MATH1131/41 Algebra
Lab Test 2 Sample Solutions

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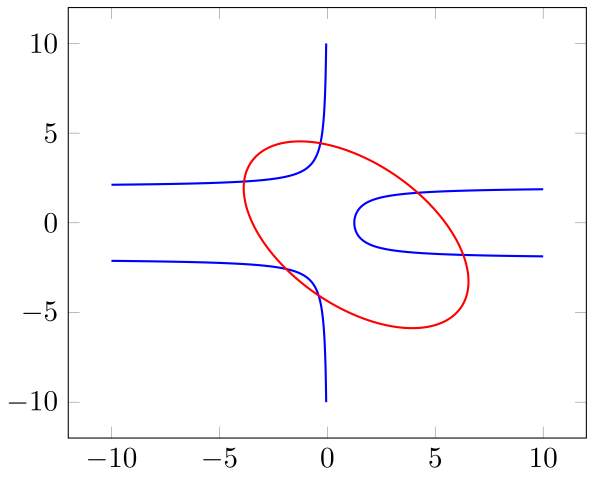
Question 1

Plot the two curves defined by $4x - xy^2 = 5$ *and* $(x-1)^2 + y^2 + xy = 20$ *and find the number of intersections in the square* $-10 \leq x \leq 10$, $-10 \leq y \leq 10$.

Solution: The answer is 6.
In Maple, activate
`with(plots)`
then assign the variables
`f1 := 4*x - x*y^2 - 5`
`f2 := (x-1)^2 + y^2 + x*y -20`
Then use the following command to plot the two curves,
`implicitplot([f1,f2], x=-10..10, y=-10..10)`
Count the number of times the curves intersect, which is the answer.



Question 2

Find the moduli of the five roots of the polynomial $p(z) = z^5 + z^4 + 5z - 4$ *and enter a decimal approximation to the largest modulus. Your approximation should be correct to 10 significant figures.*

Solution: The answer is 1.81834680794244.
Enter the following sequence of commands in Maple:

```
h := z^5 + z^4 + 5*z - 4
h1 := solve(h=0)
max(map(abs, [h1]))
evalf (%)
```


Question 3

Express $\sin^4 t$ *in terms of cosines and sines of multiples of t:*

$$\sin^4 t = a\cos 4t + b\cos 2t + c.$$

Solution: The answer is $\mathbf{a = \frac{1}{8}, b = -\frac{1}{2}, c = \frac{3}{8}}$**.**

Note that $\sin^2 t = \dfrac{1-\cos 2t}{2}$ and $\cos^2 t = \dfrac{1+\cos 2t}{2}$. So

$$\begin{aligned}
\sin^4 t &= (\sin^2 t)^2 \\
&= \left(\frac{1-\cos 2t}{2}\right)^2 \\
&= \frac{1}{4}(1 - 2\cos 2t + \cos^2 2t) \\
&= \frac{1}{4}\left(1 - 2\cos 2t + \frac{1+\cos 4t}{2}\right) \\
&= \frac{1}{8}\cos 4t - \frac{1}{2}\cos 2t + \frac{3}{8}.
\end{aligned}$$

Alternatively, we can use the formula $\sin t = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ to find $\sin^4 t$ by taking the 4th power of this formula and simplifying.

Question 4

Express $\cos(4t)$ *in terms of powers of* $\cos t$*:*

$$\cos(4t) = a\cos^4 t + b\cos^2 t + c.$$

Solution: The answer is $\mathbf{a = 8, b = -8, c = 1}$.

By De Moivre's theorem,

$$\begin{aligned}
\cos 4t + i\sin 4t &= (\cos t + i\sin t)^4 \\
&= \binom{4}{0}\cos^4 t + \binom{4}{1}\cos^3 t(i\sin t) + \binom{4}{2}\cos^2 t(i\sin t)^2 + \binom{4}{3}\cos t(i\sin t)^3 + \binom{4}{4}(i\sin t)^4 \\
&= \cos^4 t - 6\cos^2 t\sin^2 t + \sin^4 t + i(4\cos^3 t\sin t - \cos t\sin^3 t).
\end{aligned}$$

By equating the real parts of the above equation,

$$\begin{aligned}
\cos 4t &= \cos^4 t - 6\cos^2 t\sin^2 t + \sin^4 t \\
&= \cos^4 t - 6\cos^2 t(1-\cos^2 t) + (1-\cos^2 t)^2 \\
&= 8\cos^4 t - 8\cos^2 t + 1.
\end{aligned}$$

Question 5

Find the principal argument of the 5 roots of the polynomial

$$p(z) = z^5 - z^4 + z^2 - 2$$

and enter a decimal approximation of the largest principle argument. Your approximation should be correct to 10 significant figures.

Solution: The answer is 2.556918833.

In Maple, enter the following commands:

```
g := z^5 - z^4 + z^2 - 2
fsolve(g,z,complex).
```

This will output 5 different solutions, separated by commas. Copy and paste each solution into the `argument()` function to find which argument is largest.

Question 6

Let z and w be complex numbers with the following properties:

$$|z| = 2, |w| = 5, \text{Arg}(z) = -\frac{2\pi}{3}, \text{Arg}(w) = -\frac{3\pi}{4}.$$

Find the following quantities: $|zw|$, $\frac{|z|}{|w|}$, $\text{Arg}(zw)$, $\text{Arg}\left(\frac{z}{w}\right)$, $\text{Arg}(z^2w^5)$.

Solution: The answer is 10, $\mathbf{\frac{2}{5}}$, $\mathbf{\frac{7\pi}{12}}$, $\mathbf{\frac{\pi}{12}}$, $\mathbf{\frac{11\pi}{12}}$.

To find $|zw|$, $|z| \times |w| = 10$.
To find $\frac{|z|}{|w|}$, divide $|z|$ and $|w|$, which equals $\frac{2}{5}$.
To find $\text{Arg}(zw)$, add $\text{Arg}(z)$ and $\text{Arg}(w)$ to get -$\frac{17\pi}{12}$. Since $\text{Arg}(zw)$ must be between $-\pi$ and π, we can add multiples of 2π to our solution to get $\frac{7\pi}{12}$.
To find $\text{Arg}\left(\frac{z}{w}\right)$, subtract $\text{Arg}(z)$ and $\text{Arg}(w)$ and add multiples of 2π to get $\frac{\pi}{12}$.
To find $\text{Arg}(z^2w^5)$, find $2\times$ $\text{Arg}(z)$ + $5\times$ $\text{Arg}(w)$ + $2k\pi$ to get $\frac{11\pi}{12}$.

Question 7

Let $p(x)$ be a real polynomial of degree 8. You are given that $p(-6+4i)=0$. Find a real quadratic factor of $p(x)$.

Solution: The answer is $\mathbf{x^2+12x+52}$.

By the complex conjugate root theorem, if $-6+4i$ is a root then $-6-4i$ is also a root.
Therefore $(x-(-6+4i))$ and $(x-(-6-4i))$ are factors of $p(x)$.
Multiplying these,

$$\begin{aligned}(x-(-6+4i))(x-(-6-4i)) &= x^2+6x+4xi+6x-4xi+36+24i-24i-16i^2\\ &= x^2+12x+52.\end{aligned}$$