Integration Bee 2023

UNSW Mathematics Society

Questions

Round 1

1. [2] $\int 2x \sec x + x^2 \sec x \tan x \ dx$
2. [2] $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$
3. [3] $\int \frac{2x+3}{x^2-9} dx$
4. [2] $\int_1^3 \frac{2x^2-x+1}{x^2-4} dx$
5. [5] $\int e^{\sin^{-1} x} dx$
6. [4] $\int_{-1}^0 \frac{x^5+5x^4+10x^3+10x^2+5x+1}{x^4-4x^3+6x^2-4x+1} dx$
7. [4] $\int \sin x \sin(\cos x) \sin(\cos(\cos x)) dx$
8. [3] $\int x^{\frac{x}{2023\ln(x)}} dx$
9. [4] $\int \frac{e^{2023x}-1}{e^{2023x}+1} dx$
10. [4] $\int \frac{3\sin x + 5\cos x}{8\sin x - 13\cos x} dx$
11. [5] $\int \frac{dx}{\sqrt{1-x^2}\sqrt{1-\arcsin^2 x}\sqrt{1-\arccos^2(\arcsin x)}}$
12. [4.5] $\int \frac{\tan^{-1} x}{x^2} dx$
13. [4] $\int x^2 \sin^2 x \ dx$
14. [5] $\int \frac{dx}{2+\tan x}$
15. [2] $\int_{1914}^{2023} x \ dx$
16. [1.5] $\int_0^{\pi} \lceil \lfloor \sin x \rfloor \rceil \ dx$
17. [3.5] $\int \frac{dx}{x\sqrt{x^{2023}-1}}$

18. [3] $\int_0^{2\pi} \cos^{2023}(x)dx$

19. [5] $\int \frac{dx}{\sin^2(x)+2023}$

20. [5] $\int e^{x^{\frac{1}{8}}}dx$

Round 2

1. [3] $\int \sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}\ dx$

2. [4] $\int \frac{dx}{x+x^e}$

3. [3] $\int \frac{dx}{x^2-4x+1}$

4. [4] $\int \sqrt{1+\sin 2x}\ dx$

5. [5.5] $\int_{-1}^{1} \sin^2 x \cos^{-1} x\ dx$

6. [4] $\int \frac{x^2-1}{x^4+1}dx$

7. [3] $\int \frac{x^2+3x+1}{(x-1)(x-2)^2(x+1)}dx$

8. [5] $\int \frac{dx}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$

9. [4] $\int \frac{dx}{x+x^5}$

10. [4.5] $\int \frac{6}{\sin x+\sin 2x}dx$

11. [4.5] $\int xe^x(\sin x+\cos x)dx$

12. [2] $\int_0^{\frac{\pi}{2}} \sqrt{1+\sec x}\ dx$

13. [3] $\int \frac{1+e^{2x}(1-2x)}{1+2e^{2x}+e^{4x}}dx$

14. [4] $\int_0^{\infty} \frac{(1+x)^{2023}}{(2+x)^{2025}}dx$

15. [4.5] $\int_0^1 \sqrt[20]{1-x^{23}} - \sqrt[23]{1-x^{20}}dx$

16. [3] $\int_0^{2023} \lfloor x \rfloor \lceil x \rceil dx$

17. [1] $\displaystyle\int_0^1 \lfloor x \rfloor^{\lceil x \rceil} dx$

18. [4] $\displaystyle\int \frac{x^2}{\sqrt{x^2 - 24\pi e}} dx$

19. [3.5] $\displaystyle\int \frac{\sin x}{\sin x - \cos x} dx$

20. [5] $\displaystyle\int \frac{1}{\sqrt[3]{1 - x^3}} dx$

Round 3

Semi-Finals 1

1. [1] $\displaystyle\int \frac{dx}{\sin^3 x \cos^5 x}$

2. [2] $\displaystyle\int \ln(1 - \sqrt[3]{x}) dx$

3. [3.5] $\displaystyle\int x\sqrt{x\sqrt[3]{x\sqrt[4]{x\sqrt[5]{x\cdots}}}}\, dx$

4. [3.5] $\displaystyle\int_0^{\pi/2} \ln(\sin x) dx$

Semi-Finals 2

1. [1] $\displaystyle\int \frac{5x^2 - 83x + 459}{(x-7)(x^2 - 22x + 146)} dx$

2. [2] $\displaystyle\int \frac{1}{\sin^2(x) + \cos(x) + 1} dx$

3. [3.5] $\displaystyle\int \ln\left(\sqrt{x} + \sqrt{x+1}\right) dx$

4. [3.5] $\displaystyle\sum_{x=1}^{\infty} \int \operatorname{arccot}\left(\frac{1 - x^2 + x^4}{2x}\right) dx$

Grand Final

1. [1] $\displaystyle\int \frac{dx}{(1-x)^2\sqrt{1-x^2}}$

2. [2] $\displaystyle\int_0^1 x^{1/4}(1-x)^{3/4} dx$

3. [3.5] Let $t(x)$ be the 2nd decimal place of x in its decimal expansion. Let $\overline{F}_x$ be the $t(x)$th Fibonacci number, where $F_0 = F_1 = 1$. Find $\int_0^5 \frac{\overline{F}_{x-2}}{\overline{F}_x \overline{F}_{x-1}} dx$.

4. [3.5] $\displaystyle\int_0^1 e^{-\frac{1}{2}\lceil \frac{1}{x} \rceil} \prod_{k=1}^{\lceil \frac{1}{x} \rceil} e^{kx} dx$

Answers

Round 1

1. [2] $x^2 \sec x + C$
2. [2] $\dfrac{2\pi}{\sqrt{3}}$
3. [3] $\dfrac{1}{2}\ln|x+3| + \dfrac{3}{2}\ln|x-3| + C$
4. [2] Undefined
5. [5] $\dfrac{\sqrt{1-x^2}+x}{2}e^{\sin^{-1}x} + C$, $\dfrac{x-\cos(\sin^{-1}x)}{2}e^{\sin^{-1}x} + C$
6. [4] $\dfrac{167}{6} - 40\ln(2)$
7. [4] $-\cos(\cos(\cos x)) + C$
8. [3] $2023x^{\frac{x}{2023\ln(x)}} + C$
9. [4] $\dfrac{2}{2023}\ln(e^{2023x}+1) - x + C$
10. [4] $-\dfrac{41}{233}x + \dfrac{79}{233}\ln|8\sin x - 13\cos x| + C$
11. [5] $\arccos(\arccos(\arcsin(x))) + C$
12. [4.5] $-\dfrac{\tan^{-1}x}{x} + \ln|x| - \dfrac{1}{2}\ln|1+x^2| + C$
13. [4] $\dfrac{x^3}{6} - \dfrac{x^2\sin 2x}{4} - \dfrac{x\cos 2x}{4} + \dfrac{\sin 2x}{8} + C$
14. [5] $\dfrac{1}{10}\ln|t^2-t-1| - \dfrac{1}{10}\ln(t^2+1) + \dfrac{2}{5}\tan^{-1}t + C$, where $t = \tan x$
15. [2] $\dfrac{429133}{2}$
16. [1.5] 0
17. [3.5] $\dfrac{2}{2023}\tan^{-1}(\sqrt{x^{2023}-1}) + C$
18. [3] 0
19. [5] $\dfrac{1}{\sqrt{4094552}}\tan^{-1}\left(\sqrt{\dfrac{2024}{2023}}\tan x\right) + C$
20. [5] $8e^u(u^3 - 3u^2 + 6u - 6) + C$, where $u = x^{\frac{1}{8}}$

Round 2

1. [3] $\dfrac{5x}{12} + C$
2. [4] $\dfrac{\ln|1 + x^{1-e}|}{1-e} + C$
3. [3] $\dfrac{1}{2\sqrt{3}}\big(\ln|x - \sqrt{3} - 2| - \ln|x + \sqrt{3} - 2|\big) + C$
4. [4] $\sqrt{\sin 2x + 1}\frac{\sin x - \cos x}{\sin x + \cos x} + C$
5. [5.5] $\dfrac{\pi}{2} - \dfrac{\pi}{4}\sin 2$
6. [4] $\dfrac{1}{2\sqrt{2}}\ln\left|\dfrac{x^2 - \sqrt{2}x + 1}{x^2 + \sqrt{2}x + 1}\right| + C$
7. [3] $\dfrac{5\ln|x-1|}{2} + \dfrac{23\ln|x-2|}{9} - \dfrac{11}{2(x-2)} + \dfrac{\ln|x+1|}{18} + C$
8. [5] $2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln\left|x^{\frac{1}{6}} + 1\right| + C$
9. [4] $\ln|x| - \dfrac{1}{4}\ln|1 + x^4| + C$
10. [4.5] $2\ln\left(\sin\dfrac{x}{2}\right) + 6\ln\left(\cos\dfrac{x}{2}\right) - 4\ln(2\cos x + 1) + C$
11. [4.5] $\dfrac{1}{2}e^x((2x-1)\sin x + \cos x) + C$
12. [2] π
13. [3] $\dfrac{x}{1 + e^{2x}} + C$
14. [4] $\sum_{i=0}^{2023}\binom{2023}{i}\frac{1}{2^{2025-i}(2025-i)}$
15. [4.5] 0
16. [3] 2759728048
17. [1] 0
18. [4] $\frac{1}{2}\sqrt{x^2 - 24e}x + 12e\log\left(\frac{x - \sqrt{x^2-24e}}{x + \sqrt{x^2-24e}}\right) + C$
19. [3.5] $\dfrac{x + \ln|\sin x - \cos x|}{2} + C$
20. [5] $\frac{1}{3}\ln\left(\frac{x}{\sqrt[3]{1-x^3}} + 1\right) + \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}\frac{x}{\sqrt[3]{1-x^3}} - \frac{1}{\sqrt{3}}\right) - \frac{1}{6}\log\left(-\frac{x}{\sqrt[3]{1-x^3}} + \frac{x^2}{(1-x^2)^{\frac{2}{3}}} + 1\right)$.

Round 3

Semi-Finals 1

1. $\boxed{1}$ $\dfrac{1}{4}\tan^4 x + \dfrac{3}{2}\tan^2 x + 3\ln|\tan x| - \dfrac{1}{2}\cot^2 x + C$
2. $\boxed{2}$ $-\frac{1}{2}x^{\frac{2}{3}} - \frac{1}{3}x - x^{\frac{1}{3}} + (x-1)\ln(1-\sqrt[3]{(}x))$
3. $\boxed{3.5}$ $\dfrac{x^e}{e} + C$
4. $\boxed{3.5}$ $\frac{\pi}{2}\ln\left(\frac{1}{2}\right)$

Semi-Finals 2

1. $\boxed{1}$ $\log\left(x^2 - 22x + 146\right) + 3\log\left(x-7\right) + \dfrac{19}{5}\tan^{-1}\left(\dfrac{x-11}{5}\right) + C$
2. $\boxed{2}$ $\frac{2\sqrt{3}}{9}\tan^{-1}\left(\sqrt{3}\tan(\frac{x}{2}) + \frac{1}{3}\tan(\frac{x}{2})\right)$
3. $\boxed{3.5}$ $-\frac{1}{2}\sqrt{x}\sqrt{x+1} + x\log\left(\sqrt{x}+\sqrt{x+1}\right) + \ln(\sqrt{x}+\sqrt{x+1}) + C$
4. $\boxed{3.5}$ $\frac{\pi}{2}x$

Grand Final

1. $\boxed{1}$ $-\dfrac{x-2}{4(x-1)^2} + \dfrac{1}{8}\log\left(\dfrac{x+1}{x-1}\right) + C$
2. $\boxed{2}$ $\dfrac{3\pi}{16\sqrt{2}}$
3. $\boxed{3.5}$ $\dfrac{44}{89}$
4. $\boxed{3.5}$ $1 - e^{-\frac{1}{2}}$