \sim \text{Bin}(6, 1/2)$, with $\mathbb{P}(X = j) = \binom{6}{j}/64$. In particular,

$$\mathbb{P}(X > 4) = \frac{\binom{6}{5} + \binom{6}{6}}{64} = \frac{7}{64}, \quad \mathbb{P}(X > 3) = \frac{15 + 6 + 1}{64} = \frac{22}{64}.$$

The strict tails decrease with their threshold, so the least qualifying strict threshold is $k_{>} = 4$. For an integer-valued variable, $\{X \geq 5\} = \{X > 4\}$, whereas $\{X \geq 4\} = \{X > 3\}$. The least qualifying inclusive threshold is consequently $k_{\geq} = 5$, giving D. Option A swaps the threshold order. B incorrectly includes the mass at 4 in the allowed tail, while C unnecessarily increases the already sufficient strict threshold from 4 to 5.

Question 30 — Correct answer: B.[Back to question](#)

Linearity of expectation gives $\mathbb{E}[M_n] = 3$. Independence gives $\text{Var}(M_n) = 8/n$. For the affine transformation $Z_n = 2M_n - 1$, these become

$$\mathbb{E}[Z_n] = 2 \cdot 3 - 1 = 5, \quad \text{Var}(Z_n) = 2^2 \frac{8}{n} = \frac{32}{n}.$$

Chebyshev's bound applied to M_n gives

$$\mathbb{P}(|M_n - 3| \geq 1/2) \leq \frac{8}{n(1/2)^2} = \frac{32}{n}.$$

This certificate is at most $1/10$ exactly when $n \geq 320$, so the least sufficient integer is 320 and B is correct. Option A multiplies variance by 2 instead of 2^2 . Option C imposes an unnecessary strict inequality on the risk bound. D ignores the transformation and the square of the tolerance. The calculation concerns what the bound certifies; it does not claim that the actual risk equals that upper bound.