ACCELERATE QUESTIONS

Question 1

Find the next value in the sequence:

$$11011, \ 10010, \ 11101, \ 11100, \ 01001, \ 10101, \ \ldots$$

Question 2

Find the greatest integer that is the least common multiple of two distinct sets of four non-zero digits (i.e. $1, 2, \ldots, 9$).

Question 3

An odd polynomial $\mathcal{P}$ with degree $2n+1$ has the property that $\mathcal{P}(x) = x$ for $x = 1, 2, \ldots, n$. If $\mathcal{P}(n+1) = 1$, what is $\mathcal{P}(n+2)$?

Question 4

Let M be a subset of $\{1, 2, 3, ..., 15\}$ such that the product of any three distinct elements of M is not a perfect square. Determine the maximum number of elements in M.

Question 5

What is the radius of the largest sphere that will fit inside a hollow square pyramid, all of whose edges are of length 2?

Question 6

In a time when we could do so, my partner and I participated in a badminton tournament with six other couples. A number of singles matches took place such that no one played their partner and no one played anyone more than once. When I asked everyone how many people they had played, each person gave a different answer. How many opponents did my partner play?

Question 7

Solve for x where $1 + 4x + 9x^2 + 16x^3 + ... = 12$.

Question 8

The sum of a collection of negative integers is -15. What is the maximum possible product of this collection?

Question 9

Gorden, John and Raymond are each given one of three consecutive positive integers. They each know their own number and that the three numbers are consecutive, but do not know the numbers given to the others. The following sequence of true statements is made, in order:

Gorden says: "I do not know all three numbers."
John says: "I do not know all three numbers."
Raymond says: "I do not know all three numbers."
Gorden says: "I do not know all three numbers."
John says: "I now know all three numbers."
Raymond says: "I do not know all three numbers."

What number is Raymond given?

Question 10

A positive integer N has the property that multiplying it by a prime number less than 10 always increases its number of factors by 50%. Multiplying it by any other prime number doubles its number of factors. Find N.

Question 11

What is largest possible area of a rectangle which has side ratio $3 : 1$ and can be contained inside a unit square?

Question 12

Grace makes a jug of fruit drink which contains a mixture of water and pulp. After leaving the jug in the sun for an hour, she found that the mixture was $\frac{2}{3}$ water. After another hour, the mixture was $\frac{3}{5}$ water. Assuming that the same amount of water was lost during each hour, and that the amount of pulp did not change, what was the original proportion of water in the jug?

Question 13

When calculating the product of two positive integers, Philton accidentally reduces the ten's digit in the result by 7. When he divides his answer by the smaller of the integers, he obtains the quotient 48 and the remainder 17. Find the two integers.

Question 14

Find the next number in the sequence:

$$1, \ 1, \ 16, \ 81, \ 625, \ 4096, \ \ldots$$

Question 15

Suppose $f(x) = x^2 - 4$. Determine $f(f(f(\sqrt{11})))$.

Solutions

1. 01111

2. 840

3. $2 - n - 2n^2$

4. 10

5. $\frac{\sqrt{6}-\sqrt{2}}{2}$ $\left(\text{alternatively } \frac{2}{\sqrt{6}+\sqrt{2}}, \sqrt{\frac{3}{2}} - \sqrt{\frac{1}{2}} \right)$

6. 6

7. $\dfrac{1}{2}$

8. 216

9. 4

10. 210

11. $\frac{3}{8}$

12. $\frac{5}{7}$

13. 29 and 51

14. 28561 (or 13^4)

15. 2021

Bonus Round

A knockout tournament consists of 2^{20} players who are seeded*. Any higher seed has a probability of 0.9 of winning against a lower seed. Find the probability to four decimal places that the final is between the 3rd and 4th seeds.

*1st and 4th start start in the same half of the draw and can only meet in the semifinals (penultimate round). Similarly, 2nd and 3rd start in the other half.

Bonus Round Solution

0.0137