

INTEGRATION BEE 2022

Round 1: Qualifiers

1. $\boxed{2}$ $\int \frac{1}{1 - e^{-2022x}} dx$
2. $\boxed{3}$ $\int \frac{\tan^3(1 + \ln x)}{x} dx$
3. $\boxed{2}$ $\int \frac{1}{2 - 2x + x^2} dx$
4. $\boxed{2}$ $\int_{-1}^0 \frac{15165}{2} x \sqrt{1+x} dx$
5. $\boxed{3}$ $\int_0^1 (x - 3^x)^2 dx$
6. $\boxed{3}$ $\int \frac{x^{12}}{x^2 + 1} dx$
7. $\boxed{3}$ $\int \frac{x^2}{(x-1)(x^2+x+1)} dx$
8. $\boxed{2}$ $\int \frac{\sin 2x}{1 + 2 \sin^2 x} dx$
9. $\boxed{4}$ $\int_0^1 (2x-1) \frac{\sqrt{x} + \sqrt{1-x}}{\sqrt{x} - \sqrt{1-x}} dx$
10. $\boxed{3}$ $\int \frac{2 \cos x - 2 \sin x}{2 + \sin 2x} dx$
11. $\boxed{4}$ $\int_0^e x^{\frac{1}{\ln x}} dx$
12. $\boxed{4}$ $\int \sqrt{x \sqrt{x \sqrt{x \dots}}} dx$
13. $\boxed{3}$ $\int_1^2 x^{x^2+1} (2 \ln x + 1) dx$
14. $\boxed{4}$ $\int_0^{2022\pi} \sin(\sin x - x) dx$
15. $\boxed{3}$ $\int_0^1 x \tan^{-1}((x-1)(x+1)) dx$
16. $\boxed{4}$ $\int \frac{1}{\sqrt{x \sqrt{x} - x^2}} dx$
17. $\boxed{7}$ $\int_1^{e^2} (\ln(x^{\ln x - 6}) + 5)x^3 dx$
18. $\boxed{5}$ $\int_0^{\pi/2} \frac{1}{\tan^{\sqrt{2022}}(x) + 1} dx$

19. [6] $\int_0^1 \sqrt{x + \sqrt{x + \sqrt{x + \dots}}} dx$

20. [8] $\int_0^{\frac{\pi}{2}} \prod_{k=1}^7 \cos kx dx$

Round 2

Knockout Match 1

1. [1] $\int \sqrt{12 - 3x^2} dx$

2. [2] $\int \frac{x}{1 - 5x^2} \sqrt{\frac{2}{1 + 5x^2} - 1} dx$

3. [2] $\int \frac{x^{1010}}{1 - x^{2022}} dx$

4. [2] $\int_{-\infty}^0 x^{2022} e^{4x} dx$

5. [3.5] $\int_0^1 2^{\lfloor \log_2 x \rfloor} dx$

Knockout Match 2

1. [1] $\int \frac{8x^3}{1 + x^8} dx$

2. [2] $\int_{-\pi/4}^{\pi/4} \frac{1}{1 - \sin x} dx$

3. [2] $\int_{-\pi/2}^{\pi/2} \frac{\cos x(1 + \tan^{-1} x)}{2 - \cos^2 x} dx$

4. [2] $\int_2^{\log_2 6} \frac{2^{2x+1} - 9 \times 2^x - 6}{4^x - 2^x - 6} dx$

5. [3.5] Define $f(x)$ as the $\lfloor x \rfloor$ th decimal place of x . Find $\int_1^{2022} f(x) dx$.

Knockout Match 3

1. [2] $\int_0^{\infty} \frac{1}{(x + \frac{1}{x})^2} dx$

2. [2.5] $\int_{\frac{1}{2}}^1 \sum_{k=0}^{\infty} (x^k ((k+1) \ln x + x^{k^2+k})) dx$

3. [3.5] $\int_0^{\infty} \frac{1}{(1 + x^{2022})(1 + x^2)} dx$

4. [3.5] $\int_0^{10} \lim_{s \rightarrow x^+} \left(\lim_{t \rightarrow \infty} t^{-\frac{|\sin s| + \sin s}{\sin s}} \right) dx$

5. [4] $\int_0^{\infty} \sum_{k=\lfloor x \rfloor}^{\infty} \frac{2^{-k}}{k+1} dx$

Extra Questions

1. $\boxed{1}$ $\int \frac{\sin^3 x}{\sqrt{\cos x}} dx$

2. $\boxed{1}$ $\int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx$

3. $\boxed{2}$ $\int \ln(1 + \sqrt{x}) dx$

Answers

Round 1

1. $\boxed{2}$ $\frac{1}{2022} \ln |e^{2000x} - 1| + C$
2. $\boxed{3}$ $\frac{1}{2} \tan^2(1 + \ln x) + \ln |\cos(1 + \ln x)| + C$
3. $\boxed{2}$ $\tan^{-1}(x - 1) + C$
4. $\boxed{2}$ -2022
5. $\boxed{3}$ $\frac{1}{3} - \frac{2}{\ln 3} + \frac{4}{(\ln 3)^2}$
6. $\boxed{3}$ $\frac{x^{11}}{11} - \frac{x^9}{9} + \frac{x^7}{7} - \frac{x^5}{5} + \frac{x^3}{3} - x + \tan^{-1} x + C$
7. $\boxed{3}$ $\frac{1}{3} \ln(x^3 - 1) + C$
8. $\boxed{2}$ $\frac{1}{2} \ln(2 - \cos 2x) + C$
9. $\boxed{4}$ $1 + \frac{\pi}{4}$
10. $\boxed{3}$ $2 \tan^{-1}(\sin x + \cos x) + C$
11. $\boxed{4}$ e^2
12. $\boxed{4}$ $\frac{x^2}{2} + C$
13. $\boxed{3}$ 15
14. $\boxed{4}$ 0
15. $\boxed{3}$ $-\frac{\pi}{8} + \frac{1}{4} \ln 2$
16. $\boxed{4}$ $4 \sin^{-1}(x^{1/4}) + C$
17. $\boxed{7}$ $-\frac{53}{32} - \frac{19}{32} e^8$
18. $\boxed{5}$ $\frac{\pi}{4}$
19. $\boxed{6}$ $\frac{5}{12}(1 + \sqrt{5})$
20. $\boxed{8}$ $\frac{\pi}{32}$

Knockout Match 1

1. $\boxed{1}$ $\frac{\sqrt{3}}{2}(x\sqrt{4-x^2} + 4 \sin^{-1}(\frac{x}{2})) + C$
2. $\boxed{2}$ $\frac{1}{10} \sin^{-1}(5x^2) + C$
3. $\boxed{2}$ $\frac{1}{2022} \ln \left| \frac{1+x^{1011}}{1-x^{1011}} \right| + C$
4. $\boxed{2}$ $\frac{2022!}{2^{4046}}$
5. $\boxed{3.5}$ $\frac{1}{3}$

Knockout Match 2

1. $\boxed{1}$ $2 \tan^{-1}(x^4) + C$
2. $\boxed{2}$ 2
3. $\boxed{2}$ $\frac{\pi}{2}$
4. $\boxed{2}$ $3 - 2 \log_2 3$
5. $\boxed{3.5}$ 9094.5

Finals

1. $\boxed{2}$ $\frac{\pi}{4}$
2. $\boxed{2.5}$ $3 \ln 2$
3. $\boxed{3.5}$ $\frac{\pi}{4}$
4. $\boxed{3.5}$ $10 - 2\pi$
5. $\boxed{4}$ 2

Extra Questions

1. $\boxed{1}$ $\frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + C$
2. $\boxed{1}$ $\frac{\pi^3}{24}$
3. $\boxed{2}$ $(x - 1) \ln(1 + \sqrt{x}) + \sqrt{x} - \frac{x}{2} + C$